Appendix D: Course Placement and Sequences

of the

Mathematics Framework

for California Public Schools: Kindergarten Through Grade Twelve

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Appendix D

Course Placement and Sequences

Increased Rigor of Grade Eight and Algebra I/Mathematics I Standards

Success in Algebra I or Mathematics I is crucial to students' overall academic success, their continued interest and engagement in mathematics, and the likelihood of their meeting California's a–g requirements. The California Common Core State Standards for Mathematics (CA CCSSM) represent a tight progression of skills and knowledge that is inherently rigorous and designed to provide a strong foundation for success in the new, more advanced Algebra I and Mathematics I courses that are typically taken by most students in grade nine.

Development of these skills and knowledge depends on students being placed in appropriate courses, with emphasis on foundational concepts at the appropriate time, throughout their K–8 sequence and beyond. With the help of diagnostic information that is based upon rich common assessments, placement decisions should be reviewed by a team of stakeholders that includes teachers and instructional leadership (Massachusetts Department of Elementary and Secondary Education [MDESE] 2012).

Unfortunately, misplacement of students is common, with negative consequences for students who are unable to keep pace with the incremental difficulty of mathematics content; students' weaknesses in key foundational areas that support algebra readiness frequently translate into substantial difficulty reaching proficiency in higher-level mathematics while in high school (Finkelstein et al. 2012). At the same time, students need to be appropriately challenged and engaged in order to maintain their interest and skill development in mathematics throughout high school and beyond. Some students will take college-level courses (e.g., Advanced Placement Calculus, Statistics, or International Baccalaureate) as high school seniors, and the course sequences of the earlier grade levels need to support this level of course-taking. Therefore, one particular placement consideration, discussed later in this appendix, examines when and under what conditions to accelerate students in their mathematics sequence to reach the advanced courses while in high school.

Course Sequencing Challenges Involving the Transition to the CA CCSSM

Implementation of the CA CCSSM comes with many transitions over the next several years—new instructional approaches, new instructional materials, professional support for teachers, and technology readiness, among others. Furthermore, the transition from existing course sequences to new course sequences will inevitably provide challenges at both the school-district and school-site levels. Although the fundamental design of new courses presents its own immediate challenges, so too does the linkage between courses to ensure vertical articulation between grade levels and even between school systems. For example, some K–8 school districts feed into high school–only districts. In the particular case of mathematics, there is a "vocabulary" around the names of mathematics courses that is likely to cause confusion not only for educators, but also for parents. Prior to the development of the CA CCSSM, "Algebra I" was taught in grade eight to an increasing number of students. That same course name will be the default for grade nine, as most students who move forward will complete the CA CCSSM for grade eight—and the new version of Algebra I is more rigorous and more demanding than previous versions of Algebra I. Even so, the changes are expected to cause confusion. The most practical solution is to describe the course content, in addition to giving course names, as a way to eliminate confusion until "Algebra I," as commonly used, now refers to a ninth-grade and not an eighth-grade course.

Research on Course Placement and Mathematics

The mathematics courses that students take greatly affect student achievement. The research studies briefly described below provide some additional context for the tradeoffs that are inherent in deciding how best to organize CA CCSSM course sequences and place students accordingly. "Algebra I" refers to courses that were in place under the 1997 California mathematics standards, prior to the adoption of the CA CCSSM. A big difference is that the CA CCSSM have rigorous grade-eight standards, but the California standards adopted in 1997 did not have specific standards for grade eight. Over the past decade, there has been a dramatic increase in the number and proportion of eighth-grade students enrolled in Algebra I in California.[1] Williams et al. (2011) reported that, between 2003 and 2009, the percentage of grade-eight students taking Algebra I increased from 32 percent to 54 percent. Although the increase in grade-eight enrollment in Algebra I resulted in greater percentages of grade-eight students achieving either "Proficient" or "Advanced" on the Algebra I California Standards Test, it also led to larger numbers of grade-eight students achieving "Far Below Basic" or "Below Basic" on the test (Williams et al. 2011). Williams et al. (2011) concluded that the practice of placing all eighth-graders in Algebra I, regardless of their preparation, sets up many students to fail. Kurlaender, Reardon, and Jackson (2008) looked at students in San Francisco, Fresno, and Long Beach and found that students' grade point average in grade seven and course failures in grade eight were predictive of the students' high school completion. These authors also found that the timing of when students take algebra is a strong predictor of students' high school success. In two of the three districts that they analyzed, students who completed algebra by grade eight were 30 percent more likely to graduate from high school than students who had not completed algebra by grade eight.

As expected and known for some time, course work in middle school relates closely to course work in high school. Findings from 20 years ago show that course-taking patterns in middle school are highly predictive of course-taking patterns in high school. Oakes, Gamoran, and Page (1992) stated that the courses students take in junior high school are "scholastically consequential, as the choice predicts later placement in high track classes in senior high school" (Oakes, Gamoran, and Page 1992, 574). More recently, Wang and Goldschmidt (2003) concluded that middle school mathematics achievement is significantly related to high school mathematics achievement, and "mathematics preparedness is vitally important when one enters high school—where courses begin to 'count' and significantly affect postsecondary opportunities" (Wang and Goldschmidt 2003, 15). In a study examining the National Education Longitudinal Study, Stevenson et al. (1994) found that the level of mathematics that students take in eighth grade is closely related to what they take in high school. They conclude that "students who are in an accelerated mathematics sequence beginning in eighth grade are likely to maintain that position in high school" (Stevenson et al. 1994, 196).

1. This increase was not confined to California. Similar increases in grade-eight Algebra I enrollment have occurred across the country (Walston and McCarroll 2010; Stein et al. 2011).

However, many students who finish middle school are not actually prepared to succeed in a rigorous sequence of college-preparatory mathematics courses in high school (Balfanz, McPartland, and Shaw 2002). Therefore, it is not surprising that previous research found that in the high school grades, ninth grade is a key year for students in terms of future academic success. Choi and Shin (2004) examined student transcripts from a large, urban school district in California. The authors found that most students fall off track for college eligibility in grade nine. Similarly, Finkelstein and Fong (2008) found that more than 40 percent of the students did not meet the California State University requirement of completing two semesters of college-preparatory mathematics in the ninth grade. They concluded that students who fall off the college-preparatory track early in high school tend to fall farther behind and are less likely to complete a college-preparatory program as they progress through high school. Neild, Stoner-Eby, and Furstenberg (2008) further conclude that the experience of the ninth-grade year contributes substantially to the probability of dropping out of high school, even after controlling for eighth-grade academic performance and pre–high school attitudes and ambitions.

The grade-eight standards in the CA CCSSM are significantly more rigorous than the Algebra I course that many students took in eighth grade. The CA CCSSM for grade eight address the foundations of algebra by including content that was previously part of the Algebra I course—such as more in-depth study of linear relationships and equations, a more formal treatment of functions, and the exploration of irrational numbers. For example, by the end of the CA CCSSM for grade eight, students will have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. The CA CCSSM for grade eight also include geometry standards that relate graphing to algebra in a way that was not explored previously. Additionally, the statistics presented in the CA CCSSM for grade eight are more sophisticated than those previously included in middle school and connect linear relations with the representation of bivariate data.

The new Algebra I and Mathematics I courses build on the CA CCSSM for grade eight and are therefore more advanced than the previous courses. Because many of the topics included in the former Algebra I course are in the CA CCSSM for grade eight, the new Algebra I and Mathematics I courses typically start in ninth grade with more advanced topics and include more in-depth work with linear functions and exponential functions and relationships, and they go beyond the previous high school standards for statistics. Mathematics I builds directly on the continuation of the CA CCSSM for grade eight and provides a seamless transition of content through an integrated curriculum.

Because of the rigor that has been added to the CA CCSSM for grade eight, course sequencing needs to be recalibrated to ensure students are able to master the additional content. Specifically, today's students, who are similar to those who previously may have been able to master an Algebra I course in grade eight, may find the new CA CCSSM for grade-eight content significantly more difficult. This transition to the CA CCSSM provides an opportunity to strengthen conceptual understanding by encouraging students—even strong mathematics students—to take the grade-eight CA CCSSM course instead of skipping ahead to Algebra I in grade eight.

Recalibrating the course placement process will require school-district personnel, including teachers, counselors, and instructional specialists, to rethink the information they use for assigning students to courses, particularly in middle school mathematics. Many variations may exist in the mathematics sequence from grade six to grade eight. As the CA CCSSM are implemented during the next several years, steps need to be taken at the school-district and school-site levels to ensure that the sequence of courses guides students to CA CCSSM mastery by the end of grade eight.

Mathematics Course Design and Placement Under the CA CCSSM

Designing CA CCSSM–aligned mathematics courses in middle school requires careful planning to ensure that all content and practice standards are fully addressed. Some students may move through the standards more quickly than other students. Getting the pacing right will require implementation of new courses and analysis of students' progress. As noted previously, placing students in a course pathway for which they are not adequately prepared can have negative consequences. A recent longitudinal analysis based on California statewide assessment data revealed that students who fail the state exam for algebra in grade eight have a greater chance of repeating the course and failing the exam again in grade nine compared with their peers who pass the state exam for general mathematics in grade eight (Liang, Heckman, and Abedi 2012). Similarly, Finkelstein et al. (2012) reported that as many as 33 percent of students in a representative sample of California repeated algebra between grades seven and twelve (most often from grade eight to grade nine), and most of those students did not improve their demonstrated mastery following the repeated course. In essence, under standards that were adopted prior to the more rigorous CA CCSSM, California's eighth-graders who were underprepared for algebra were still underprepared in ninth grade.

In light of these findings, school systems across the nation and in California are revising the criteria used to determine mathematics placement and the different weights assigned to each criterion. Most districts typically rely on teacher recommendations and course grades to determine course placement (Bitter and O'Day 2010, 6), with standardized mathematics test scores, student or parent preferences, and counselor recommendations considered as additional factors in the decision (Hallinan 2003). As Hallinan (1994) notes, "[s]chools vary in the constellation of factors on which they rely to assign students to tracks and in the weight they attach to each factor" (Hallinan 1994, 80). Similarly, Oakes, Muir, and Joseph (2000) note, "Increasingly, school systems do not use fixed criteria to assign students to particular course levels" (Oakes, Muir, and Joseph 2000, 16). Rather, teacher and counselor placement recommendations are used; these include subjective judgments about "students' personalities, behavior, and motivation" as well as test-score performance (Oakes, Muir, and Joseph 2000, 16).

Research has also shown discrepancies in the placement of students in "advanced" classes by race, ethnicity, or socioeconomic background. Although the decision to accelerate is almost always a joint decision between the school and the family, serious efforts must be made to consider solid, objective evidence of student learning in order to avoid unwittingly depriving particular groups of students of opportunities. Among the considerations is the need to assess near-term mathematics readiness with the student's longer-term prospects for mastering advanced mathematics content. The consideration for school districts is: When, and under what circumstances, will placing students in the grade-eight CA CCSSM course transfer to greater mathematics understanding throughout high school?

In developing a policy on course sequences and student placement at the district level, districts may also turn to guidance from other education agencies. For example, as described in Appendix A of the national Common Core State Standards for Mathematics document (National Governors Association

Center for Best Practices, Council of Chief State School Officers [NGA/CCSSO 2010a, 81]), the Achieve Pathways Group developed guidelines on how placement decisions and course sequences should be evaluated:

1. **Compacted courses should include the same Common Core State Standards as the non-compacted courses.** "Learning the mathematics prescribed by CA CCSSM requires that all students, including those most accomplished in mathematics, rise to the challenge by spending the time to learn each topic with diligence and dedication. Skimming over existing materials in order to rush ahead to more advanced topics will no longer be considered good practice" (Wu 2012). When accelerated pathways are considered, it is recommended that three years of material be compacted into two years, rather than compacting two years into one. The rationale is that mathematical concepts are likely to be omitted when two years of material are squeezed into one. This practice is to be avoided, as the standards have been carefully developed to define clear learning progressions through the major mathematical domains. Moreover, the compacted courses should not sacrifice attention to the Standards for Mathematical Practice.

2. **Decisions to accelerate students into the Common Core State Standards for higher mathematics before ninth grade should not be rushed.** Premature placement of students into an accelerated pathway should be avoided at all costs. In order to ensure that students are developmentally ready for accelerated content, it is not recommended to compact the standards before grade seven. In Appendix A of the national Common Core State Standards for Mathematics document (NGA/CCSSO 2010a, 81]), it is understood that compaction begins in seventh grade for both the traditional and integrated sequences.

3. **Decisions to accelerate students into higher mathematics before ninth grade should be based on solid evidence of student learning.** "Mathematics is by nature hierarchical. Every step is a preparation for the next one. Learning it properly requires thorough grounding at each step, and skimming over any topics will only weaken one's ability to tackle more complex material down the road" (Wu 2012). Before a student is placed on an accelerated pathway, serious efforts must be made to consider solid evidence of the student's conceptual understanding, knowledge of procedural skills, fluency, and ability to apply mathematics.

4. **A menu of challenging options should be available for students after their third year of mathematics—and all students should be strongly encouraged to take mathematics in all years of high school.** Traditionally, students taking higher mathematics in the eighth grade are expected to take Precalculus in their junior year and then Calculus in their senior year. This is a good and worthy goal, but it should not be the only option for students. Advanced courses may also include Statistics, Discrete Mathematics, or Mathematical Decision Making via mathematical modeling. An array of challenging options will keep mathematics relevant for students and give them a new set of tools for their future in college and careers.

Students Who May Be Ready for Acceleration

Although the CA CCSSM are more rigorous than California's previous standards for mathematics, there will still be some students who are able to move through the mathematics quickly. Those students may choose to take an accelerated or enhanced mathematics program beginning in eighth grade (or even earlier) so they can take college-level mathematics in high school. However, the previous course sequences for acceleration will need to be updated because of the increased rigor of the CA CCSSM. Students who are capable of moving more quickly deserve thoughtful attention, both to ensure that they are challenged and that they master the full range of mathematical content and skills—without omitting critical concepts and topics. Care must be taken to ensure that students fully understand all important topics in the mathematics curriculum, and that the continuity of the mathematics learning progression is not disrupted. There should be a variety of opportunities for students to advance to mathematics courses beyond those included in this publication (NGA/CCSSO 2010a).

Maintaining motivation and engagement in advanced mathematics is essential for some students who enjoy work in mathematics and excel in mathematics and consequently in school. Slowing down instruction or restricting access to accelerated sequences may discourage and disengage some students from their progress in math, and potentially other courses as well. Therefore, some students may look forward to Advanced Placement (AP) Calculus or Multivariate Calculus as real options for their senior year of high school. For high schools that do not offer these courses on a regular basis, concurrent enrollment in local colleges and universities may provide some students with an alternative to high school courses.

Districts are encouraged to work with mathematics leadership, teachers, parents, and curriculum coordinators to design pathways that best meet the needs of students. Enrichment opportunities should allow students to increase their depth of understanding by developing expertise in the modeling process and applying mathematics to novel and complex contexts (MDESE 2012).

In the CA CCSSM, students begin preparing for algebra in kindergarten, as they start learning about the properties of operations. Furthermore, much of the content central to Algebra I courses of the past—namely, linear equations, inequalities, and functions—is now found in the grade-eight CA CCSSM. Mastery of the algebra content, including the Standards for Mathematical Practice, is fundamental for success in further mathematics and on college entrance examinations. Skipping over material to get students to a particular point in the curriculum will create gaps in the students' mathematical background. To accelerate, students must prove that they are proficient in the CA CCSSM for kindergarten through grade eight (NGA/CCSSO 2010a).

It is essential that multiple measures are used to determine a student's readiness for acceleration. Districts should create a system for gathering evidence of a student's readiness for an accelerated pathway. Placement assessments that include constructed responses should be used to determine a student's conceptual understanding. The assessments should incorporate performance items that address multiple domains. Additionally, the assessments should measure a student's ability to demonstrate the skills included in the Standards for Mathematical Practice. Many schools and districts in California use commercially produced assessments; however, others use valid and reliable exams created by districts themselves. A portfolio of student work may be collected as evidence of readiness, in addition to student grade reports and assessment data from their previous mathematics courses.

One example of a widely available cognitive diagnostic assessment is the Mathematics Diagnostic Testing Project (MDTP), a statewide effort involving the California State University, the University of California, California Community Colleges, and California K–12 mathematics teachers to develop readiness tests and constructed-response materials. The MDTP provides students and teachers with diagnostic information about student readiness for a broad range of math courses from Prealgebra through Calculus. This information can help students identify specific areas where additional study or review is needed and can help teachers identify topics and skills that need more attention in courses. The MDTP readiness tests can be administered online, and the results are immediately available after test completion. In addition to using MDTP test results formatively to adapt instruction, some districts use the test results to assist with course placement decisions.

Examples of Accelerated Middle School Pathways

If the precautions noted above are considered, a middle school acceleration pathway could compact grade seven, grade eight, and Algebra I or Mathematics I in middle school. The term *compacted* means to compress content, which requires a faster pace to complete; it does not involve skipping content. To prepare eighth-grade students for higher mathematics, districts are encouraged to have a well-crafted sequence of compacted courses. The Achieve Pathways Group has provided "compacted" pathways in which the standards from grade seven, grade eight, and the Algebra I or Mathematics I course could be compressed into an accelerated pathway for students in grades seven and eight, allowing students to enter the Geometry (or Mathematics II) course in grade nine. Details of this "Compacted Pathway" example can be found in Appendix A of the national Common Core State Standards for Mathematics document (NGA/CCSSO 2010a). The appendix is posted at http://www.corestandards.org/Math/ (accessed September 30, 2015).

Examples of Accelerated High School Pathways

Because of the importance of middle school mathematics, districts may choose to offer high school acceleration options instead of, or in addition to, an accelerated pathway that begins in middle school. Some students may not have the necessary preparation to enter a Compacted Pathway but may still develop an interest in taking advanced mathematics, such as AP Calculus or AP Statistics in their senior year. Districts are encouraged to work with mathematics leadership, teachers, and curriculum coordinators to design pathways that best meet the abilities and needs of students. For students who study the eighth-grade standards in grade eight, there are pathways that will lead them to advanced mathematics courses in high school (e.g., Calculus). As shown with the numbered list that follows, compressed and accelerated pathways for high school students may follow a range of models. Note that the accelerated high school pathways delay decisions about which students to accelerate while still allowing access to advanced mathematics in grade twelve (MDESE 2012); see the course-sequence illustrations at the end of this appendix.

1. Students could "double up" by enrolling in the Geometry course during the same year that they take Algebra I or Algebra II.
2. Allow students in schools with block scheduling to take a mathematics course in both semesters of the same academic year.

3. Offer summer courses that are designed to provide the equivalent experience of a full course in all aspects, including attention to the Standards for Mathematical Practice.[2]
4. Create different compaction ratios, compressing four years of high school content into three years, beginning in ninth grade.
5. Create a hybrid Algebra II/Precalculus or Mathematics III/Precalculus course that allows students to go straight to Calculus in grade twelve (see the Enhanced Pathway).
6. Standards that focus on a sub-topic such as trigonometry or statistics could be pulled out and taken alongside the traditional or integrated courses so that students would only need to "double up" for one semester.
7. Standards from Mathematics I, Mathematics II, and Mathematics III courses could be compressed into an accelerated pathway for students for two years, allowing students to enter the Precalculus course in the third year.

A combination of these methods and the suggested compacted sequences in Appendix A of the national Common Core State Standards for Mathematics (NGA/CCSSO 2010a) would allow for the most mathematically inclined students to take advanced mathematics courses during high school.

Students Who May Need Additional Support

It is expected that students across the state will find the CA CCSSM challenging at all grade levels. For students who needed additional support to meet previously adopted mathematics standards, the CA CCSSM will likely provide even greater teaching and learning challenges. A common structural solution in California's public schools has been to encourage students to repeat courses where they have not demonstrated mastery of content. This has been done frequently between eighth and ninth grade, when concerns about the mastery of pre-algebraic and algebraic content have arisen. Under the CA CCSSM, it is intended that course repetition be reduced. An alternative is to rethink the content of existing courses in grades six, seven, and eight. Alignment with earlier grade levels in elementary school is essential, as is the need to examine how mathematics standards from early grade levels are mastered.

Some school districts in California have developed course structures that allow mathematics content to be reinforced over multiple years through *expansion*—the opposite of compaction. Under the CA CCSSM, it is possible that this approach will be helpful, particularly with the assistance of formative testing under the Smarter Balanced Assessment Consortium and other diagnostic testing. Districts should consider how scheduling within the school day, within the school year, and across school years might facilitate increased mastery on the *combined* CA CCSSM from grades six through eight.

Support for K–12 Teachers

The increased rigor of the CA CCSSM and the demands of fully addressing the MP standards create additional opportunities and challenges for California's K–12 teachers. Accelerating students who are prepared for advanced course work will add a new layer to this set of challenges. Students who follow a compacted pathway undertake advanced work at an accelerated pace. This creates a great challenge

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2. As with other methods of accelerating students, enrolling students in summer courses should be handled with care, as the pace of the courses will likely be fast.

for those students as well as their teachers, who will teach eighth-grade standards and Algebra I or Mathematics I standards that are significantly more rigorous than in the past and within a compressed timeframe. Teachers must be prepared not only to address new and more challenging content, but they will also need to build on their repertoire of acceleration strategies. Teacher-preparation programs must respond to this call for additional training and support for teachers. Support and professional learning for experienced teachers should be provided by school districts, county offices of education, and the California Mathematics Project.

Acceleration

Figures D-1 through D-5 show some possibilities for accelerating students so that they may take an Advanced Placement mathematics course in grade twelve (e.g., Calculus or Statistics and Probability). Readers should note that the decision points for acceleration vary with each plan; some decision points occur as early as grade six. These are merely examples; there are other ways to accelerate students. The important thing to keep in mind is to avoid skipping any content.

Figure D-1. No Acceleration

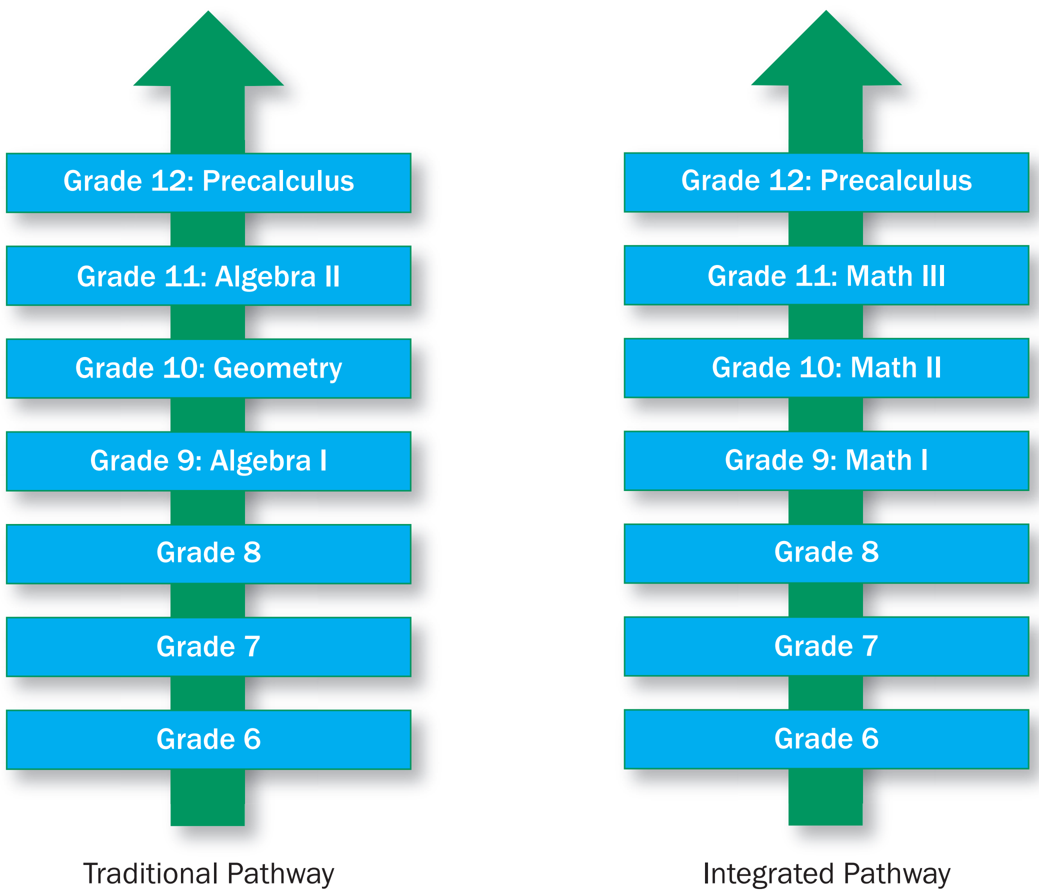


Figure D-2. Acceleration in Middle School

The decision point is at the end of grade six.

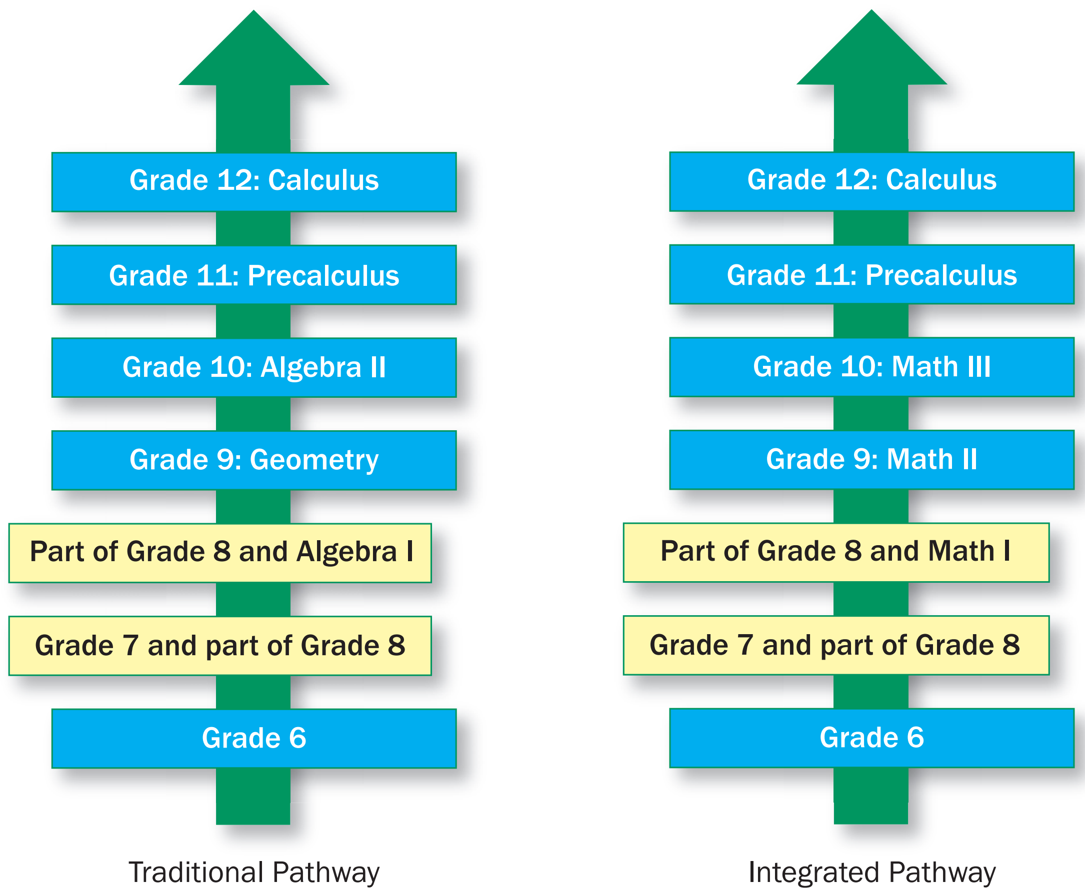


Figure D-3. Acceleration in High School

The decision point is at the end of grade eight.

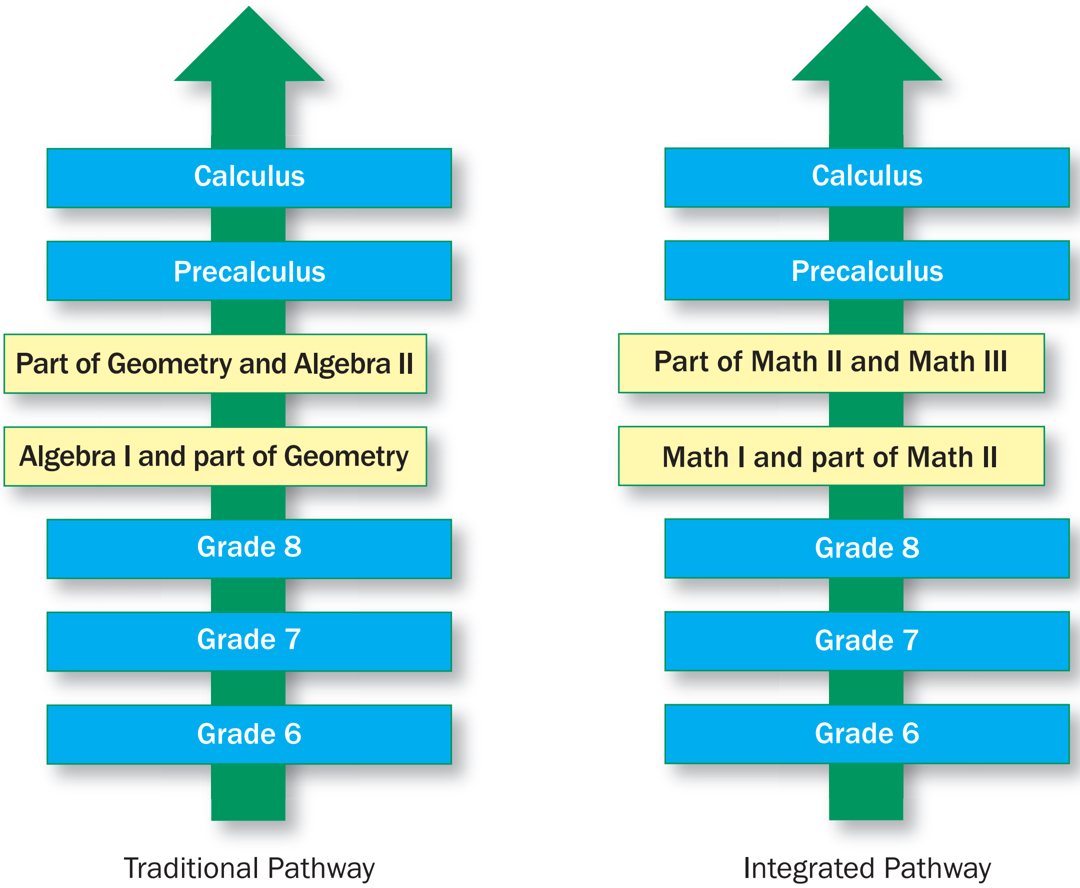


Figure D-4. Enhanced High School Sequence

The decision point is at the end of grade eight.

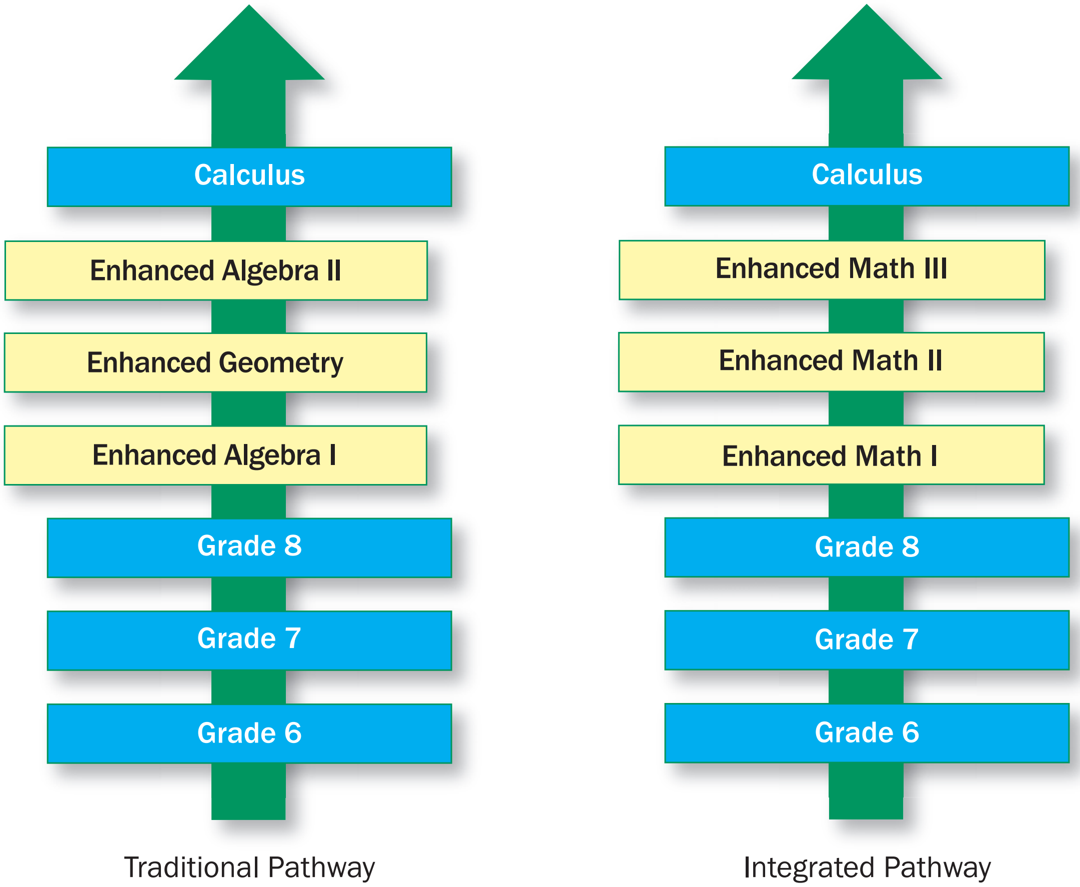
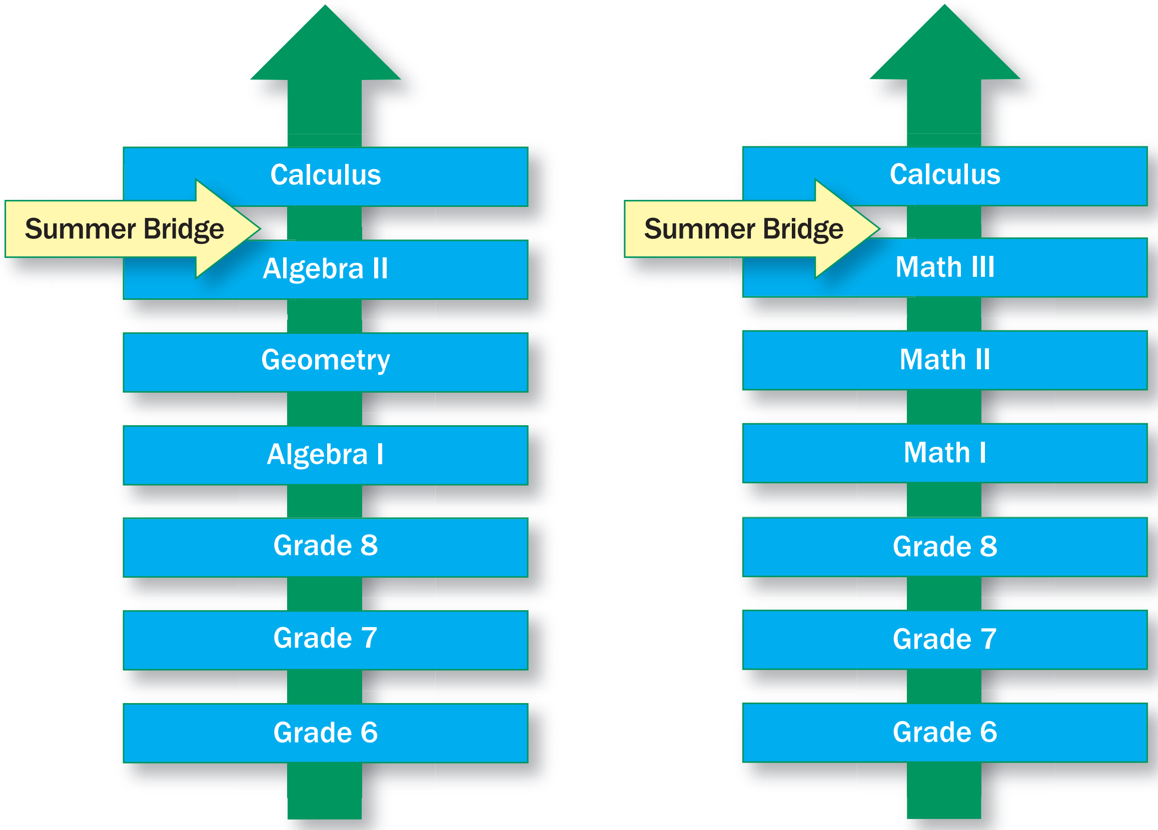


Figure D-5. Summer Bridge Sequence

The decision point is at the end of grade eleven.



Traditional Pathway Integrated Pathway

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