Glossary: Mathematical Terms, Tables, and Illustrations

of the

Mathematics Framework

for California Public Schools: Kindergarten Through Grade Twelve

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The glossary also includes terms defined in the Common Core State Standards Initiative's Mathematics Glossary (available at http://www.corestandards.org/ [accessed May 15, 2014]), as well as many additional terms.

AA similarity (**angle–angle similarity**). When two angles of one triangle are congruent to two angles of a second triangle, the triangles are similar.

absolute value. The absolute value of a number x is the non-negative number that represents its distance from 0 on a number line. Equivalently, $|x| = x$ if $x \geq 0$, or $-x$ *if* $x < 0$.

addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole-number answers, and with sum or minuend in the range 0–5, 0–10, 0–20, or 0–100, respectively. *Example:* $8 + 2 = 10$ is an addition within 10; $14 - 5 = 9$ is a subtraction within 20; and $55 - 18 = 37$ is a subtraction within 100.

additive identity property of 0. See table GL-1 in this glossary.

additive inverses. Two numbers whose sum is 0 are additive inverses of one another.
Example: $\frac{3}{4}$ and $-\frac{3}{4}$ are additive inverses of one another because $\frac{3}{4} + \left(-\frac{3}{4}\right) = \left(-\frac{3}{4}\right) + \frac{3}{4} = 0$
Source: National Governors Association Center for Best Practices, Council of Chief State School Officers (NGA/CCSSO) 2010c.

algorithm. A set of predefined steps applicable to a class of problems that give the correct result in every case when the steps are carried out correctly.

analog. Having to do with data represented by continuous variables—for example, a clock with hour, minute, and second hands (Merriam-Webster 2013).

analytic geometry. The branch of mathematics that uses functions and relations to study geometric phenomena (e.g., the description of ellipses and other conic sections in the coordinate plane by quadratic equations).

argument of a complex number. The angle θ when a complex number is represented in polar form, as in $r(\cos\theta + i\sin\theta)$.

ASA congruence (**angle–side–angle congruence**). When two triangles have corresponding angles and the included side that are congruent, the triangles themselves are congruent (Mathwords 2013).

associative property of addition. See table GL-1 in this glossary.

associative property of multiplication. See table GL-1 in this glossary.

assumption. A fact or statement (as a proposition, axiom, postulate, or notion) accepted as true.

attribute. A common feature of a set of figures.

benchmark fraction. A common fraction against which other fractions can be measured, such as $\frac{1}{2}$.

binomial theorem. The theorem that gives the polynomial expansion of each whole-number power of a binomial.

bivariate data. Pairs of linked numerical observations. An example is a list of the height and weight for each player on a football team.

box plot. A graphic that shows the distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50 percent of the data. (*Source:* Wisconsin Department of Public Instruction [WDPI] 2013.)

calculus. The mathematics of change and motion. The main concepts of calculus are limits, instantaneous rates of change, and areas enclosed by curves.

capacity. The maximum amount or number that can be contained or accommodated. *Examples:* The jug has a one-gallon *capacity*; the auditorium was filled to *capacity*.

cardinal number. A number (as 1, 5, 15) that is used in simple counting and that tells how many elements there are in a set but not the order in which they are arranged. Compare with **ordinal number**.

Cartesian plane. A coordinate plane with perpendicular coordinate axes.

causation. The act of causing or inducing. If one action causes another, then the actions are certainly correlated; however, just because two things occur together does not mean that one caused the other (STATS 2013). See also **correlation**.

Cavalieri's principle. If, in two solids of equal altitude, the sections made by planes parallel to and at the same distance from their respective bases are always equal, the volumes of the two solids are equal (Kern and Bland 1948).

coefficient. The numerical factor in a product. *Example:* In the term $3ab$, 3 is the coefficient of ab.

commutative property. See table GL-1 in this glossary.

complex fraction. A fraction $\frac{A}{B}$ where A and/or B are fractions (B non-zero).

complex number. A number that can be written in the form $a+bi$, where a and b are real numbers and $i^2=-1$.

complex plane. A Cartesian plane in which the point (a,b) represents the complex number $a+bi$.

compose numbers. To form a new number by "putting together" other numbers, paying special attention to the number 10. *Example:* 1 ten and 6 ones compose the number 16, or 10 + 6 = 16. See also **decompose numbers**.

compose shapes. To join geometric shapes without overlaps and form other shapes.

composite number. A whole number that has more than two distinct positive factors (Harcourt School Publishers 2013).

computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also **algorithm** and **computation strategy**.

computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also **computation algorithm**.

congruent. Two plane or solid figures are congruent if one can be obtained from the other by a rigid motion (i.e., a sequence of rotations, reflections, and translations).

conjugate. The result of writing a sum of two terms as a difference, or vice versa. *Example:* The conjugate of $x-2$ is $x+2$. (*Source:* Mathwords 2013.)

constant of proportionality. The constant k in the equation $y=kx$ that shows that y is directly proportional to x. The unit rate associated with a ratio is an example of a constant of proportionality. See also **proportional relationship**.

coordinate plane. A plane in which points are designated using two coordinates. In the Cartesian or rectangular coordinate plane, the two coordinates correspond to numbers on two perpendicular numbers lines, called *axes*, which intersect at the zero of each axis.

correlation. A measure of the amount of positive or negative relationship existing between two measures. *Example:* If the height and weight of a set of individuals were measured, it could be said that there is a positive correlation between height and weight if the data showed that larger weights tended to be paired with larger heights and smaller weights tended to be paired with smaller heights. The stronger those tendencies, the closer the measure is to –1 or 1. See also **causation**. (*Source:* WDPI 2013.)

cosine. A trigonometric function that for an acute angle of a right triangle is the ratio between a leg adjacent to the angle and the hypotenuse.

counting number. A number used in counting objects (e.g., a number from the set 1, 2, 3, 4, 5, . . .). See figure GL-1 at the end of this glossary.

counting on. A strategy for finding the number of objects in a group without having to count every member of the group. *Example:* If a stack of books is known to have 8 books, and 3 more books are added to the top, it is not necessary to count the stack all over again; one can find the total by counting on—pointing to the top book and saying "Eight," following this with "nine, ten, eleven. There are eleven books now."

decimal expansion. The representation of a real number using base-ten notation (e.g., the decimal expansion of the number $\frac{1}{4}$ is 0.25).

decimal fraction. A fraction (such as $0.25=\frac{25}{100}$ or $0.025=\frac{25}{1000}$) or mixed number (such as $3.025=3\frac{25}{1000}$) in which the denominator is a power of 10. Decimal fractions are usually expressed in base-ten notation with a decimal point.

decimal number. Any real number expressed in base-ten notation, such as 2.673.

decompose numbers. To "break apart" a number and represent it as a sum or difference of two or more other numbers. *Example:* 16 = 10 + 6.

decompose shapes. Given a geometric shape, to identify geometric shapes that meet without overlap to form the given shape.

digit. (**a**) Any of the Arabic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. (**b**) One of the elements that combine to form numbers in a system other than the decimal system.

digital. Having to do with data that are represented in the form of numerical digits; providing a readout in numerical digits (e.g., a *digital* watch).

dilation. A transformation that moves each point along the ray through the point emanating from a fixed center and multiplies distances from the center by a common scale factor (NGA/CCSSO 2010c); a transformation in which a figure is made proportionally larger or smaller.

directrix. A straight line the distance to which from any point of a conic section is in fixed ratio to the distance from the same point to the conic's focus.

discrete mathematics. The branch of mathematics that includes combinatorics, recursion, Boolean algebra, set theory, and graph theory.

dot plot. See **line plot**.

double number line diagram. A diagram in which two number lines subdivided in the same way are set one on top of the other with zeros lined up. Although the number lines are subdivided in the same way, the units in each may be different, which allows for the illustration of ratio relationships. Double number lines can also be constructed vertically.

expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. *Example:* $643 = 600 + 40 + 3$.

expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

exponent. For positive integer values, the number that indicates how many times the base is used as a factor. *Example:* In $4^3 = 4 \times 4 \times 4 = 64$, the exponent is 3, indicating that 4 is repeated as a factor three times.

exponential function. An exponential function is a function of the form $y(x) = a \bullet b^x$, where $a \neq 0$ and either $0 < b < 1$, or $1 < b$. The variables do not have to be x and y. *Example:* $A = 3.2 \bullet (1.02)^t$ defines A as an exponential function of t.

expression. A mathematical phrase that combines operations, numbers, and/or variables—for example, $3^2 \div a$ (Harcourt School Publishers 2013).

extreme values of a polynomial. The graph of a polynomial of degree n has at most $n-1$ extreme values (local minima and/or maxima). The total number of extreme values could be $n-1$, $n-3$, $n-5$, and so forth. *Example:* A degree 9 polynomial could have 8, 6, 4, 2, or 0 extreme values. A degree 2 (quadratic) polynomial must have 1 extreme value. (*Source:* Mathwords 2013.)

Fibonacci sequence. The sequence of numbers beginning with 1, 1, in which each number that follows is the sum of the previous two numbers (e.g., 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 . . .).

first quartile. For a data set with median M, the first quartile is the median of the data values less than M. *Example:* For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[1] See also **median**, **third quartile**, and **interquartile range**.

fluency.

- ***conceptual fluency.*** Being able to use the relevant ideas or procedures in a wide range of contexts (Smarter Balanced Assessment Consortium [Smarter Balanced] 2012a).
- ***contextual fluency.*** The ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems (Smarter Balanced 2012a).
- ***fluency as a special case of assessing individual content standards.*** *Fluent*, according to the Common Core State Standards for Mathematics, means fast and accurate. The word *fluency* was used judiciously in the standards to mark the endpoints of progressions of learning that begin with solid underpinnings and then pass upward through stages of growing maturity. Assessing the full range of the standards means assessing fluency where it is called for in the standards. (*Source:* Smarter Balanced 2012a.)
- ***procedural fluency.*** Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately (NGA/CCSSO 2010c).

focus (**plural** ***foci***)**.** One of the fixed points from which the distances to any point of a conic curve, such as an ellipse or parabola, are connected by a linear relation (Oxford Dictionaries 2013).

fraction. A number expressible in the form $\frac{a}{b}$ where a is a whole number and b is a positive whole number. (In the context of this publication, the word *fraction* always refers to a non-negative number.) See also **rational number**.

function. (**a**) A mathematical relation for which each element of the domain corresponds to exactly one element of the range. (**b**) A rule that assigns to every element of one set (the domain) exactly one element from another set (the co-domain).

function notation. A notation that describes a function. For a function f when x is a member of the domain, the symbol $f(x)$ denotes the corresponding member of the range (e.g., $f(x)=x+3$).

fundamental theorem of algebra. A theorem which states that when using complex numbers, all polynomials can be factored into a product of linear terms. An alternative form of the theorem asserts that any polynomial of degree n has exactly n complex roots (counting multiplicity).

geometric sequence (**progression**)**.** An ordered list of numbers that has a common ratio between consecutive terms (e.g., 2, 6, 18, 54, and so on) [Harcourt School Publishers 2013].

histogram. A special type of bar graph used to display the distribution of measurement data across a continuous range.

imaginary number. A complex number of the form bi, where $i=\sqrt{-1}$.

independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

1. Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Eric Langford, "Quartiles in Elementary Statistics," *Journal of Statistics Education* 14, no. 3 (2006).

initial value (of a function). (**a**) For a function f with domain of the interval $[a, b]$, the *initial value* of f is the value $f(a)$. If the domain of f is discrete, then the initial value of f is $f(n)$, where n is the smallest value in the domain of f (should such a smallest value exist). (**b**) For a function f with domain all real numbers, the initial value is also taken to mean $f(0)$.

integers (set of). The set of numbers that includes whole numbers and their opposites—for example, {. . . –3, –2, –1, 0, 1, 2, 3 . . .}.

interquartile range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. *Example:* For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is $15-6=9$. See also **first quartile**, **third quartile**.

inverse. See **additive inverses**, **multiplicative inverses**.

inverse function. Two functions, $y=h(x)$ and $x=g(y)$, are said to be inverses when $g(h(x))=x$ and $h(g(y))=y$. The function inverse to $f(x)$ is denoted $f^{-1}(x)$.

irrational number. A real number that cannot be expressed as a quotient of two integers (e.g., $\sqrt{2}$). A number is irrational only if it cannot be written as a repeating or terminating decimal.

law of cosines. An equation relating the cosine of an interior angle and the lengths of the sides of a triangle (Mathwords 2013).

law of sines. Equations relating the sines of the interior angles of a triangle and the corresponding opposite sides (Mathwords 2013).

linear association. Two variables have a linear association if a scatter plot of the data can be well approximated by a line. See **correlation**.

linear equation. Any equation that can be written in the form $Ax+By+C=0$, where A and B are not both 0. The graph of such an equation is a line.

linear function. A function, f, which may be brought into the form $f(x)=mx+b$. *Example:* $f(t)=2(t-7)$ represents a linear function.

line of symmetry. A line that divides a figure into two congruent parts, so that the reflection of either part across the line maps precisely onto the other part.

line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a *dot plot* (WDPI 2013).

logarithm. The exponent that indicates the power to which a base number is raised to produce a given number. *Example:* The logarithm of 100 to the base 10 is 2 (Merriam-Webster 2013).

logarithmic function. A function $f(x)=\log_b(x)$ which is inverse to the function $g(x)=b^x$.

matrix (plural *matrices*). A rectangular array of numbers or variables.

mean. A measure of central tendency in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[2] *Example:* For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

2. To be more precise, this defines the *arithmetic mean*.

mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. *Example:* For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 19.9.

measure of variability. A determination of how much the performance of a group deviates from the mean or median. The most frequently used measure is standard deviation.

median. A measure of central tendency in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list, or the mean of the two central values if the list contains an even number of values. *Example:* For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

midline. In the graph of a sine or cosine function, the horizontal line halfway between its maximum and minimum values.

minuend. A number from which another number is to be subtracted (Merriam-Webster 2013).

model. A mathematical representation (e.g., number, graph, matrix, equation[s], geometric figure) for real-world or mathematical objects, properties, actions, or relationships (WDPI 2013).

modulus of a complex number. The distance between a complex number and the origin on the complex plane. The modulus of $a+bi$ is written $|a+bi|$ and computed as $|a+bi|=\sqrt{a^2+b^2}$. For a complex number in polar form, $r(\cos\theta+i\sin\theta)$, the modulus is $|r|$.

multiplication and division within 100. Multiplication or division of two whole numbers with whole-number answers, and with product or dividend in the range 0–100. *Examples:* $4\times 21=84$ and $72\div 8=9$.

multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. *Example:* $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because $\frac{3}{4}\times\frac{4}{3}=\frac{4}{3}\times\frac{3}{4}=1$.
Source: NGA/CCSSO 2010c.

network. (**a**) A figure consisting of vertices and edges that shows how objects are connected. (**b**) A collection of points (vertices) with certain connections (edges) between them.

non-linear association. The relationship between two variables is non-linear if the value of each variable changes with the value of the other, but the change in one is not simply proportional to the change in the other variable.

number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

numeral. A symbol or mark used to represent a number. For example, 16, 3, and IV are all numerals.

order of operations. Convention adopted to perform mathematical operations in a consistent order: (1) perform all operations inside parentheses, brackets, and/or above and below a fraction bar in the order specified in steps 3 and 4; (2) find the value of any powers or roots; (3) multiply and divide from left to right; (4) add and subtract from left to right. (*Source:* National Council of Teachers of Mathematics [NCTM] 2013.)

ordinal number. A number designating the place (such as *first*, *second*, or *third*) occupied by an item in an ordered sequence (Merriam-Webster 2013).

partition. The process or result of dividing an object, set of objects, or a number into non-overlapping parts.

partitive division (or *fair-share division*). A division that determines how many are in each group when some quantity is portioned equally into groups. *Example:* Partitive division is used to determine how many pencils each child gets if a parent divides a dozen pencils equally among three children. The calculation is accomplished with counters by parceling 12 counters into 3 piles ("One for Adam, one for Beth, one for Charlie; two for Adam, two for Beth, two for Charlie," and so on) and checking how many counters are in each pile.

Pascal's triangle. A triangular arrangement of numbers in which each row starts and ends with 1 and each other number is the sum of the two numbers above it (Harcourt School Publishers 2013).

percent rate of change. A rate of change expressed as a percentage. *Example:* If a population grows from 50 to 55 in a year, it grows by $\frac{5}{50}$, or 10% per year.

periodic phenomena. Events that recur at regular, fixed intervals—for example, the solstices.

picture graph. A graph that uses pictures to show and compare information. Also known as a *pictograph*.

piecewise-defined function. A function defined by multiple subfunctions, each of which applies to a certain interval of the main function's domain.

polar form. The polar form of the complex number $a+bi$ is either of the following forms: $r\cos\theta + ri\sin\theta$; or $r(\cos\theta + i\sin\theta)$, which is often simplified to $r\text{cis}\theta$ when (r,θ) are polar coordinates of $a+bi$ on the complex plane. In either of these forms, $|r|$ is called the modulus and θ is called the argument.

polynomial. The sum or difference of terms which have variables raised to non-negative integer powers and which have coefficients that may be real or complex. Each of the following examples is a polynomial: $5x^3-2x^2+x-13$, x^2y^3+xy, x^2y^3 and $(1+i)a^2+ib^2$. (*Source:* Mathwords 2013.)

polynomial function. Any function whose values are determined by evaluating a polynomial.

prime factorization. A number written as the product of all its prime factors (Harcourt School Publishers 2013).

prime number. A positive integer that has only 1 and the number itself as factors. For example, 2, 3, 5, 7, 11, 13 (and so on) are all primes. By convention, the number 1 is not prime. (*Source:* Mathwords 2013.)

probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

probability distribution. The set of possible values of a random variable with a probability assigned to each.

probability model. A mathematical representation used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the *sample space*, and their probabilities sum to 1. See also **uniform probability model**.

proof. A method of using deductive reasoning to construct a valid argument.

properties of equality. See table GL-2 in this glossary.

properties of inequality. See table GL-3 in this glossary.

properties of integers. See tables GL-1, GL-2, and GL-3 in this glossary.

properties of operations. See table GL-1 in this glossary.

proportion. (**a**) Another term for a fraction of a whole. *Example:* The "proportion of the population that prefers product A" might be 60 percent. (**b**) An equation that states that two ratios are equivalent. *Example:* $\frac{4}{8}=\frac{1}{2}$ or $4:8=1:2$.

proportional relationship. A collection of pairs of numbers that are in equivalent ratios. A ratio $A:B$ determines a proportional relationship—namely, the collection of pairs (cA, cB) for c positive. A proportional relationship is described by an equation of the form $y=kx$, where k is a positive constant (often called a constant of proportionality). (*Source:* University of Arizona [UA] Progressions Documents for the Common Core Math Standards 2011c.)

Pythagorean Theorem. For any right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

quadratic equation. An equation that includes second-degree (and possibly lower-degree) polynomials. Some examples are $y=3x^2-5x^2+1$, $x^2+5xy+y^2=1$, and $1.6a^2+5.9a-3.14=0$.

quadratic expression. A polynomial expression that contains a term of degree 2, but no term of higher degree.

quadratic function. A function that can be represented by an equation of the form $y=ax^2+bx+c$, where a, b, and c are arbitrary (but fixed) numbers and $a\neq 0$. The graph of this function is a parabola (WDPI 2013).

quadratic polynomial. A polynomial where the highest degree of any of its terms is 2.

quotitive division (or *measurement division*). A division that determines how many equal-size groups can be formed from a given quantity. For example, quotitive division is used to determine how many pies can be purchased for $12 when each pie costs $3. The calculation can be accomplished with counters by parceling 12 counters into piles of size 3 each ("One, two, three for the first pie; one, two, three for the second pie," and so on) and checking how many piles there are.

radical. The $\sqrt{\ }$ symbol, which is used to indicate square roots or nth roots (Mathwords 2013).

random sampling. A smaller group of people or objects chosen from a larger group or population by a process giving equal chance of selection to all possible people or objects, and all possible subsets of the same size (Harcourt School Publishers 2013).

random variable. An assignment of a numerical value to each outcome in a sample space (Merriam-Webster 2013).

range (of a set of data). The numerical difference between the largest and smallest values in a set of data (WDPI 2013).

rate. A *rate* associated with the ratio $A:B$ is $\frac{A}{B}$ units of the first quantity per 1 unit of the second quantity. The two quantities may have different units. (*Source:* UA Progressions Documents 2011c.)

ratio. A multiplicative comparison of two numbers or quantities (e.g., 4 to 7 or 4:7 or $\frac{4}{7}$).

rational expression. A quotient of two polynomials with a non-zero denominator.

rational number. A number expressible in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. See figure GL-1 at the end of this glossary.

real number. A number that corresponds to a point on a number line. See figure GL-1 at the end of this glossary.

rectangular array. An arrangement of mathematical elements into rows and columns.

rectilinear figure. A polygon whose every angle is a right angle.

recursive pattern or sequence. A pattern or sequence wherein each successive term can be computed from some or all of the preceding terms by an algorithmic procedure.

reflection. A type of transformation that flips points about a line or a point.

relative frequency. The empirical counterpart of a probability. If an event occurs N' times in N trials, its relative frequency is $\frac{N'}{N}$ (Merriam-Webster 2013).

remainder theorem. If $f(x)$ is a polynomial in x, then the remainder on dividing $f(x)$ by $x-a$ is $f(a)$ [Merriam-Webster 2013].

repeated reasoning. The reasoning involved in solving one mathematical problem that is used again in a different mathematical problem or problems. Mathematically proficient students notice if calculations are repeated and look for general methods as well as shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process while attending to the details. They continually evaluate the reasonableness of their intermediate results. See Standard for Mathematical Practice 8. (*Source:* NGA/CCSSO 2010c.)

repeating decimal. A decimal expansion of a number in which, after a certain point, a particular digit or sequence of digits repeats itself indefinitely; the decimal form of a rational number (Merriam-Webster 2013). See also **terminating decimal**.

rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions preserve distances and angle measures.

rotation. A type of transformation that turns a figure about a fixed point (called the *center of rotation*).

sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

SAS congruence (side–angle–side congruence). When two triangles have two pairs of corresponding sides that are congruent, and the included angle formed by those sides is congruent, then the triangles are congruent.

scatter plot. A graph in the coordinate plane representing a set of bivariate data. *Example:* The heights and weights of a group of people could be displayed on a scatter plot (WDPI 2013).

scientific notation. A widely used system in which numbers are expressed as products consisting of a number between 1 and 10 multiplied by an appropriate power of 10 (e.g., $562 = 5.62 \times 10^2$) [Mathwords 2013].

sequence, progression. An ordered set of elements (e.g., 1, 3, 9, 27, 81). In this sequence, 1 is the *first term*, 3 is the *second term*, 9 is the *third term*, and so on.

similarity transformation. A transformation that can be represented as a rigid motion followed by a dilation.

simultaneous equations. Two or more equations containing common variables (Mathwords 2013).

sine. The trigonometric function that, for an acute angle of a right triangle, is the ratio between the leg opposite the angle and the hypotenuse.

SSS congruence (**side–side–side congruence**)**.** When two triangles have all three pairs of corresponding sides congruent, then the triangles are congruent.

strategy. (**a**) A plan of action designed to achieve a long-term or overall aim. (**b**) An approach to teaching and learning (Oxford Dictionaries 2013).

subitize. To immediately, and without counting, perceive a quantity.

tangent. (**a**) A line passing perpendicular to a radius at the point lying on the circle is said to be tangent to the circle. (**b**) The trigonometric function that, for an acute angle of a right triangle, is the ratio between the leg opposite the angle and the leg adjacent to the angle.

tape diagram. A drawing that looks like a segment of tape and is used to illustrate number relationships. Also known as a *strip diagram*, *bar model*, *fraction strip*, or *length model*.

terminating decimal. A repeating decimal number whose repeating digit is 0. See also **repeating decimal**.

third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. *Example:* For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. See also **median**, **first quartile**, and **interquartile range**.

transformation. A prescription, or rule, that sets up a one-to-one correspondence between the points in a geometric object (the *pre-image*) and the points in another geometric object (the *image*). Reflections, rotations, translations, and dilations are examples of transformations.

transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other attributes as well, such as time, weight, area, and volume.

translation. A type of transformation that moves every point in a graph or geometric figure by the same distance in the same direction without a change in orientation or size (Mathwords 2013).

trigonometric function. Any of the six functions (sine, cosine, tangent, cotangent, secant, cosecant) that, for an acute angle of a right triangle, may be expressed in terms of ratios of sides of the right triangle.

trigonometry. The study of triangles and trigonometric functions.

uniform probability model. A probability model that assigns equal probability to all outcomes (NGA/CCSSO 2010c). See also **probability model**.

unit fraction. A fraction with a numerator of 1, such as $\frac{1}{3}$ or $\frac{1}{5}$.

unit rate. The numerical part of the rate; the word *unit* in the term *unit rate* is used to highlight the 1 in "per 1 unit" of the second quantity. *Example:* If 3 melons cost $4.50, then the unit rate is simply the number $\frac{4.50}{3} = 1.5$ [without units]. (*Source:* UA Progressions Documents 2011c.)

univariate. Relating to a single variable.

valid. (**a**) Well grounded or justifiable; being at once relevant and meaningful (e.g., a *valid* theory). (**b**) Logically correct. (*Source:* Mathwords 2013.)

variable. A quantity that can change or that may take on different values. Refers to the letter or symbol representing such a quantity in an expression, equation, inequality, or matrix. (*Source:* Mathwords 2013.)

vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

visual fraction model. A tape diagram, number line diagram, or area model.

whole numbers. The numbers 0, 1, 2, 3, . . . See figure GL-1 at the end of this glossary.

Tables and Illustrations of Key Mathematical Properties, Rules, and Number Sets

Table GL-1. The Properties of Operations

In this table, a, b, and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a+b)+c=a+(b+c)$
Commutative property of addition	$a+b=b+a$
Additive identity property of 0	$a+0=0+a=a$
Existence of additive inverses	For every a there exists $-a$, so that $a+(-a)=(-a)+a=0$.
Associative property of multiplication	$(a\times b)\times c=a\times(b\times c)$
Commutative property of multiplication	$a\times b=b\times a$
Multiplicative identity property of 1	$a\times 1=1\times a=a$
Existence of multiplicative inverses	For every $a\neq 0$ there exists $\frac{1}{a}$, so that $a\times\frac{1}{a}=\frac{1}{a}\times a=1$.
Distributive property of multiplication over addition	$a\times(b+c)=a\times b+a\times c$

Table GL-2. The Properties of Equality

In this table, a, b, and c stand for arbitrary numbers in the rational number system, the real number system, and the complex number system.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table GL-3. The Properties of Inequality

In this table, a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$, then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Table GL-4. Common Addition and Subtraction Situations[*]

	Result Unknown	**Change Unknown**	**Start Unknown**
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2+3=\square$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were 5 bunnies. How many bunnies hopped over to the first two? $2+\square=$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were 5 bunnies. How many bunnies were on the grass before? $\square+3=$
Take from	Five apples were on the table. I ate 2 apples. How many apples are on the table now? $5-2=\square$	Five apples were on the table. I ate some apples. Then there were 3 apples. How many apples did I eat? $5-\square=$	Some apples were on the table. I ate 2 apples. Then there were 3 apples. How many apples were on the table before? $\square-2=$
Put together/ Take apart[‡]	**Total Unknown** Three red apples and 2 green apples are on the table. How many apples are on the table? $3+2=\square$	**Addend Unknown** Five apples are on the table. Three are red, and the rest are green. How many apples are green? $3+\square=\quad,\quad -3=\square$	**Both Addends Unknown**[†] Grandma has 5 flowers. How many can she put in her red vase and how many in her blue vase? $5=0+5,\ 5=5+0$ $5=1+4,\ 5=4+1$ $5=2+3,\ 5=3+2$
Compare[§]	**Difference Unknown** ("How many more?" version): Lucy has 2 apples. Julie has 5 apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has 2 apples. Julie has 5 apples. How many fewer apples does Lucy have than Julie? $2+\square=\quad,\quad -2=\square$	**Bigger Unknown** (Version with *more*): Julie has 3 more apples than Lucy. Lucy has 2 apples. How many apples does Julie have? (Version with *fewer*): Lucy has 3 fewer apples than Julie. Lucy has 2 apples. How many apples does Julie have? $2+3=\square,\ 3+2=\square$	**Smaller Unknown** (Version with *more*): Julie has 3 more apples than Lucy. Julie has 5 apples. How many apples does Lucy have? (Version with *fewer*): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $\quad -3=\square,\ \square+\quad =5$

[*]Adapted from Boxes 2–4 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (National Research Council, Committee on Early Childhood Mathematics 2009, 32–33).

[‡] Either addend can be unknown, so there are three variations of these problem situations. "Both Addends Unknown" is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[†] These take-apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign (=), help children understand that the equal sign does not always mean *makes* or *results in* but does always mean *is the same number as*.

[§] For the "Bigger Unknown" or "Smaller Unknown" situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

Table GL-5. Common Multiplication and Division Situations*

	Unknown Product	Group Size Unknown	Number of Groups Unknown
	$\times 6 = \square$	$3 \times \square =$ and $\div 3 = \square$	$\square \times 6 =$ and $\div \ = \square$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there altogether? **Measurement example** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally and packed inside 3 bags, then how many plums will be in each bag? **Measurement example** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed, with 6 plums to a bag, then how many bags are needed? **Measurement example** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,† Area‡	There are 3 rows of apples with 6 apples in each row. How many apples are there? **Area example** What is the area of a rectangle that measures 3 centimeters by 6 centimeters?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? **Area example** A rectangle has an area of 18 square centimeters. If one side is 3 centimeters long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? **Area example** A rectangle has an area of 18 square centimeters. If one side is 6 centimeters long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? **Measurement example** A rubber band is 6 centimeters long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18, and that is three times as much as a blue hat costs. How much does a blue hat cost? **Measurement example** A rubber band is stretched to be 18 centimeters long and that is three times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? **Measurement example** A rubber band was 6 centimeters long at first. Now it is stretched to be 18 centimeters long. How many times as long is the rubber band now as it was at first?
General	$\times b = \square$	$\times \square =$ and $\div a = \square$	$\square \times b = p$ and $p \div b = \square$

*The first examples in each cell focus on discrete things. These examples are easier for students and should be given before the measurement examples.

† The language in the array examples shows the easiest form of array problems. A more difficult form of these problems uses the terms *rows* and *columns*, as in this example: *"The apples in the grocery window are in 3 rows and 6 columns. How many apples are there?"* Both forms are valuable.

‡ Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps; thus array problems include these especially important measurement situations.

Figure GL-1. The Number System

The number system consists of number sets beginning with *counting numbers* and culminating in the more complete *complex numbers*. The name of each set is written on the boundary of the set, indicating that each increasing oval encompasses the sets contained within. Note that the *real number set* is composed of two parts: rational numbers and irrational numbers.