Appendix A: Financial Literacy and Mathematics Education

of the

Mathematics Framework

for California Public Schools: Kindergarten Through Grade Twelve

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Appendix A

Financial Literacy and Mathematics Education

Financial literacy is defined as the knowledge, tools, and skills that are essential for effective management of personal fiscal resources and financial well-being. Gaining mathematical knowledge is the first step toward developing financial literacy, which in turn provides early opportunities for meaningful mathematical modeling. The global economic downturn that occurred in the late 2000s highlighted the need for increased financial education for school-age students as well as adults. A 2009 survey conducted by the Financial Industry Regulatory Authority (FINRA) showed that about half of the Americans surveyed had trouble keeping up with their monthly expenses. Members of the same survey group were unable to save a portion of their income for emergencies or retirement (FINRA Investor Education Foundation 2009). This inability to save money has even greater implications for the future as the average life expectancy increases and people need more money to sustain themselves throughout their lives.

The President's Advisory Council on Financial Capability (2012) states, "Research shows that low levels of financial literacy are associated with high levels of indebtedness, lower wealth accumulation, and less retirement savings." Individuals with low levels of financial literacy are also particularly vulnerable to predatory lending. In response to these troubling social trends, there have been movements in many states across the country to increase the financial education of Americans, beginning in elementary school and continuing through postsecondary education.

California has not adopted its own standards for financial literacy; however, there are two sets of national standards that teachers may use to influence their instruction. The Jump$tart Coalition for Personal Financial Literacy created and maintains the *National Standards in K–12 Personal Finance Education,* available at http://jumpstart.org/assets/files/standard_book-ALL.pdf (accessed May 28, 2014). These standards describe financial knowledge and skills that students should be able to exhibit. The Jump$tart standards are organized under six major categories of personal finance:

- Financial Responsibility and Decision Making
- Income and Careers
- Planning and Money Management
- Credit and Debt
- Risk Management and Insurance
- Saving and Investing

The second set of national standards available to teachers is the *National Standards for Financial Literacy* published by the Council for Economic Education (CEE). The CEE standards are available at http://www.councilforeconed.org/wp/wp-content/uploads/2013/02/national-standards-for-financial-literacy.pdf (accessed May 28, 2014) and, like the Jump$tart standards, are organized under six major categories of personal finance:

- Earning Income
- Buying Goods and Services
- Saving
- Using Credit
- Financial Investing
- Protecting and Insuring

The standards in each category provide expectations for students' financial knowledge and skills at each grade level, but leave it to stakeholders to determine the methods for delivery. For example, under Jump$tart's Planning and Money Management category, the standards call for students to develop a plan for spending and saving, keep and use a system for financial records, apply consumer skills to purchasing decisions, and use other important money-management tools. A recent study supported by the National Endowment for Financial Education and the Citi Foundation shows that students who receive cumulative (repeated) financial education demonstrate more positive financial behaviors as adults. In addition, the study documents that early exposure to financial education has a positive impact on people's lives (Serido and Shim 2011). In some cases, students have very positive role models at home when it comes to financial decision making; however, other students may greatly benefit from learning about these concepts and tools at school.

There are numerous Web resources available to teachers free of charge to support financial education in schools. Links to these resources can be found at the end of this appendix, as well as on the CDE's K–12 Financial Literacy Resources Web page at http://www.cde.ca.gov/eo/in/fl/finlitk12.asp (accessed May 28, 2014).

Time constraints in the regular school day often make it impossible to offer a separate course in financial education; however, financial literacy concepts can be integrated into other core content areas. Mathematics courses are often considered a natural fit for the integration of financial literacy exercises and skills. The California Common Core State Standards for Mathematics (CA CCSSM) provide multiple entry points for the incorporation of problems or exercises that teach important financial literacy concepts and skills—and the opportunity to teach financial literacy concepts is even more evident in the real-world problems emphasized by the CA CCSSM. This includes the use of mathematical modeling, which is included in every grade level in the standards (see appendix B for more information). In addition, the Standards for Mathematical Practice (MP) emphasize the analytical skills that students will use when solving problems that involve financial literacy concepts. Although financial literacy has a place in the mathematics classroom, analyzing financial situations and making decisions based on the analysis is not pure mathematics per se. There are *computational* aspects of finance

problems and the various terms, definitions, and mathematical meanings that constitute the pure mathematics in these problems. In addition, financial literacy problems provide students with rich opportunities to hone their mathematical problem-solving skills within real-world contexts.

Teachers must find an appropriate balance when considering the integration of financial literacy into the mathematics classroom. Standard **2.OA.1** asks students to "Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions." Students in second grade may have some experience with doing chores at home to earn an allowance. They also may have had to make choices about how to spend their money. Together with students' experiences, standard **2.OA.1** provides opportunities to discuss concepts in the CEE's financial literacy standards (Earning Income and Buying Goods and Services) as well as Jump$tart's financial literacy categories (Income and Careers and Planning and Money Management). Consider the following word problem:

Lucy earns an allowance of $5 per week. She also walks her neighbor's dog every day, earning $15 per week. In addition, Lucy received a birthday gift of $20 from her aunt. In all, Lucy has three different sources of income this week. Use column A in the following table to input Lucy's income.

Column A: Income		Column B: Expenses	
Source	**Amount ($)**	**Item**	**Amount ($)**
Allowance	$5		
Dog walking	$15		
Birthday money	$20		
		Savings	$5
	Total: $40		Total:
How much is left over? (Column A total minus Column B total): $ _____			

Lucy must decide how to spend her money. At the end of the week, she would also like to have $5 left over to deposit into her savings account. She comes up with a list of possible ways to spend her money:

- Trip to the movies: $14
- Birthday gift for her brother: $10
- Favorite magazine: $4
- Donation to the local food bank: $5
- Materials for a school assignment: $7
- Money owed to her sister for a previous loan: $6

Follow-up financial literacy questions:

1. Does Lucy have enough money for all of these things?
2. How would you suggest that she spend her money?

Use column B in the previous table to show your suggestions for Lucy's expenses (adapted from Federal Reserve Bank of Cleveland 2007).

The standards for mathematics in middle school allow for more in-depth exercises that address financial literacy concepts. For example, the seventh-grade standard **7.EE.4b** reads as follows:

> Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

This standard has the potential to address several financial literacy categories in the Jump$tart standards (Financial Responsibility and Decision Making; Income and Careers; Planning and Money Management; and Saving and Investing) as well as three of the CEE's financial literacy topics (Earning Income; Buying Goods and Services; and Saving). Consider the following problem:

Darla works as a salesperson. Her monthly salary is $1000, and she can make an additional $25 per sale. In order to pay her monthly bills, she must make at least $2750 per month. How many sales will she need to make per month to meet her monthly financial obligations? The following table lists Darla's monthly expenses:

Expenses	
Item	**Amount ($)**
Rent	$1200
Utility bills	$280
Cell phone bill	$75
Entertainment	$150
Bus fares	$105
Groceries	$450
Credit card payments	$240
Charitable contributions	$100
Clothing purchases	$150
	Total: $2750

[Write an inequality for the number of sales she will need to make, and describe the solutions.]

$$1000 + 25x \geq 2750$$

Darla would like to save an additional $200 per month over the next year for a down payment on a car. Considering the previous information, how many sales will she need to make per month? [Write an inequality for the number of sales she will need to make, and describe the solutions.]

$$1000 + 25x \geq 2750 + 200$$

Once Darla purchases the car, her monthly car payment will be $300. How many sales will she need to make each month in order to meet her monthly financial obligations, including the car payment? [Write an inequality for the number of sales she will need to make, and describe the solutions.]

$$1000 + 25x \geq 2750 + 300$$

Follow-up financial literacy questions:

1. What additional costs associated with car ownership must Darla consider?
2. What risk is Darla taking when basing the total income she needs on the number of sales she makes beyond her base salary?
3. Using the previous table that shows Darla's monthly expenses, if her sales are lower than expected for any given month, where might she cut some costs in order to afford her car?

In higher mathematics, the CA CCSSM allow for a more sophisticated discussion of financial management and decision making. For instance, the early use of recursively defined sequences (standard **F-IF.3**) allows for a simpler and more intuitive discussion of compound interest (i.e., $a_0 = P$ (principal), $a_{n+1} = a_n \bullet (1+r)$ for $n \geq 0$); discount (i.e., $a_0 = P$ (principal), $a_{n+1} = a_n \bullet (1-r)$ for $n \geq 0$); and amortization of debt (i.e., $a_0 = P$ (principal), $a_{n+1} = a_n \bullet (1+r) -$(payment) for $n \geq 0$). These types of interest and payment calculations can help students understand the origin of common formulas such as $A(t) = P_0(1+r)^t$.

In the conceptual category of Functions, standard **F-BF.1** calls for students to "Build a function that models a relationship between two quantities" (cluster heading) and specifically to "Determine an explicit expression, a recursive process, or steps for calculation from a context" (**F-BF.1a**). Consider the following problem concerning the cost of credit:

James arrived at college and was given two credit cards. He didn't really know much about managing his money, but he did understand how to use the cards—so he bought a few things for his dorm room, including a television for $1200 and a microwave for $200. Each of the items was purchased with a different credit card, and each card had a different interest rate. The television was purchased with a card that had an 18% annual interest rate; the microwave was purchased with a card that had a 28% annual interest rate. James has a job. He earns $1500 per month and spends $900 per month on school-related and living expenses.

1. What questions do you have about each credit card that would help you advise James on how to pay off each of his debts? (For example, students might ask about the minimum payments required for each card, late charges, and so forth.)

2. If James takes the amount of money he has left after paying his other expenses and splits it between the two cards, how long would it take him to pay off each account?
3. What other options does James have for paying off the debts?
4. Which option would result in James paying the least amount of interest?
   a) Write one or more equations to model the situation and support your answer.
   b) What is the total amount of interest James will end up paying for each credit card?

CA CCSSM Alignment

Building Functions (F-BF)

Build a function that models a relationship between two quantities.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.

When students are introduced to financial literacy education early in their academic lives, they can develop a lifelong foundation for making intelligent decisions about how to earn, save, and invest money. In many cases there is not enough time in the regular school day to offer a course in financial literacy, and thus it is important to leverage opportunities in other core subject areas to include financial literacy lessons where appropriate. The CA CCSSM open many doors for examining and practicing financial literacy topics, especially through the application of the Standards for Mathematical Practice and real-world problems. It is important for mathematics instructors to find the appropriate balance between teaching the mathematical standards and concepts and financial literacy skills.

The following table provides information about financial literacy word problems created by the Math Forum @ Drexel that align with both sets of national financial literacy standards, as well as the content and mathematical practices standards of the CA CCSSM. To view the actual word problems, please visit http://mathforum.org/pow/financialed/ (accessed May 28, 2014).

Grade Level or Course	Name of Word Problem	Standards for Mathematical Practice	Standards for Mathematical Content[1]	CEE National Standard	Jump$tart Standard
Kindergarten	The Yard Sale	**MP.1:** Make sense of problems and persevere in solving them. **MP.3:** Construct viable arguments and critique the reasoning of others.	**Operations and Algebraic Thinking:** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. (**K.OA.2**)	**Standard II:** Buying Goods and Services	**Planning and Money Management, Standard 4:** Apply consumer skills to purchase decisions.[2]
Grade 1	Jordan's Jobs	**MP.1:** Make sense of problems and persevere in solving them. **MP.3:** Construct viable arguments and critique the reasoning of others.	**Operations and Algebraic Thinking:** Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (**1.OA.2**)	**Standard I:** Earning Income	**Planning and Money Management, Standard 4:** Apply consumer skills to purchase decisions.
Grade 2	Money Matters	**MP.1:** Make sense of problems and persevere in solving them. **MP.3:** Construct viable arguments and critique the reasoning of others.	**Operations and Algebraic Thinking:** Represent and solve problems involving addition and subtraction. (**2.OA** cluster heading)	**Standard II:** Buying Goods and Services	**Planning and Money Management, Standard 4:** Apply consumer skills to purchase decisions.
Grade 3	Miracle Miranda and the Mascot	**MP.1:** Make sense of problems and persevere in solving them. **MP.3:** Construct viable arguments and critique the reasoning of others. **MP.5:** Use appropriate tools strategically. **MP.7:** Look for and make use of structure.	**Operations and Algebraic Thinking:** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (**3.OA.8**)	**Standard I:** Earning Income	**Income and Careers, Standard 2:** Identify sources of personal income.

1. For some examples in this column, cluster headings are listed rather than standards.

2. The correct term is *purchasing decisions,* but the Jump$tart standards use the term *purchase decisions.*

Grade Level or Course	Name of Word Problem	Standards for Mathematical Practice	Standards for Mathematical Content[1]	CEE National Standard	Jump$tart Standard
Grade 4	Building Bouquets	**MP.1:** Make sense of problems and persevere in solving them. **MP.3:** Construct viable arguments and critique the reasoning of others.	**Operations and Algebraic Thinking:** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. (**4.OA.4**)	**Standard I:** Earning Income	**Income and Careers, Standard 2:** Identify sources of personal income.
Grade 5	Super Salsa Deal	**MP.1:** Make sense of problems and persevere in solving them. **MP.3:** Construct viable arguments and critique the reasoning of others. **MP.5:** Use appropriate tools strategically. **MP.6:** Attend to precision.	**Number and Operations in Base Ten:** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (**5.NBT.7**)	**Standard II:** Buying Goods and Services	**Planning and Money Management, Standard 4:** Apply consumer skills to purchase decisions.
Grade 6	Buying Cola	**MP.1:** Make sense of problems and persevere in solving them. **MP.3:** Construct viable arguments and critique the reasoning of others.	**Ratios and Proportional Relationships:** Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* (**6.RP.2**)	**Standard II:** Buying Goods and Services	**Planning and Money Management, Standard 4:** Apply consumer skills to purchase decisions.

Grade Level or Course	Name of Word Problem	Standards for Mathematical Practice	Standards for Mathematical Content[1]	CEE National Standard	Jump$tart Standard
Grade 7	That's Interest-ing!	**MP.1:** Make sense of problems and persevere in solving them. **MP.3:** Construct viable arguments and critique the reasoning of others. **MP.4:** Model with mathematics. **MP.5:** Use appropriate tools strategically.	**Ratios and Proportional Relationships:** Use proportional relationships to solve multi-step ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* (**7.RP.3**)	**Standard III** (Saving) and **Standard IV** (Financial Investing)	**Saving and Investing, Standard 3:** Evaluate investment alternatives.
Grade 8	Saving Your Raise	**MP.1:** Make sense of problems and persevere in solving them. **MP.2:** Reason abstractly and quantitatively. **MP.3:** Construct viable arguments and critique the reasoning of others. **MP.4:** Model with mathematics. **MP.5:** Use appropriate tools strategically.	**Expressions and Equations:** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$. (**8.EE.1**)	**Standard III:** Saving	**Saving and Investing, Standard 2:** Explain how investing builds wealth and helps meet financial goals.
Algebra I, Mathematics I	Dinner at Pepe's	**MP.1:** Make sense of problems and persevere in solving them. **MP.3:** Construct viable arguments and critique the reasoning of others. **MP.4:** Model with mathematics.	**Algebra, Reasoning with Equations and Inequalities:** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (**A-REI.1**) **Algebra, Creating Equations:** Create equations and inequalities in one variable **including ones with absolute value** and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* **CA★** (**A-CED.1**)	**Standard II:** Buying Goods and Services	**Financial Responsibility and Decision Making, Standard 5:** Develop communication strategies for discussing financial issues.

Grade Level or Course	Name of Word Problem	Standards for Mathematical Practice	Standards for Mathematical Content[1]	CEE National Standard	Jump$tart Standard
Algebra I, Mathematics II	Credit Card Payoff Options	**MP.1:** Make sense of problems and persevere in solving them. **MP.2:** Reason abstractly and quantitatively. **MP.3:** Construct viable arguments and critique the reasoning of others. **MP.4:** Model with mathematics. **MP.5:** Use appropriate tools strategically.	**Algebra, Seeing Structure in Expressions:** Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* 1.15^t *can be rewritten as* $\left(1.15^{1/12}\right)^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*★ (**A-SSE.3c**) **Algebra, Building Functions:** Build a function that models a relationship between two quantities. (**F-BF** cluster heading)	**Standard IV:** Using Credit	**Credit and Debt, Standard 1:** Identify the costs and benefits of various types of credit.
Statistics and Probability	A Great Way to Make Money	**MP.1:** Make sense of problems and persevere in solving them. **MP.3:** Construct viable arguments and critique the reasoning of others.	**Statistics and Probability:** Using Probability to Make Decisions: (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. ★ (**S-MD.2**)	**Standard I:** Earning Income	**Saving and Investing, Standard 3:** Evaluate investment alternatives.
Calculus	College Savings	**MP.1:** Make sense of problems and persevere in solving them. **MP.2:** Reason abstractly and quantitatively. **MP.3:** Construct viable arguments and critique the reasoning of others. **MP.4:** Model with mathematics.	**Calculus:** Students demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change. Students can use derivatives to solve a variety of problems from physics, chemistry, economics, and so forth that involve the rate of change of a function. (**Calculus 4.2**)	**Standard III:** Saving	**Saving and Investing, Standard 2:** Explain how investing builds wealth and helps meet financial goals.

Grade Level or Course	Name of Word Problem	Standards for Mathematical Practice	Standards for Mathematical Content[1]	CEE National Standard	Jump$tart Standard
The following lesson plan was produced by the Mathematics Assessment Resource Service (MARS). See http://map.mathshell.org/materials/download.php?fileid=1250 (accessed July 24, 2014) for more information.					
Algebra II, Mathematics II/III	Comparing Investments	**MP.1:** Make sense of problems and persevere in solving them. **MP.2:** Reason abstractly and quantitatively. **MP.4:** Model with mathematics. **MP.7:** Look for and make use of structure.	**Algebra:** Seeing Structure in Expressions **Functions:** Linear, Quadratic, and Exponential Models		

Financial Literacy Curriculum Resources

California Department of Education. 2014. Grades K–12 Financial Literacy Resources. http://www.cde.ca.gov/eo/in/fl/finlitk12.asp (accessed May 28, 2014).

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