imab-cfird-jul23item01

Attachment 3

Page 1 of 9

# Summary of Revisions by Chapter

## Overall Revision Summary

The public comments generally underscored the importance of clear phrasing and reader-friendly structuring and formatting of information throughout the framework, as well as reducing the volume of text. Recognizing this, each chapter was edited for clear and concise wording, to remove redundancies, shortening where possible, and improve graphics. In some places text was rearranged, subheads or paragraph heads were added, and/or transitions were created to improve reader friendliness. To improve the flow and readability of the text, longer vignettes were moved to an appendix, with hyperlinks. To support framework-wide cohesion, discussion of the instructional design approach, which recurs across chapters, was clarified and standardized. Chapters 1, 2, 5, 8, 9, and 13 reflect more significant content changes. Those changes are summarized in attachment 2.

## Chapter 1—Mathematics for All: Purpose, Understanding, and Connection

### Chapter Overview

Chapter 1 introduces and sets up the *Mathematics Framework,* which lays out the curricular and instructional approaches that evidence suggests will afford all students the opportunities they need to learn meaningful and rigorous mathematics, meet state mathematics standards, access pathways to high level mathematics courses, and achieve success. The California Common Core State Standards for Mathematics (CA CCSSM) were adopted in 2010 and modified in 2013. The standards map out what California students need to know and be able to do, grade by grade, in mathematics. Built upon principles of *focus*, *coherence*, and *rigor*, they hold the promise of enabling all California students to become powerful users of mathematics in order to better understand and positively impact the world. To fulfill that promise and reach the goal of deep, active learning of mathematics for all, the framework is centered around the investigation of Big Ideas in mathematics, connected to authentic, real-world contexts and taught in multidimensional ways that meet varied learning needs. While this approach to mathematics education may be a tall order, research shows that it is the means to both provide access for all students and teach mathematics effectively. Chapter 1 provides an overview of how subsequent chapters are organized.

### Chapter Revision Summary

Addressing public comments in the second field review included updating and revising the neuroscientific discussions to avoid overgeneralizations and ensure that citations are correct and research findings are clearly and accurately captured. To clarify the core ideas introduced in this first chapter, some sections were moved and section headers reworded to more effectively communicate the central ideas of the framework: an understanding of the Big Ideas that organize the standards, the use of multidimensional teaching approaches and open tasks, and a focus on equity. Material from other chapters was incorporated to more clearly explain the Drivers of Investigation (DIs), Standards for Mathematical Practice, and Content Connections (CCs) and how they interact.

## Chapter 2—Teaching for Equity and Engagement

### Chapter Overview

Chapter 2 focuses on how teachers can create equitable and engaging mathematics classroom environments, transitional kindergarten through grade twelve (TK–12), that support all students and improve mathematics access and outcomes. The chapter describes dimensions of the system that contribute to access and equity and expands on the five components of instructional design, introduced in chapter 1, that encourage equitable outcomes and active student engagement: teaching Big Ideas; using open tasks; teaching for social justice; inviting students’ questions and conjectures; and prioritizing reasoning and justification. Instruction that incorporates these components can enable a diverse group of students to see themselves as mathematically capable individuals whose curiosity and a love of mathematics learning will be sustained throughout their schooling.

### Chapter Revision Summary

In responding to public comment, this chapter revision began with data on mathematics achievement and inequities in student outcomes that warrant instructional change. It then presents the five instructional strategies described in the chapter, incorporating additional discussion and research evidence to help readers understand what the strategy is and how its use may support equitable student outcomes. The chapter was shortened by rearranging text and removing several tables not previously discussed in the narrative as well as some information specific to implementation of the instructional strategies.

## Chapter 3—Number Sense

### Chapter Overview

By way of an in-depth discussion of number sense, chapter 3 illustrates how the progression of mathematical concepts occurs across TK–12 as elementary, middle, and high school teachers use investigations and connections to teach the mathematical Big Ideas of each grade level. The chapter shows that number sense can itself be described as a progression of Big Ideas, which include, for example: in transitional kindergarten through grade two, organizing and counting with numbers; in grades three through five, extending flexibility with numbers; in grades six through eight, number line understanding; and in grades nine through twelve, seeing parallels between numbers and functions. The chapter emphasizes the growth of number fluency—the ability to use strategies that are flexible, efficient, and accurate—and highlights the value of math talks and games, which encourage students’ mental problem solving and communication as well as playful exploration and skill practice.

### Chapter Revision Summary

Changes in chapter 3 consisted of light editing for clarity and reader friendliness, and the addition of an introductory paragraph.

## Chapter 4—Exploring, Discovering, and Reasoning With and About Mathematics

### Chapter Overview

Chapter 4 goes deeply into California’s Standards for Mathematical Practice, providing important background for discussing how elementary, middle, and high school teachers teach the Big Ideas of mathematics (see chapters 6, 7, and 8). The Standards for Mathematical Practice embed the habits of mind and habits of interaction—for example, persevering in problem solving, explaining one’s thinking, constructing arguments—that form the basis of mathematics learning. Using three interrelated Standards for Mathematical Practice for illustration, the chapter demonstrates how key mathematical practices, integrated with each other, can help teachers across grade levels create powerful mathematics experiences centered on exploring, discovery, and reasoning—thus enabling students to develop and deepen those skills, in relation to progressions in mathematics content, as they move through the grades.

### Chapter Revision Summary

Changes in chapter 4 consisted of light editing throughout, along with movement of text from the conclusion to earlier sections, per public comment, rewording of several subheads, addition of an introductory paragraph, and addition of the “Why, How, and What of Mathematics” table from chapter 1 to provide the reader with a visual of what the text explains.

## Chapter 5—Mathematical Foundations for Data Science

### Chapter Overview

This chapter is a first step in helping educators identify how the CA CCSSM can support K–12 students to develop foundational knowledge and skills for the use of data as a tool for mathematical problem solving. Like number sense, discussed in chapter 3, foundations of data science are embedded in mathematics at all grade levels. Making sense of data, identifying misleading uses of data, and using data to make decisions are all important skills for students in their roles as global citizens. Developing these abilities requires that students generate questions and work with data beginning in kindergarten (or before). This work should continue and increase in depth and complexity throughout their school careers. Across all grade levels, students should be encouraged and supported to understand and describe tendencies and variability in data and data distributions; consider data collection, sampling, and random processes; and compare distributions and identify associations between variables. Students who wish to focus extra attention on data science should have an opportunity to pursue advanced courses in high school.

### Chapter Revision Summary

In responding to public comments in the second field review, the revisions of this chapter seek to help readers understand how the foundations of data science currently reside within existing content standards across grade levels, the chapter was reorganized around three thematic topics derived directly from the CA CCSSM: understanding and describing variability in data and data distributions; data collection, sampling, and random processes; and comparing distributions and identifying associations between variables. The revised chapter also seeks to clarify how data literacy and data science are part of a continuum. While data literacy—the ability to understand and use data to answer questions—is part of data science, the field of data science also includes advanced mathematics, and computational skills that build upon—and go far beyond—the content contained in the kindergarten through grade twelve CA CCSSM. The chapter also contains new content that highlights how the use of computational tools and bigger data sets can support students’ understanding of statistics in middle school and high school.

## Chapter 6— Mathematics: Investigating and Connecting, Transitional Kindergarten through Grade Five

### Chapter Overview

This chapter discusses how the framework’s approach to mathematics teaching unfolds throughout elementary school. The framework envisions mathematics in transitional kindergarten through grade five as a vibrant, interactive, student-centered endeavor of investigating and connecting the Big Ideas of mathematics. In these grades, children experience enormous growth in maturity, reasoning, and conceptual understanding. They develop an understanding of concepts that include place value, arithmetic operations, fractions, geometric shapes and properties, and measurement. Building on this deep understanding, they also develop fluency, including the ability to retrieve and use facts and procedures as they undertake more complex topics. The chapter examines how teachers can use meaningful mathematics activities that nourish curiosity and develop reasoning skills, at the same time connecting content and mathematical practices within and across grade levels. Students who have gained an understanding of elementary mathematics and enter sixth grade viewing themselves as mathematically capable are positioned for success in middle school and beyond.

### Chapter Revision Summary

Revisions in chapter 6 were editorial, consisting of rearranging of some information for purposes of flow, coherence, and reader friendliness. Editing also added the standardized explanation of the instructional design model (used in multiple chapters), reworded several subheads, and added an introductory paragraph.

## Chapter 7—Mathematics: Investigating and Connecting, Grades Six through Eight

### Chapter Overview

This chapter discusses how the framework’s approach to mathematics teaching unfolds throughout middle school. Building on the foundational understanding of mathematics concepts developed in transitional kindergarten through grade five, middle school’s major topics include proportional reasoning, rational numbers, measurement in geometrical and data science scenarios—all developed through activities that are situated in intriguing, authentic contexts and require students to build connections among ideas. The critical element of success continues to be piquing students’ curiosity and interest through engagement with meaningful and relevant mathematics activities and experiences. As the chapter discusses, students’ middle school experiences are pivotal in shaping their attitudes toward mathematics and self-perceptions as mathematics learners. Those experiences, combined with the guidance they receive, determine whether or not students get on a pathway to high-level mathematics, crucially affecting their mathematics futures in high school and beyond.

### Chapter Revision Summary

Revisions in chapter 7 were editorial and intended to ensure a parallel structure with chapter 6, since these two chapters (grade band chapters for elementary and middle school) are analogous and likely to be read by an overlapping audience. As in chapter 6, editing added the standardized explanation of the instructional design model (used in multiple chapters), reworded several subheads, and added an introductory paragraph.

## Chapter 8—Mathematics: Investigating and Connecting, High School

### Chapter Overview

This chapter discusses how the framework’s approach to mathematics teaching unfolds throughout high school. California’s high school mathematics content standards are organized into conceptual categories: number and quantity; algebra; functions; modeling; geometry; and statistics and probability. This chapter provides an overview of the traditional and integrated pathways schools may offer, and describes the mathematical content that students learn in the courses associated with each pathway. It also describes the range of third- and fourth-year high school mathematics course options from which students may choose to reflect their interests and future aspirations and clarifies the expectations for coursework leading to different higher education options, including STEM careers. (See also appendix A of the framework.)

Examples are included of how course content can support the Standards for Mathematical Practice. The pathways integrate content with mathematics practices—that is, the framework’s investigation/connection approach enables students to develop a disposition toward reasoning and communication in mathematics, knowledge of mathematical ideas and skills, and the ability to think both critically and creatively in solving problems. As this chapter discusses, teaching this way requires careful consideration of many issues in addition to learning goals, including motivation, coherence, students’ and teachers’ cultural and linguistic assets, access and equity, context, and sustainability.

### Chapter Revision Summary

Chapter 8 was revised to address public comments in the second field review, particularly to remove all references to the Mathematics: Investigating and Connecting (MIC) pathway, which had encouraged more intensive use of data science. Instead the framework encourages the greater use of data in all course pathways. The revision also pulled portions of appendix A into the chapter to better explain the course content within each of the two pathways. The pathways graphic (figure 8.4) was also updated to clarify that students should be able to choose from more course offerings during their third- and fourth-year mathematics classes, as well as to clarify which courses are needed to be ready to pursue a STEM pathway in college. Redundancies within the chapter were removed, and the research citations were added to support the discussion of topics such as integration, algebra enrollment, and acceleration. Information about University of California mathematics (area C) course criteria was checked and clarified.

## Chapter 9—Structuring School Experiences for Equity and Engagement

### Chapter Overview

Chapter 9 describes methods of teaching and coursework designs that can enable all students to be appropriately challenged and many more to reach advanced mathematics—without requiring that all students work on the same mathematics or be placed in inflexible course sequences that make it difficult for them to move into or between STEM or non-STEM pathways if they so choose. As with appropriate course sequencing (described in chapter 8), the goal is to expand access to rigorous mathematics for all students, allowing each student to experience the joy and excitement of well-taught mathematics in ways that stimulate their learning and engagement.

### Chapter Revision Summary

Chapter 9 was revised to address public comments in the second field review, specifically to clarify and ensure accuracy in discussions of the research on tracking, provide additional explanation and research evidence related to personalization, one-on-one tutoring, and technology integration, and eliminate redundancies within the chapter.

## Chapter 10—Supporting Educators in Offering Equitable and Engaging Mathematics Instruction

### Chapter Overview

Chapter 10 discusses how to plan and design structured, ongoing, high-quality programs of professional learning that support teachers throughout their careers as they enact the framework’s approach to teaching mathematics. Teaching for equity and engagement is as rewarding as it is complex. Teachers who focus on Big Ideas and connections and teach mathematics by way of carefully designed, intriguing investigations see their students come alive through exploration and discovery. But especially since most teachers did not learn mathematics this way, they need support to rethink mathematics teaching and acquire skills and strategies that result in the changes in practice vital to improving student learning.

### Chapter Revision Summary

Changes in chapter 10 consisted of light editing throughout along with movement of one section (“Collaboration Among Partners and Communities”) where it fit better with the flow, rewording of several subheads, addition of an introductory paragraph, and addition of the “Why, How, and What of Mathematics” table from chapter 1 to provide the reader with a visual of what the text explains.

## Chapter 11—Technology and Distance Learning in the Teaching of Mathematics

### Chapter Overview

This chapter provides guidance to help educators enrich students’ learning of the mathematics content and practice standards through the use of technology. Today, a host of technologies have the potential to support rich and deep mathematical learning for all students. Introducing students to technology is important in itself, given its increasingly integral role in our lives. At the same time, this chapter emphasizes that using technology in the teaching of mathematics should support instructional objectives and student learning of mathematics, deeply understood. Based on principles for technology use in mathematics learning, the chapter recommends adopting technology in combination with changes to teaching practices that make the technology an integral and sustained component of the instruction—accompanied by high-quality, ongoing professional learning for teachers. The chapter also discusses distance learning, features of effective distance learning, and tips for success.

### Chapter Revision Summary

Changes in chapter 11 consisted of light editing for clarity and reader friendliness, along with the addition of an introductory paragraph.

## Chapter 12—Mathematics Assessment in the 21st Century

### Chapter Overview

This chapter encourages educators, administrators, and policymakers to focus on assessment that engages students in continuous improvement efforts by using mastery-based approaches—notably, by assessing with rubrics and using self, peer, and teacher feedback. Such an approach reflects the goal of achieving conceptual understanding, problem-solving capacity, and procedural fluency. It also promises to maximize the amount of learning each child is capable of while minimizing the socio-cultural effects of narrow testing. The chapter discusses California’s evolving comprehensive assessment system, with its two primary forms of mathematics assessments—formative and summative—and how they relate to mathematics instruction and learning. In California as nationwide, mathematics assessment is in transition, shifting from rote tests of fact-based skills to multi-dimensional measures of procedural skills, problem-solving capacity, and evidence-based reasoning. The shift reflects a growing alignment between how mathematics is being taught and how it is being tested—in turn reflecting shifting classroom, school, district, and state priorities.

### Chapter Revision Summary

Changes in chapter 12 consisted of light editing for clarity and reader friendliness, along with the addition of an introductory paragraph.

## Chapter 13—Instructional Materials to Support Equitable and Engaging Learning of the California Common Core State Standards for Mathematics

### Chapter Overview

Chapter 13 is directed to the developers and publishers of instructional materials and conveys information needed to participate in the State Board of Education’s instructional materials adoption process. It explains the need for developers and publishers to provide the content (including coverage, depth, and delivery) to address standards while at the same time maintaining coherence as learning progresses across grade levels. Materials need to reflect grade-level shifts in Big Ideas, and activities need to continue to be relevant to student lives. Resources that bridge content and context and provide guidance to teachers—while also allowing the flexibility necessary for supporting all students—will be critical for the implementation of the framework. Notably, developers and publishers should consider the lens of equity when developing lessons and units to serve California’s diverse student population. The chapter also provides guidance for local districts on the adoption of instructional materials for students in grades nine through twelve, the social content review process, supplemental instructional materials, and accessible instructional materials.

### Chapter Revision Summary

To align with public comments throughout the framework in the second field review, the revisions of this chapter included adding and adapting language from the 2013 evaluation criteria and piloting guidelines into a new introduction with context around state and local adoptions with added emphasis on alternatives to traditional textbooks as technology evolves. The State Adoption of Instructional Materials section has added emphasis of the standards organization around Big Ideas along the learning progressions, and addresses local educational agencies’ responsibility and authority to evaluate instructional materials and adopt those that best meet the needs of their students. Changes to this section further clarify the categories of criteria for the evaluation of instructional resources.

## Appendix A—Mathematical Progressions within the High School Pathways and [Key Mathematical Ideas to Promote Student Success in Introductory University Courses in Quantitative Fields](#IntroMathQuantitativeFields)

While chapter 8 describes the course content in the two high school pathways—Traditional and Integrated—this appendix describes ways in which the content ideas progress from one course to the next in each pathway’s sequence. It then presents key mathematical ideas to promote student success in introductory university courses in quantitative fields.

## Appendix B—Works Cited

This appendix lists all references cited in the framework, by chapter.

## Appendix C––Vignettes

This appendix contains all vignettes referenced in the framework, by chapter, that are longer than two pages. Hyperlinks in the chapters give readers easy access to each vignette.

California Department of Education, June 2023