Grade-Eight Chapter

of the

Mathematics Framework

for California Public Schools: Kindergarten Through Grade Twelve

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Grade Eight

Prior to entering grade eight, students wrote and interpreted expressions, solved equations and inequalities, explored quantitative relationships between dependent and independent variables, and solved problems involving area, surface area, and volume. Students who are entering grade eight have also begun to develop an understanding of statistical thinking (adapted from Charles A. Dana Center 2012).

Critical Areas of Instruction

In grade eight, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, as well as solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; and (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem (National Governors Association Center for Best Practices, Council of Chief State School Officers [NGA/CCSSO] 2010o). Students also work toward fluency in solving sets of two simple equations with two unknowns by inspection.

Standards for Mathematical Content

The Standards for Mathematical Content emphasize key content, skills, and practices at each grade level and support three major principles:

- **Focus**—Instruction is focused on grade-level standards.
- **Coherence**—Instruction should be attentive to learning across grades and to linking major topics within grades.
- **Rigor**—Instruction should develop conceptual understanding, procedural skill and fluency, and application.

Grade-level examples of focus, coherence, and rigor are indicated throughout the chapter.

The standards do not give equal emphasis to all content for a particular grade level. Cluster headings can be viewed as the most effective way to communicate the **focus** and **coherence** of the standards. Some clusters of standards require a greater instructional emphasis than others based on the depth of the ideas, the time needed to master those clusters, and their importance to future mathematics or the later demands of preparing for college and careers.

Table 8-1 highlights the content emphases at the cluster level for the grade-eight standards. The bulk of instructional time should be given to "Major" clusters and the standards within them, which are indicated throughout the text by a triangle symbol (▲). However, standards in the "Additional/Supporting" clusters should not be neglected; to do so would result in gaps in students' learning, including skills and understandings they may need in later grades. Instruction should reinforce topics in major clusters by using topics in the additional/supporting clusters and including problems and activities that support natural connections between clusters.

Teachers and administrators alike should note that the standards are not topics to be checked off after being covered in isolated units of instruction; rather, they provide content to be developed throughout the school year through rich instructional experiences presented in a coherent manner (adapted from Partnership for Assessment of Readiness for College and Careers [PARCC] 2012).

Table 8-1. Grade Eight Cluster-Level Emphases

The Number System — 8.NS

Additional/Supporting Clusters

- Know that there are numbers that are not rational, and approximate them by rational numbers.[1] (**8.NS.1–2**)

Expressions and Equations — 8.EE

Major Clusters

- Work with radicals and integer exponents. (**8.EE.1–4▲**)
- Understand the connections between proportional relationships, lines, and linear equations. (**8.EE.5–6▲**)
- Analyze and solve linear equations and pairs of simultaneous linear equations. (**8.EE.7–8▲**)

Functions — 8.F

Major Clusters

- Define, evaluate, and compare functions. (**8.F.1–3▲**)

Additional/Supporting Clusters

- Use functions to model relationships between quantities.[2] (**8.F.4–5**)

Geometry — 8.G

Major Clusters

- Understand congruence and similarity using physical models, transparencies, or geometry software. (**8.G.1–5▲**)
- Understand and apply the Pythagorean Theorem. (**8.G.6–8▲**)

Additional/Supporting Clusters

- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. (**8.G.9**)

Statistics and Probability — 8.SP

Additional/Supporting Clusters

- Investigate patterns of association in bivariate data.[3] (**8.SP.1–4**)

Explanations of Major and Additional/Supporting Cluster-Level Emphases

Major Clusters (▲) — Areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than others based on the depth of the ideas, the time needed to master them, and their importance to future mathematics or the demands of college and career readiness.

Additional Clusters — Expose students to other subjects; may not connect tightly or explicitly to the major work of the grade.

Supporting Clusters — Designed to support and strengthen areas of major emphasis.

Note of caution: Neglecting material, whether it is found in the major or additional/supporting clusters, will leave gaps in students' skills and understanding and will leave students unprepared for the challenges they face in later grades.

Adapted from Smarter Balanced Assessment Consortium 2011, 88.

1. Work with the number system in this grade is intimately related to work with radicals, and both of these may be connected to the Pythagorean Theorem as well as to volume problems (e.g., in which a cube has known volume but unknown edge lengths).

2. The work in this cluster involves functions for modeling linear relationships and a rate of change/initial value, which supports work with proportional relationships and setting up linear equations.

3. Looking for patterns in scatter plots and using linear models to describe data are directly connected to the work in the Expressions and Equations clusters. Together, these represent a connection to the fourth Standard for Mathematical Practice, **MP.4** (Model with mathematics).

Connecting Mathematical Practices and Content

The Standards for Mathematical Practice (**MP**) are developed throughout each grade and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject. The MP standards represent a picture of what it looks like for students to understand and do mathematics in the classroom and should be integrated into every mathematics lesson for all students.

Although the description of the MP standards remains the same at all grades, the way these standards look as students engage with and master new and more advanced mathematical ideas does change. Table 8-2 presents examples of how the MP standards may be integrated into tasks appropriate for students in grade eight. (Refer to the Overview of the Standards Chapters for a description of the MP standards.)

Table 8-2. Standards for Mathematical Practice—Explanation and Examples for Grade Eight

Standards for Mathematical Practice	Explanation and Examples
MP.1 Make sense of problems and persevere in solving them.	In grade eight, students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking questions such as these: "What is the most efficient way to solve the problem?" "Does this make sense?" "Can I solve the problem in a different way?"
MP.2 Reason abstractly and quantitatively.	Students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students *contextualize* to understand the meaning of the number(s) or variable(s) related to the problem and *decontextualize* to manipulate symbolic representations by applying properties of operations.
MP.3 Construct viable arguments and critique the reasoning of others.	Students construct arguments with verbal or written explanations accompanied by expressions, equations, inequalities, models, graphs, tables, and other data displays (e.g., box plots, dot plots, histograms). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions such as these: "How did you get that?" "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
MP.4 Model with mathematics.	Students in grade eight model real-world problem situations symbolically, graphically, in tables, and contextually. Working with the new concept of a *function*, students learn that relationships between variable quantities in the real world often satisfy a dependent relationship, in that one quantity determines the value of another. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students use scatter plots to represent data and describe associations between variables. They should be able to use any of these representations as appropriate to a particular problem context. Students should be encouraged to answer questions such as "What are some ways to represent the quantities?" or "How might it help to create a table, chart, graph, or _____?"

Table 8-2 *(continued)*

Standards for Mathematical Practice	Explanation and Examples
MP.5 Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when particular tools might be helpful. For instance, students in grade eight may translate a set of data given in tabular form into a graphical representation to compare it with another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal that intersects parallel lines. Teachers might ask, "What approach are you considering?" or "Why was it helpful to use _____?"
MP.6 Attend to precision.	In grade eight, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays. Teachers might ask, "What mathematical language, definitions, or properties can you use to explain _____?"
MP.7 Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. In grade eight, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.
MP.8 Look for and express regularity in repeated reasoning.	In grade eight, students use repeated reasoning to understand the slope formula and to make sense of rational and irrational numbers. Through multiple opportunities to model linear relationships, they notice that the slope of the graph of the linear relationship and the rate of change of the associated function are the same. For example, as students repeatedly check whether points are on the line with a slope of 3 that goes through the point (1, 2), they might abstract the equation of the line in the form $\frac{y-2}{x-1}=3$. Students divide to find decimal equivalents of rational numbers (e.g., $\frac{2}{3}=0.\overline{6}$) and generalize their observations. They use iterative processes to determine more precise rational approximations for irrational numbers. Students should be encouraged to answer questions such as "How would we prove that _____?" or "How is this situation like and different from other situations using these operations?"

Adapted from Arizona Department of Education (ADE) 2010 and North Carolina Department of Public Instruction (NCDPI) 2013b.

Standards-Based Learning at Grade Eight

The following narrative is organized by the domains in the Standards for Mathematical Content and highlights some necessary foundational skills from previous grade levels. It also provides exemplars to explain the content standards, highlight connections to Standards for Mathematical Practice (**MP**), and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. A triangle symbol (▲) indicates standards in the major clusters (see table 8-1).

Domain: The Number System

In grade seven, adding, subtracting, multiplying, and dividing rational numbers was the culmination of numerical work with the four basic operations. The number system continues to develop in grade eight, expanding to the real numbers with the introduction of irrational numbers, and develops further in higher mathematics, expanding to become the complex numbers with the introduction of imaginary numbers (adapted from PARCC 2012).

The Number System	**8.NS**

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

In grade eight, students learn that not all numbers can be expressed in the form $\frac{a}{b}$, where a and b are positive or negative whole numbers with $b \neq 0$. Such numbers are called *irrational*, and students explore cases of both rational and irrational numbers and their decimal expansions to begin to understand the distinction. The fact that rational numbers eventually result in repeating decimal expansions is a direct result of the way in which long division is used to produce a decimal expansion.

Why Rational Numbers Have Terminating or Repeating Decimal Expansions	**8.NS.1**

In each step of the standard algorithm to divide a by b, a partial quotient and a remainder are determined; the requirement is that each remainder is smaller than the divisor (b). In simpler examples, students will notice (or be led to notice) that once a remainder is repeated, the decimal repeats from that point onward, as in $\frac{1}{6} = 0.16666... = 0.1\overline{6}$ or $\frac{3}{11} = 0.272727... = 0.\overline{27}$. If a student imagines using long division to convert the fraction $\frac{3}{13}$ to a decimal without going through the tedium of actually producing the decimal, it can be reasoned that the possible remainders are 1 through 12. Consequently, a remainder that has already occurred will present itself by the thirteenth remainder, and therefore a repeating decimal results.

The full reasoning for why the converse is true—that eventually repeating decimals represent numbers that are rational—is beyond the scope of grade eight. However, students can use algebraic reasoning to show that repeating decimals eventually represent rational numbers in some simple cases (**8.NS.1**).

Example: Converting the Repeating Decimal $0.\overline{18}$ into a Fraction of the Form $\frac{a}{b}$ **8.NS.1**

Solution: One method for converting such a decimal into a fraction is to set $N=0.\overline{18}=0.18181818\ldots$. If this is the case, then $100N=18.\overline{18}$. Subtracting $100N$ and N yields $99N$. This means that $99N=18.\overline{18}-.\overline{18}=18$. Solving for N, students see that $N=\frac{18}{99}=\frac{2}{11}$.

Since every decimal is either repeating or non-repeating, this leaves irrational numbers as those numbers whose decimal expansions do not have a repeating pattern. Students understand this informally in grade eight, and they approximate irrational numbers by rational numbers in simple cases. For example, π is irrational, so it is approximated by $\frac{22}{7}$ or 3.14.

Example: Finding Better and Better Approximations of $\sqrt{2}$ **8.NS.2**

The following reasoning may be used to approximate simple irrational square roots.

- Since $1^2<2<2^2$, then $\sqrt{1^2}<\sqrt{2}<\sqrt{2^2}$, which leads to $1<\sqrt{2}<2$. This means that $\sqrt{2}$ must be between 1 and 2.
- Since $1.4^2=1.96$ and $1.5^2=2.25$, students know by guessing and checking that $\sqrt{2}$ is between 1.4 and 1.5.
- Through additional guessing and checking, and by using a calculator, students see that since $1.41^2=1.9881$ and $1.42^2=2.0164$, $\sqrt{2}$ is between 1.41 and 1.42.

Continuing in this manner yields better and better approximations of $\sqrt{2}$. When students investigate this process with calculators, they gain some familiarity with the idea that the decimal expansion of $\sqrt{2}$ never repeats. Students should graph successive approximations on number lines to reinforce their understanding of the number line as a tool for representing real numbers.

Ultimately, students should come to an informal understanding that the set of real numbers consists of rational numbers and irrational numbers. They continue to work with irrational numbers and rational approximations when solving equations such as $x^2=18$ and in problems involving the Pythagorean Theorem. In the Expressions and Equations domain that follows, students learn to use radicals to represent such numbers (adapted from California Department of Education [CDE] 2012d, ADE 2010, and NCDPI 2013b).

Focus, Coherence, and Rigor

In grade eight, the standards in The Number System domain support major work with the Pythagorean Theorem (**8.G.6–8▲**) and connect to volume problems (**8.G.9**)—for example, a problem in which a cube has known volume but unknown edge lengths.

Domain: Expressions and Equations

In grade seven, students formulated expressions and equations in one variable, using these equations to solve problems and fluently solving equations of the form $px+q=r$ and $p(x+q)=r$. In grade eight, students apply their previous understandings of ratio and proportional reasoning to the study of linear equations and pairs of simultaneous linear equations, which is a critical area of instruction for this grade level.

Expressions and Equations	**8.EE**

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Students in grade eight add the following properties of integer exponents to their repertoire of rules for transforming expressions, and they use these properties to generate equivalent expressions (**8.EE.1▲**).

Properties of Integer Exponents	**8.EE.1▲**

For any non-zero numbers a and b and integers n and m:

1. $a^n a^m = a^{n+m}$
2. $(a^n)^m = a^{nm}$
3. $a^n b^n = (ab)^n$
4. $a^0 = 1$
5. $a^{-n} = \dfrac{1}{a^n}$

Source: University of Arizona (UA) Progressions Documents for the Common Core Math Standards 2011d.

Students in grade eight have focused on place-value relationships in the base-ten number system since elementary school, and therefore working with powers of 10 is a natural place for students to begin investigating the patterns that give rise to these properties. However, powers of numbers other than 10 should also be explored, as these foreshadow the study of exponential functions in higher mathematics courses.

Example: Reasoning About Patterns to Explore the Properties of Exponents **8.EE.1▲**

Students fill in the blanks in the table below and discuss with a neighbor any patterns they find.

	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
Expanded	$2\times2\times2$	2×2	2	?	?	?	?
Evaluate	8	4	2	?	?	?	?

Students can reason about why the value of 2^0 should be 1, based on patterns they may see—for example, in the bottom row of the table, each value is $\frac{1}{2}$ of the value to the left of it. Students should explore similar examples with other bases to arrive at the general understanding that:

$$a^n = a\times a\times\cdots\times a \text{ (}n\text{ factors)}, a^0 = 1, \text{ and } a^{-n} = \frac{1}{a^n}.$$

Generally, Standard for Mathematical Practice **MP.3** calls for students to construct mathematical arguments; therefore, reasoning should be emphasized when it comes to learning the properties of exponents. For example, students can reason that $5^3\times5^2=(5\times5\times5)\times(5\times5)=5^5$. Through numerous experiences of working with exponents, students generalize the properties of exponents before using them fluently.

Students do not learn the properties of rational exponents until they reach the higher mathematics courses. However, in grade eight they start to work systematically with the symbols for square root and cube root—for example, writing $\sqrt{64}=8$ and $\sqrt[3]{5^3}=5$. Since $\sqrt{p}$ is defined to mean only the positive solution to the equation $x^2=p$ (when the square root exists), it is not correct to say that $\sqrt{64}=\pm8$. However, a correct solution to $x^2=64$ would be $x=\pm\sqrt{64}=\pm8$. Most students in grade eight are not yet able to prove that these are the only solutions; rather, they use informal methods such as "guess and check" to verify the solutions (UA Progressions Documents 2011d).

Students recognize perfect squares and cubes, understanding that square roots of non-perfect squares and cube roots of non-perfect cubes are irrational (**8.EE.2▲**). Students should generalize from many experiences that the following statements are true (**MP.2, MP.5, MP.6, MP.7**):

- Squaring a square root of a number returns the number back (e.g., $\left(\sqrt{5}\right)^2=5$).
- Taking the square root of the square of a number *sometimes* returns the number back (e.g., $\sqrt{7^2}=\sqrt{49}=7$, while $\sqrt{(-3)^2}=\sqrt{9}=3\neq-3$).
- Cubing a number and taking the cube root can be considered inverse operations.

Students expand their exponent work as they perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Students use scientific notation to express very large or very small numbers. Students compare and interpret scientific notation quantities in the context of the situation, recognizing that the powers of 10 indicated in quantities expressed in scientific notation follow the rules of exponents shown previously (**8.EE.3–4▲**) [adapted from CDE 2012d, ADE 2010, and NCDPI 2013b].

Example: Ants and Elephants **8.EE.4▲**

An ant has a mass of approximately 4×10^{-3} grams, and an elephant has a mass of approximately 8 metric tons. How many ants does it take to have the same mass as an elephant?

(*Note:* 1 kg = 1000 grams, 1 metric ton = 1000 kg.)

Solution: To compare the masses of an ant and an elephant, we convert the mass of an elephant into grams:

$$8 \text{ metric tons} \times \frac{1000\text{kg}}{1\text{ metric ton}} \times \frac{1000\text{g}}{1\text{kg}} = 8 \times 10^3 \times 10^3 \text{ grams} = 8\times10^6 \text{ grams}$$

If N represents the number of ants that have the same mass as an elephant, then $(4\times10^{-3})N$ is their total mass in grams. This should equal 8×10^6 grams, which yields a simple equation:

$$(4\times10^{-3})N = 8\times10^6, \text{ which means that}$$

$$N = \frac{8\times10^6}{4\times10^{-3}} = 2\times10^{6-(-3)} = 2\times10^9$$

Therefore, 2×10^9 ants would have the same mass as an elephant.

Adapted from Illustrative Mathematics 2013f.

Focus, Coherence, and Rigor

As students work with scientific notation, they learn to choose units of appropriate size for measurement of very large or very small quantities (**MP.2, MP.5, MP.6**).

Students build on their work with unit rates from grade six and proportional relationships from grade seven to compare graphs, tables, and equations of proportional relationships (**8.EE.5▲**). In grade eight, students connect these concepts to the concept of the slope of a line.

Understand the connections between proportional relationships, lines, and linear equations.

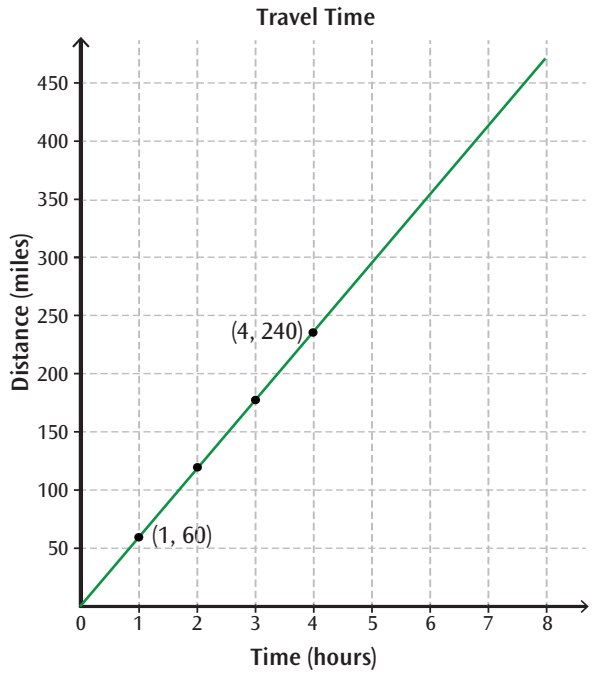
5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Students identify the unit rate (or slope) to compare two proportional relationships represented in different ways (e.g., as a graph of the line through the origin, a table exhibiting a constant rate of change, or an equation of the form $y=kx$). Students interpret the unit rate in a proportional relationship (e.g., r miles per hour) as the slope of the graph. They understand that the slope of a line represents a constant rate of change.

Example **8.EE.5▲**

Compare the scenarios below to determine which represents a greater speed. Include in your explanation a description of each scenario that discusses unit rates.

Scenario 1:



Scenario 2: The equation for the distance y in miles as a function of the time x in hours is:

$$y = 55x$$

Solution: "The unit rate in Scenario 1 can be read from the graph; it is 60 miles per hour. In Scenario 2, I can see that this looks like an equation $y = kx$, and in that type of equation the unit rate is the constant k. Therefore, the speed in Scenario 2 is 55 miles per hour. So the person traveling in Scenario 1 is moving at a faster rate."

Adapted from CDE 2012d, ADE 2010, and NCDPI 2013b.

Table 8-3 presents a sample classroom activity that connects the Standards for Mathematical Content and Standards for Mathematical Practice.

Table 8-3. Connecting to the Standards for Mathematical Practice—Grade Eight

Standards Addressed	Explanation and Examples
Connections to Standards for Mathematical Practice **MP.1.** Students are encouraged to persevere in solving the entire problem and make sense of each step. **MP.4.** Students model a very simple real-life cost situation. **Standards for Mathematical Content** **8.EE.5▲.** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in two different ways.	**Task:** Below is a table that shows the cost of various amounts of almonds. Almonds (pounds): 3, 5, 8, 10, 15 Cost (dollars): 15.00, 25.00, 40.00, 50.00, 75.00 1. Graph the cost versus the number of pounds of almonds. Place the number of pounds of almonds on the horizontal axis and the cost of the almonds on the vertical axis. 2. Use the graph to find the cost of 1 pound of almonds. Explain how you arrived at your answer. 3. The table shows that 5 pounds of almonds cost $25.00. Use your graph to find out how much 6 pounds of almonds cost. 4. Suppose that walnuts cost $3.50 per pound. Draw a line on your graph to represent the cost of different numbers of pounds of walnuts. 5. Which are cheaper: almonds or walnuts? How do you know? ***Solution:*** 1. A graph is shown. 2. To find the cost of 1 pound of almonds, students would locate the point that has 1 as the first coordinate; this is the point (1, 5). Thus the *unit cost* is $5 per pound. 3. Students can do this by locating 6 pounds on the horizontal axis and finding the point on the graph associated with this number of pounds. However, the teacher might also urge students to notice that when moving to the right by 1 unit along the graph, the next point on the graph is found 5 units up from the previous point. This idea is the genesis of the *slope* of a line and should be explored. 4. Ideally, students draw a line that passes through (0, 0) and the approximate point (1, 3.50). Proportional thinkers might notice that 2 pounds of walnuts cost $7, so they can plot a point with whole-number coordinates. 5. Walnuts are cheaper. Students may explore several different ways to see this, including the unit cost, the steepness of the line, a comparison of common quantities of nuts, and so on. **Classroom Connections.** The concept of slope can be approached in its simplest form with directly proportional quantities. In this case, when two quantities x and y are directly proportional, they are related by an equation $y = kx$, which is equivalent to $\frac{y}{x} = k$, where k is a constant known as the *constant of proportionality*. In the example involving almonds, the k in an equation would represent the unit cost of almonds. Students should have several experiences graphing and exploring directly proportional relationships to build a foundation for understanding more general linear equations of the form $y = mx + b$.

Focus, Coherence, and Rigor

The connection between the unit rate in a proportional relationship and the slope of its graph depends on a connection with the geometry of similar triangles (see standards **8.G.4–5▲**). The fact that a line has a well-defined slope—that the ratio between the rise and run for any two points on the line is always the same—depends on similar triangles.

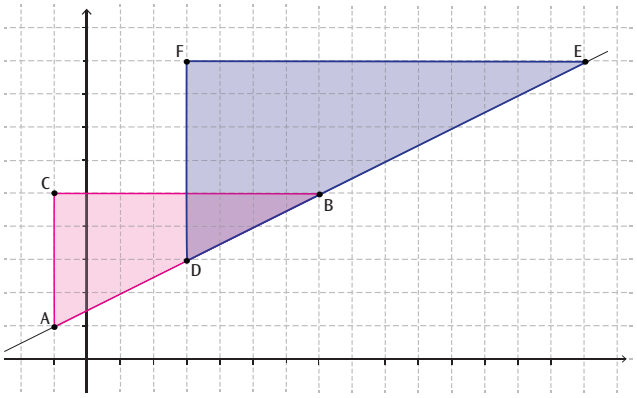
Adapted from UA Progressions Documents 2011d.

Standard **8.EE.6▲** represents a convergence of several ideas from grade eight and previous grade levels. Students have graphed proportional relationships and found the slope of the resulting line, interpreting it as the unit rate (**8.EE.5▲**). It is here that the language of "rise over run" comes into use. In the Functions domain, students see that any linear equation $y = mx + b$ determines a function whose graph is a straight line (a *linear function*), and they verify that the slope of the line is equal to m (**8.F.3▲**). Standard **8.EE.6▲** calls for students to go further and explain why the slope m is the same through any two points on a line. Students justify this fact by using similar triangles, which are studied in standards **8.G.4–5▲**.

Example of Reasoning **8.EE.6▲**

Show that the slope is the same between any two points on a line.

In grade seven, students made scale drawings of figures and observed the proportional relationships between side lengths of such figures (**7.G.1▲**). In grade eight, students generalize this idea and study *dilations* of plane figures, and they define figures as being *similar* in terms of dilations (see standard **8.G.4▲**). Students discover that similar figures share a proportional relationship between side lengths, just as scale drawings did: there is a *scale factor* $k > 0$ such that corresponding side lengths of similar figures are related by the equation $s_1 = k \bullet s_2$. Furthermore, the ratio of two sides in one shape is equal to the ratio of the corresponding two sides in the other shape. Finally, standard **8.G.5▲** calls for students to informally argue that triangles with two corresponding angles of the same measure must be similar, and this is the final piece of the puzzle for using similar triangles to show that the slope is the same between any two points on the coordinate plane (**8.EE.6▲**).



Explain why the slopes between points A and B and points D and E are the same.

Solution: "$\angle A$ and $\angle D$ are equal because they are corresponding angles formed by the transversal crossing the vertical lines through points A and D. Since $\angle C$ and $\angle F$ are both right angles, the triangles are similar.

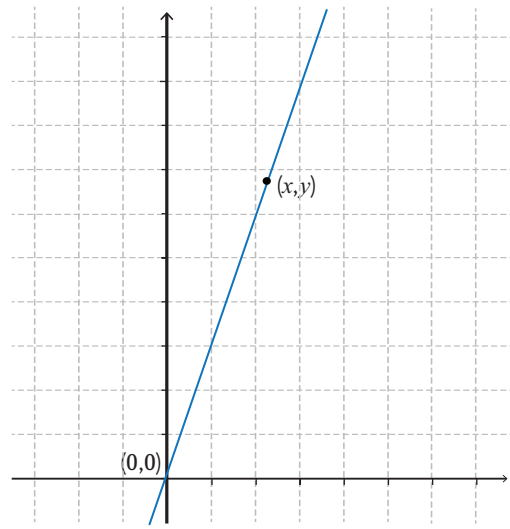
This means the ratios $\frac{AC}{BC}$ and $\frac{DF}{EF}$ are equal. But when you find the 'rise over the run,' these are the exact ratios you find, so the slope is the same between these two sets of points."

In grade eight, students build on previous work with proportional relationships, unit rates, and graphing to connect these ideas and understand that the points (x,y) on a non-vertical line are the solutions of the equation $y = mx + b$, where m is the slope of the line as well as the unit rate of a proportional relationship in the case $b = 0$.

Additional Examples of Reasoning **8.EE.6▲**

Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Example 1: Explain how to derive the equation $y = 3x$ for the line of slope $m = 3$ shown at right.

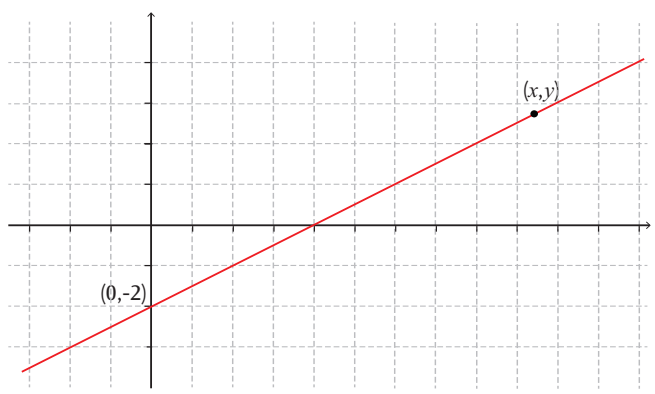


Solution: "I know that the slope is the same between any two points on a line. So I'll choose the origin $(0,0)$ and a generic point on the line, calling it (x,y). By choosing a generic point like this, I know that any point on the line will fit the equation I come up with. The slope between these two points is found by

$$3 = \frac{\text{rise}}{\text{run}} = \frac{y-0}{x-0} = \frac{y}{x}$$

This equation can be rearranged to $y = 3x$."

Example 2: Explain how to derive the equation $y = \frac{1}{2}x - 2$ for the line of slope $m = \frac{1}{2}$ with intercept $b = -2$.



Solution: "I know the slope is $\frac{1}{2}$, so I'll determine the equation of the line using the slope formula, with the point $(0,-2)$ and the generic point (x,y). The slope between these two points is found by

$$\frac{1}{2} = \frac{\text{rise}}{\text{run}} = \frac{y-(-2)}{x-0} = \frac{y+2}{x}$$

This can be rearranged to $y + 2 = \frac{1}{2}x$, which is the same as $y = \frac{1}{2}x - 2$."

Students have worked informally with one-variable linear equations as early as kindergarten. This important line of development culminates in grade eight, as much of the students' work involves analyzing and solving linear equations and pairs of simultaneous linear equations.

Expressions and Equations 8.EE

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8. Analyze and solve pairs of simultaneous linear equations.
   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
   b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
   c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Grade-eight students solve linear equations in one variable, including cases with one solution, an infinite number of solutions, and no solutions (**8.EE.7▲**). Students show examples of each of these cases by successively transforming an equation into simpler forms ($x = a$, $a = a$, and $a = b$, where a and b represent different numbers). Some linear equations require students to expand expressions by using the distributive property and to collect like terms.

Solutions to One-Variable Equations **8.EE.7a▲**

- When an equation has only one solution, there is only one value of the variable that makes the equation true (e.g., $12 - 4y = 16$).
- When an equation has an infinite number of solutions, the equation is true for all real numbers and is sometimes referred to as an *identity*—for example, $7x + 14 = 7(x + 2)$. Solving this equation by using familiar steps might yield $14 = 14$, a statement that is true regardless of the value of x. Students should be encouraged to think about why this means that any real number solves the equation and relate it back to the original equation (e.g., the equation is showing the distributive property).
- When an equation has no solutions, it is sometimes described as *inconsistent*—for example, $5x - 2 = 5(x + 1)$. Attempting to solve this equation might yield $-2 = 5$, which is a false statement regardless of the value of x. Students should be encouraged to reason why there are no solutions to the equation; for example, they observe that the original equation is equivalent to $5x - 2 = 5x + 5$ and reason that it is never the case that $N - 2 = N + 5$, no matter what N is.

Adapted from ADE 2010.

Grade-eight students also analyze and solve pairs of simultaneous linear equations (**8.EE.8 a–c▲**). Solving pairs of simultaneous linear equations builds on the skills and understandings students used to solve linear equations with one variable, and systems of linear equations may also have one solution, an infinite number of solutions, or no solutions. Students will discover these cases as they graph systems of linear equations and solve them algebraically. For a system of linear equations, students in grade eight learn the following:

- If the graphs of the lines meet at one point (the lines intersect), then there is one solution, the ordered pair of the point of intersection representing the solution of the system.
- If the graphs of the lines do not meet (the lines are parallel), the system has no solutions, and the slopes of these lines are the same.
- If the graphs of the lines are *coincident* (the graphs are exactly the same line), then the system has an infinite number of solutions, the solutions being the set of all ordered pairs on the line (adapted from ADE 2010).

Example: Introducing Systems of Linear Equations **8.EE.8a▲**

To introduce the concept of a system of linear equations, a teacher might ask students to assemble in small groups and think about how they would start a business selling smoothies at school during lunch. Then each group would create a budget that details the cost of the items that would have to be purchased each month (students could use the Internet to acquire pricing or use their best estimate), as well as a monthly total. Each group would also establish a price for its smoothies. Students can also discuss a model (equation) for the profit their business will make in a month. The teacher might ask the students questions such as these:

1. What are some variable quantities in our situation? (The number of smoothies sold and monthly profit are important.)
2. What is the profit at the beginning of the month? (This would be a negative number corresponding to the monthly total of items purchased.)
3. How many smoothies will you need to sell to make a profit? (The teacher instructs students to make a table that shows profit versus the number of smoothies sold, for multiples of 10 smoothies to 200. Students are also asked to create a graph from the data in their table. The teacher can demonstrate the graphs of the lines $y = 0$ and $y = 50$, and then ask students to draw the same lines on their graph. Students should also be asked to explain the meaning of those lines.)

Solution: The line $y = 0$ represents the point when the business is no longer losing money; the line $y = 50$ represents the point at which the company makes a $50 profit. The teacher can demonstrate the points of intersection and discuss the importance of these two coordinates. Finally, the teacher asks two students from different groups (groups whose graphs will intersect should be selected) to graph their data on the same axis for the whole class to see. Students discuss the significance of the point of intersection of the two lines, including the concept that the number of smoothies sold and the profit will be the same at that point. As a class, students write equations for both lines and demonstrate by substitution that the coordinates of the intersection point are solutions to both equations.

By making connections between algebraic and graphical solutions and the context of the system of linear equations, students are able to make sense of their solutions.

Students in grade eight also solve real-world and mathematical problems leading to two linear equations in two variables (**8.EE.8c▲**). Below is an example of how reasoning about real-world situations can be used to introduce and make sense out of solving systems of equations by elimination. The technique of elimination may be used in general cases to solve systems of equations.

Example: Solving a System of Equations by Elimination **8.EE.8c▲**

Suppose you know that the total cost of 3 gift cards and 4 movie tickets is \$168, while 2 gift cards and 3 movie tickets cost \$116.

1. Explain how to use this information to find the cost of 1 gift card and 1 movie ticket.
2. Next, explain how you could find the cost of 1 movie ticket.
3. Explain how you would find the cost of 1 gift card.

Solution:

1. If g represents the cost of a gift card and t represents the cost of a movie ticket, then I know that $3g+4t=168$ and $2g+3t=116$. I can represent this in a diagram:

If I subtract the 2 gift cards and 3 movie tickets from the 3 gift cards and 4 movie tickets, I get \$168 – \$116 = \$52. This means the cost of 1 of each item together is \$52. I can represent this by

$$\begin{array}{c} 3g+4t=168 \\ 2g+3t=116 \\ \hline g+t=52 \end{array}$$

2. Now I can see that 2 of each item would cost \$104. If I subtract this result from the second equation above, I am left with 1 movie ticket, and it costs \$12.

$$\begin{array}{c} 2g+3t=116 \\ 2g+2t=104 \\ \hline 1t=12 \end{array}$$

3. Now it is easy to see that if 1 gift card and 1 ticket together cost \$52, then 1 gift card alone would cost \$52 – \$12 = \$40.

Domain: Functions

In grade seven, students learned to determine if two quantities represented a proportional relationship. Proportional reasoning is a transitional topic that falls between arithmetic and algebra. Underlying the progression from proportional reasoning through algebra and beyond is the idea of a *function*—a rule that assigns to each input exactly one output. The concept of a function is a critical area of instruction in grade eight. Students are introduced to functions and learn that proportional relationships are part of a broader group of linear functions.

Functions	**8.F**

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[4]
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4), and (3,9), which are not on a straight line.*

In grade eight, students understand two main points regarding functions (**8.F.1▲**):

- A *function* is a rule that assigns to each input exactly one output.
- The *graph* of a function is the set of ordered pairs consisting of an input and the corresponding output.

In general, students understand that functions describe situations in which one quantity determines another. The main work in grade eight concerns linear functions, though students are exposed to non-linear functions to contrast them with linear functions. Thus, students may view a linear equation such as $y = -0.75x + 12$ as a rule that defines a quantity y whenever the quantity x is given. In this case, the function may describe the amount of money remaining after a number of turns, x, when a student who starts with \$12 plays a game that costs \$0.75 per turn. Alternatively, students may view the formula for the area of a circle, $A = \pi r^2$, as a (non-linear) function in the sense that the area of a circle is dependent on its radius. Student work with functions at grade eight remains informal but sets the stage for more formal work in higher mathematics courses.

4. Function notation is not required in grade eight.

Example: Introduction to Functions **8.F.1▲**

To introduce the concept of a function, a teacher might have students contrast two workers' wages at two different jobs, one with an hourly wage and the other based on a combination of an hourly wage and tips. Students read through scenarios and make a table for each of the two workers, listing hours worked and money earned during 20 different shifts varying from 3 to 8 hours in length. Students answer questions about the data, including the level of predictability of the wage of each worker, based on the number of hours worked. Students graph the data and observe the patterns of the graph. Next, the teacher could introduce the concept of a function and relate the tables and graphs from the activity to the idea of a function, emphasizing that an input value completely determines an output value. Students could then be challenged to find other quantities that are functions and to create and discuss corresponding tables and/or graphs.

Students are able to connect foundational understandings about functions to their work with proportional relationships. The same kinds of tables and graphs that students used in grade seven to recognize and represent proportional relationships between quantities are used in grade eight when students compare the properties of two functions that are represented in different ways (e.g., numerically in tables or visually in graphs). Students also compare the properties of two functions that are represented algebraically or verbally (**8.F.2▲**).

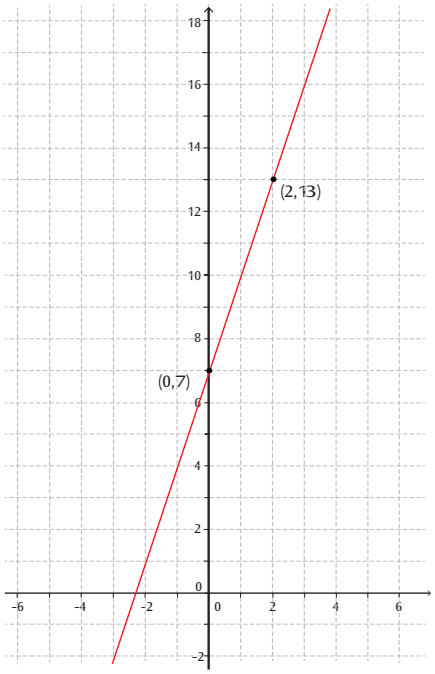
Example: Functions Represented Differently **8.F.2▲**

Which function has a greater rate of change? (**MP.1, MP.2, MP.4, MP.8**)

Function 1: The function represented by the graph shown.

Function 2: The function whose input x and output y are related by the equation $y = 4x + 7$.

Solution: The graph of the function shows that when $x = 0$, the value of the function is $y = 7$, and when $x = 2$ the value of the function is $y = 13$. This means that function 1 increases by 6 units when x increases by 2 units. Function 2 also has an output of $y = 7$ when $x = 0$, but when $x = 2$, the value of function 2 is $y = 15$. This means that function 2 increases by 8 units when x increases by 2 units. Therefore, function 2 has a greater rate of change.



Adapted from ADE 2010.

Students' understanding of the equation $y = mx + b$ deepens as they learn that the equation defines a linear function whose graph is a straight line (**8.F.3▲**), a concept closely related to standard **8.EE.6▲**. To avoid the mistaken impression that all functional relationships are linear, students also work with non-linear functions and provide examples of non-linear functions, recognizing that the graph of a non-linear function is not a straight line.

Functions 8.F

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or non-linear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

In grade eight, students learn to use functions to represent relationships between quantities. This work is also closely tied to **MP.4** (Model with mathematics). There are many real-world problems that can be modeled with linear functions, including instances of constant payment plans (e.g., phone plans), costs associated with running a business, and relationships between associated bivariate data (see standard **8.SP.3**). Students also recognize that linear functions in which $b = 0$ are proportional relationships, something they have studied since grade six.

Standard **8.F.4** refers to students finding the *initial value* of a linear function. Thus, if f represents a linear function with a domain $[a, b]$—that is, the input values for f are between the values a and b—then the initial value for f would be $f(a)$. The term *initial value* takes its name from an interpretation of the independent variable as representing time, although the term can apply to any function. Note that formal introduction of the term *domain* does not occur until the higher mathematics courses, but teachers may wish to include this language if it clarifies these ideas for students. The following example illustrates the definition of *initial value*.

Example: Modeling With a Linear Function **8.F.4**

A car rental company charges \$45 per day to rent a car as well as a one-time \$25 fee for the car's GPS navigation system. Write an equation for the cost in dollars, c, as a function of the number of days the car is rented, d. What is the initial value for this function?

Solution: There are several aids that may help students determine an equation for the cost:

- A verbal description: "Each day adds \$45 to the cost, but there is a one-time \$25 GPS fee. This means that the cost should be \$25 plus \$45 times the number of days you rent the car, or $c = 25 + 45d$. Since a customer must rent the car for a minimum of 1 day, the initial value is $25 + 45 = 70$, which means it costs \$70 to rent the car for 1 day."

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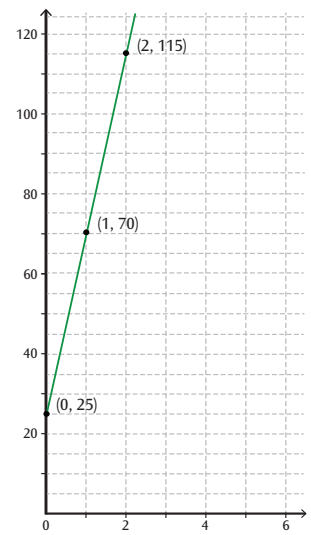
Example: 8.F.4 *(continued)*

- A table: "I made a table to give me a feel for how much the car rental might cost after d days.

d (days)	c (dollars)
1	$70 = 25 + (1)45$
2	$115 = 25 + (2)45$
3	$160 = 25 + (3)45$
4	$205 = 25 + (4)45$
d	$c = 25 + (d)45$

The table helped me see that the cost in dollars is represented by $c = 25 + (d)45$. Since the car must be rented for 1 day or more, the initial value is when $d = 1$, which is $c = 70$, or $70."

- A graph: "I made a rough graph and saw that the relationship between the cost and the days rented appeared to be linear. I found the slope of the line to be 45 (which is the cost per day) and the y intercept to be 25. This means the equation is $c = 45d + 25$. It is important to see that even though the y intercept of the graph is 25, that is not the initial value—because the initial value is when someone rents the car for 1 day. The point on the graph is $(1, 70)$ so the initial value is $70."



Students analyze graphs and then describe qualitatively the functional relationship between two quantities (e.g., where the function is increasing or decreasing, linear or non-linear). They are able to sketch graphs that illustrate the qualitative features of functions that are described verbally (**8.F.5**).

Focus, Coherence, and Rigor

Work in the cluster "Use functions to model relationships between quantities" involves functions for modeling linear relationships and computing a rate of change or initial value, which supports major work at grade eight with proportional relationships and setting up linear equations (**8.EE.5–8▲**).

Domain: Geometry

In grade seven, students solved problems involving scale drawings and informal geometric constructions, and they worked with two- and three-dimensional shapes to solve problems involving area, surface area, and volume. Students in grade eight complete their work on volume by solving problems involving cones, cylinders, and spheres. They also analyze two- and three-dimensional space and figures using distance, angle, similarity, and congruence and by understanding and applying the Pythagorean Theorem, which is a critical area of instruction at this grade level.

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
   a. Lines are taken to lines, and line segments to line segments of the same length.
   b. Angles are taken to angles of the same measure.
   c. Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

In this grade-eight Geometry domain, a major shift in the traditional curriculum occurs with the introduction of basic transformational geometry. In particular, the notion of *congruence* is defined differently than it has been in the past. Previously, two shapes were understood to be congruent if they had the "same size and same shape." This imprecise notion is exchanged for a more precise one: that a two-dimensional figure is congruent to another if the second figure can be obtained from the first by a sequence of rotations, reflections, and translations. Students need ample opportunities to explore these three geometric transformations and their properties. The work in the Geometry domain is designed to provide a seamless transition to the Geometry conceptual category in higher mathematics courses, which begins by approaching transformational geometry from a more advanced perspective.

With the aid of physical models, transparencies, and geometry software, students in grade eight gain an understanding of transformations and their relationship to congruence of shapes. Through experimentation, students verify the properties of rotations, reflections, and translations, including discovering that these transformations change the position of a geometric figure but not its shape or size (**8.G.1a–c▲**). Finally, students come to understand that congruent shapes are precisely those that can be "mapped" one onto the other by using rotations, reflections, or translations (**8.G.2▲**).

Characteristics of Rotations, Reflections, and Translations	8.G.1a–c▲

Students come to understand that the following transformations result in shapes that are *congruent* to one another.

- Students understand a *rotation* as the spinning of a figure around a fixed point known as the *center of rotation*. Unless specified otherwise, rotations are usually performed counterclockwise according to a particular angle of rotation.
- Students understand a *reflection* as the flipping of an object over a line known as the *line of reflection*.
- Students understand a *translation* as the shifting of an object in one direction for a fixed distance, so that any point lying on the shape moves the same distance in the same direction.

An example of an interactive online tool that shows transformation is Shodor Education's "Interactivate Transmographer" (http://www.shodor.org/interactivate/activities/Transmographer/ [Shodor Education Foun-dation, Inc. 2015]), which allows students to work with rotation, reflection, and translation.

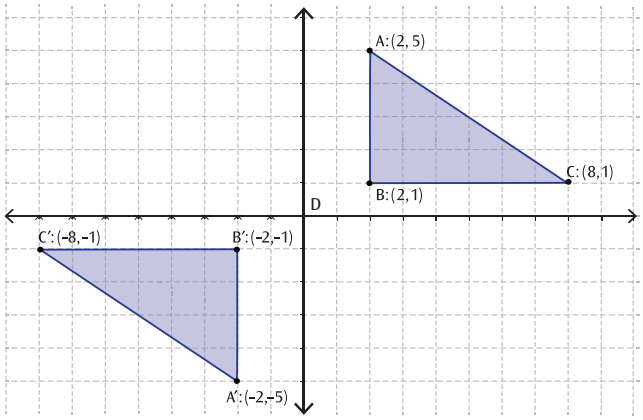
Standard **8.G.3▲** also calls for students to study dilations. A *dilation* with scale factor $k > 0$ can be thought of as a stretching (if $k > 1$) or shrinking (if $k < 1$) of an object. In a dilation, a point is specified from which the distance to the points of a figure is multiplied to obtain new points, and hence a new figure.

Examples of Four Geometric Transformations	8.G.2▲

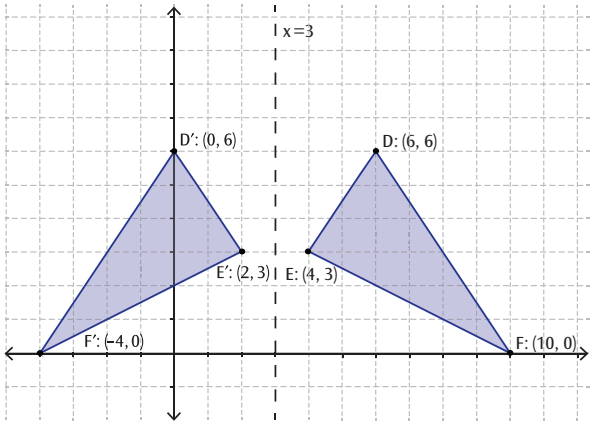
(Note that the original figure is called the *preimage*, and the new figure is called the *image*.)

Rotation: A figure can be rotated up to 360° about the center of rotation.

Consider when ΔABC is rotated 180° clockwise about the origin. The coordinates of ΔABC are $A(2,5)$, $B(2,1)$, and $C(8,1)$. When rotated 180°, the image triangle $\Delta A'B'C'$ has coordinates $A'(-2,-5)$, $B'(-2,-1)$, $C'(-8,-1)$. Each coordinate of the image is the opposite of its preimage point's coordinate.



Reflection: In the picture shown, ΔDEF has been reflected across the line $x = 3$. Notice the change in the orientation of the points, in the sense that the counterclockwise order of the preimage D–E–F is reversed in the image to D'–F'–E'. Notice also that each point on the image is at the same distance from the line of reflection as its corresponding point on the preimage.



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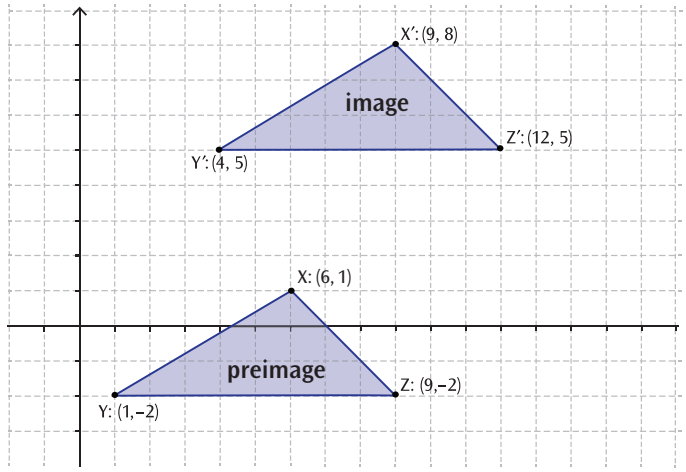
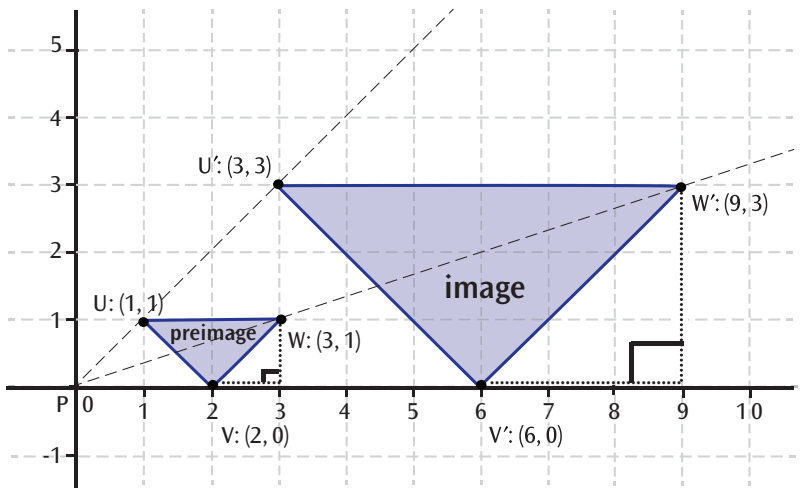
Example: 8.G.2▲ *(continued)*

Translation: Here, ΔXYZ has been translated 3 units to the right and 7 units up. Orientation is preserved. It is not too difficult to see that under this transformation, a preimage point (x, y) yields the image point $(x+3, y+7)$.



Dilation: In the picture, ΔUVW has been dilated from the origin $P: (0,0)$ by a factor of $k = 3$. The picture shows that the segments PU, PV, and PW have all been multiplied by the factor $k = 3$, which results in a new triangle, $\Delta U'V'W'$. By definition, ΔUVW and $\Delta U'V'W'$ are similar triangles. Students should experiment and find that the ratios of corresponding side lengths satisfy $\frac{U'V'}{UV} = \frac{V'W'}{VW} = \frac{U'W'}{UW} = 3$,



which corresponds to k. Students can apply the Pythagorean Theorem (**8.G.7–8▲**) to find the side lengths and justify this result. For example, they may find the lengths of VW and $V'W'$:

$$\overline{VW} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\text{and } \overline{V'W'} = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$$

Students can check informally that $\sqrt{18} = 3\sqrt{2}$, as formal work with radicals has not yet begun in grade eight.

Geometry 8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle–angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

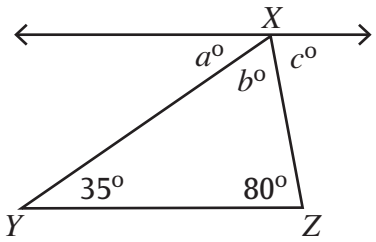
The definition of *similar shapes* is analogous to the new definition of *congruence*, but it has been refined to be more precise. Previously, shapes were said to be similar if they had the "same shape but not necessarily the same size." Now, two shapes are said to be similar if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations (**8.G.4▲**). By investigating dilations and using reasoning such as in the previous example, students learn that the following statements are true:

1. When two shapes are similar, the length of a segment AB in the first shape is multiplied by the scale factor k to give the length of the corresponding segment $A'B'$ in the second shape: $A'B' = k \bullet AB$.
2. Because the previous fact is true for all sides of a dilated shape, the ratio of the lengths of any two corresponding sides of the first and second shape is equal to k.
3. It is also true that the ratio of any two side lengths from the first shape is the same as the ratio of the corresponding side lengths from the second shape—for example, $\frac{AB}{BC} = \frac{A'B'}{B'C'}$. (Students can justify this algebraically, because fact 2 yields that $\frac{AB}{A'B'} = \frac{BC}{B'C'}$.)

Students use informal arguments to establish facts about the angle sum and exterior angles of triangles (e.g., consecutive exterior angles are supplementary), the angles created when parallel lines are cut by a transversal (e.g., corresponding angles are congruent), and the angle–angle criterion for similarity of triangles (if two angles of a triangle are congruent to two angles of another triangle, the two triangles are similar) [**8.G.5▲**]. When coupled with the previous three properties of similar shapes, the angle–angle criterion for triangle similarity allows students to justify the fact that the slope of a line is the same between any two points on the line (see discussion of standard **8.EE.6▲**).

Example: The sum of the measures of the angles of a triangle is 180°. **8.G.5▲**

In the figure shown, the line through point X is parallel to segment YZ.



We know that $a = 35$ because it is the measure of an angle that is alternating with $\angle Y$. For a similar reason, $c = 80$. Because all lines have an angle measure of 180°, we know that $a + b + c = 180$, which leads to $b = 180 - (35 + 80) = 65$. So the sum of the measures of the angles in this triangle is 180°.

Adapted from ADE 2010.

Geometry 8.G

Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

The Pythagorean Theorem is useful in practical problems, relates to grade-level work with irrational numbers, and plays an important role mathematically in coordinate geometry in higher mathematics. Students in grade eight explain a proof of the Pythagorean Theorem (**8.G.6▲**)—and there are many different and interesting proofs of this theorem.[5] In grade eight, students apply the Pythagorean Theorem to determine unknown side lengths in right triangles (**8.G.7▲**) and to find the distance between two points in a coordinate system (**8.G.8▲**). Application of the Pythagorean Theorem supports students' work in higher-level coordinate geometry.

Focus, Coherence, and Rigor

Understanding, modeling, and applying (**MP.4**) the Pythagorean Theorem and its converse require that students look for and make use of structure (**MP.7**) and express repeated reasoning (**MP.8**). Students also construct and critique arguments as they explain a proof of the Pythagorean Theorem and its converse to others (**MP.3**).

Adapted from Charles A. Dana Center 2012.

Geometry **8.G**

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

In grade seven, students learned about the area of a circle. Students in grade eight learn the formulas for calculating the volumes of cones, cylinders, and spheres and use the formulas to solve real-world and mathematical problems (**8.G.9**). When students learn to solve problems involving volumes of cones, cylinders, and spheres—together with their previous grade-seven work in angle measure, area, surface area, and volume—they acquire a well-developed set of geometric measurement skills. These skills, along with proportional reasoning and multi-step numerical problem solving, can be combined and used in flexible ways as part of mathematical modeling during high school and in college and careers (adapted from PARCC 2012).

Domain: Statistics and Probability

Building on work in earlier grades with univariate measurement data and analyzing data on line plots and histograms, grade-eight students begin to work with bivariate measurement data and use scatter plots to represent and analyze the data.

Bivariate measurement data represent two separate (but usually related) measurements. Scatter plots can show the relationship between the two measured variables. Collecting and analyzing bivariate measurement data help students to answer questions such as "How does more time spent on homework affect test grades?" and "What is the relationship between annual income and the number of years of formal education a person has?"

5. One example is the geometric "Proof without Words" of the Pythagorean Theorem available at http://illuminations.nctm.org/activitydetail.aspx?id=30 (NCTM Illuminations 2013a).

Statistics and Probabllity 8.SP

Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and non-linear association.
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Students in grade eight construct and interpret scatter plots to investigate patterns of association between two quantities (**8.SP.1**). They also build on their previous knowledge of scatter plots to examine relationships between variables. Grade-eight students analyze scatter plots to determine positive and negative associations, the degree of association, and type of association. Additionally, they examine outliers to determine if data points are valid or represent a recording or measurement error.

Example: Creating Scatter Plots **8.SP.1**

Customer satisfaction is vital to the success of fast-food restaurants, and speed of service is a key component of that satisfaction. In order to determine the best staffing level, the owners of a local fast-food restaurant have collected the data below showing the number of staff members and the average time for filling an order. Describe the association between the number of staff and the average time for filling an order, and make a recommendation as to how many staff should be hired.

Number of staff members	3	4	5	6	7	8
Average time to fill order (seconds)	180	138	120	108	96	84

Students can use tools such as those offered by the National Center for Education Statistics (http://nces.ed.gov/nceskids/createagraph/default.aspx [National Center for Education Statistics 2013]) to create a graph or generate data sets.

Grade-eight students know that straight lines are widely used to model relationships between two quantitative variables (**8.SP.2**). For scatter plots that appear to show a linear association, students informally fit a line (e.g., by drawing a line on the coordinate plane between data points) and informally assess the fit by judging the closeness of the data points to the straight line.

Example: Informally Determining a Line of Best Fit **8.SP.2**

The capacity of the fuel tank in a car is 13.5 gallons. The table below shows the number of miles traveled and the amount of gasoline used (in gallons). Describe the relationship between the variables. If the data are linear, determine a line of best fit. Do you think the line represents a good fit for the data set? Why or why not? What is the average fuel efficiency of the car in miles per gallon?

Miles traveled	0	75	120	160	250	300
Amount of gasoline used (gallons)	0	2.3	4.5	5.7	9.7	10.7

Students in grade eight solve problems in the context of bivariate measurement data by using the equation of a linear model (**8.SP.3**). They interpret the slope and the y-intercept in the context of the problem. For example, in a linear model for a biology experiment, students interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 centimeters in the height of the plant.

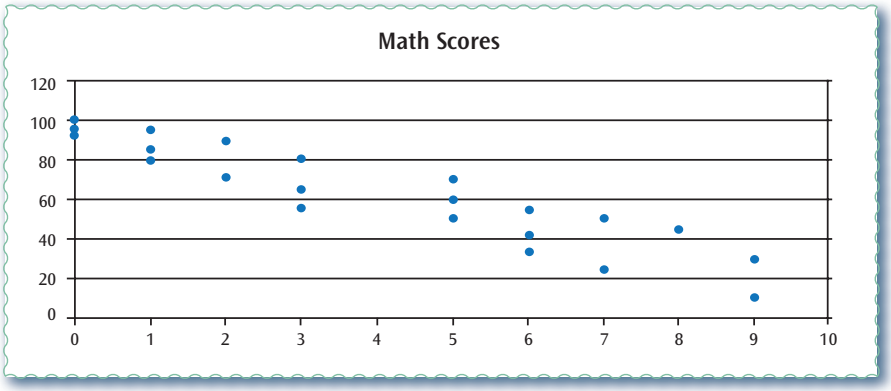
Example: Finding a Linear Model for a Data Set **8.SP.3**

Make a scatter plot by using data from students' math scores and absences. Informally fit a line to the graph and determine an approximate linear function that models the data. What would you expect to be the score of a student with 4 absences?

Solution:

Absences	3	5	1	1	3	6	5	3	0	7	8
Math scores	65	50	95	85	80	34	70	56	100	24	45
Absences	2	9	0	6	6	2	0	5	7	9	1
Math scores	71	30	95	55	42	90	92	60	50	10	80

Students would most likely use simple data software to make a scatter plot, finding a graph that looks like the following:



Students can use graphing software to find a line of best fit. Such a line might be $y = -8x + 95$. They interpret this equation as defining a function that gives the approximate score of a student based on the number of his or her absences. Thus, a student with 4 absences should have a score of approximately $y = -8(4) + 95 = 63$.

Adapted from CDE 2012d, ADE 2010, and NCDPI 2013b.

Focus, Coherence, and Rigor

Students in grade eight apply their experience with coordinate geometry and linear functions to plot bivariate data as points on a plane and to make use of the equation of a line in analyzing the relationship between two paired variables. Students develop mathematical practices as they build statistical models to explore the relationship between two variables (**MP.4**) and look for and make use of structure to describe possible associations in bivariate data (**MP.7**).

Adapted from UA Progressions Documents 2011e.

Students learn to see patterns of association in bivariate categorical data in a two-way table (**8.SP.4**). They construct and interpret a two-way table that summarizes data on two categorical variables collected from the same subjects. The two-way table displays frequencies and relative frequencies. Students use relative frequencies calculated from rows or columns to describe a possible association between the two variables. For example, students collect data from their classmates about whether they have a curfew and whether they do chores at home. The two-way table allows students to easily see if students who have a curfew also tend to do chores at home.

Example: Two-Way Tables for Categorical Data **8.SP.4**

The table at right illustrates the results when 100 students were asked these survey questions: (1) Do you have a curfew? (2) Do you have assigned chores? Students can examine the survey results to determine if there is evidence that those who have a curfew also tend to have chores.

		Curfew	
Chores		40	10
		10	40

Solution: Of the students who answered that they had a curfew, 40 had chores and 10 did not. Of the students who answered they did not have a curfew, 10 had chores and 40 did not. From this sample, there seems to be a positive correlation between having a curfew and having chores: it appears that most students with chores have a curfew and most students without chores do not have a curfew.

Adapted from CDE 2012d, ADE 2010, and NCDPI 2013b.

Focus, Coherence, and Rigor

Work in the Statistics and Probability cluster "Investigate patterns of association in bivariate data" involves looking for patterns in scatter plots and using linear models to describe data. This is directly connected to major work in the Expressions and Equations clusters (**8.EE.1–8▲**) and provides opportunities for students to model with mathematics (**MP.4**).

A detailed discussion of statistics and probability is provided online at https://commoncoretools.files.wordpress.com/2011/12/ccss_progression_sp_68_2011_12_26_bis.pdf (UA Progressions Documents 2011e).

Essential Learning for the Next Grade

In grades six through eight, multiplication and division develop into powerful forms of ratio and proportional reasoning. The properties of operations take on prominence as arithmetic matures into algebra. The theme of quantitative relationships also becomes explicit in grades six through eight, developing into the formal notion of a function by grade eight. Meanwhile, the foundations for later courses in deductive geometry are laid in grades six through eight. The gradual development of data representations in kindergarten through grade five leads to statistics in middle school: the study of shape, center, and spread of data distributions; possible associations between two variables; and the use of sampling in making statistical decisions.

In higher mathematics courses, algebra, functions, geometry, and statistics develop with an emphasis on modeling. Students continue to take a thinking approach to algebra, learning to see and make use of structure in algebraic expressions of growing complexity (adapted from PARCC 2012). To be prepared for courses in higher mathematics, students should be able to demonstrate that they have acquired particular mathematical concepts and procedural skills by the end of grade eight. Prior to grade eight, some standards identify fluency for the grade level, but there are no explicit grade-level fluency expectations for grades seven and beyond. In grade eight, linear algebra is an instructional focus, and although the grade-eight standards do not specifically identify fluency expectations, students in grade eight who can fluently solve linear equations (**8.EE.7▲**) and pairs of simultaneous linear equations (**8.EE.8▲**) will be better prepared to complete courses in higher mathematics. These fluencies and the conceptual understandings that support them are foundational for work in higher mathematics. Many students have worked informally with one-variable linear equations since kindergarten. This important line of development culminates in grade eight with the solution of general one-variable linear equations, including cases with an infinite number of solutions or no solutions, as well as cases requiring algebraic manipulation using properties of operations.

It is particularly important for students in grade eight to obtain skills and understandings to work with radical and integer exponents (**8.EE.1–4▲**); understand connections between proportional relationships, lines, and linear equations (**8.EE.5–6▲**); analyze and solve linear equations and pairs of simultaneous linear equations (**8.EE.7–8▲**); and define, evaluate, and compare functions (**8.F.1–3▲**). In addition, the skills and understandings to use functions to model relationships between quantities (**8.F.4–5**) will better prepare students to use mathematics to model real-world problems in higher mathematics.

Guidance on Course Placement and Sequences

The California Common Core State Standards for Mathematics (CA CCSSM) support a progression of learning. Many culminating standards that remain important far beyond the particular grade level appear in grades six through eight. As stated in the national Common Core State Standards Initiative documents, "some of the highest priority content for college and career readiness comes from [g]rades 6–8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume" (NGA/CCSSO 2010g).

The CA CCSSM for grades six through eight are comprehensive, rigorous, and non-redundant. Instruction in an accelerated sequence of courses will require compaction—not the former strategy of deletion. Therefore, careful consideration needs to be made before placing a student in higher-mathematics course work in grades six through eight. Acceleration may get students to advanced course work, but it may create gaps in students' mathematical background. Careful consideration and systematic collection of multiple measures of individual student performance on both the content and practice standards are required. For additional information and guidance on course placement, see appendix D (Course Placement and Sequences).

Grade 8 Overview

The Number System

- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

- Work with radicals and integer exponents.
- Understand the connection between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Geometry

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Statistics and Probability

- Investigate patterns of association in bivariate data.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

8 Grade 8

The Number System 8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Expressions and Equations 8.EE

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.*

2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*

4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.

   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8. Analyze and solve pairs of simultaneous linear equations.
   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
   b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x+2y=5$ *and* $3x+2y=6$ *have no solution because* $3x+2y$ *cannot simultaneously be 5 and 6.*
   c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Functions 8.F

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[6]
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
3. Interpret the equation $y=mx+b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A=s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4), and (3,9), which are not on a straight line.*

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or non-linear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry 8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
   a. Lines are taken to lines, and line segments to line segments of the same length.
   b. Angles are taken to angles of the same measure.
   c. Parallel lines are taken to parallel lines.

6. Function notation is not required in grade eight.

2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Statistics and Probability 8.SP

Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and non-linear association.
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

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