Kindergarten Chapter

of the

Mathematics Framework

for California Public Schools:
Kindergarten Through Grade Twelve

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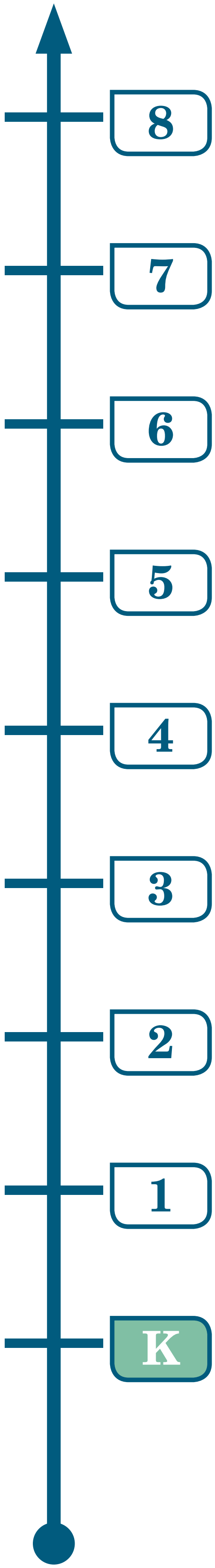


Kindergarten

Students in preschool and transitional kindergarten programs who have been exposed to important mathematical concepts—such as representing, relating, and operating on whole numbers and identifying and describing shapes—will be better prepared for kindergarten mathematics and for later learning.

Critical Areas of Instruction

In kindergarten, instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; and (2) describing shapes and space. More learning time in kindergarten should be devoted to numbers rather than to other topics (National Governors Association Center for Best Practices, Council of Chief State School Officers [NGA/CCSSO] 2010p). Kindergarten students also work toward fluency with addition and subtraction of whole numbers within 5.

Standards for Mathematical Content

The Standards for Mathematical Content emphasize key content, skills, and practices at each grade level and support three major principles:

- **Focus**—Instruction is focused on grade-level standards.
- **Coherence**—Instruction should be attentive to learning across grades and to linking major topics within grades.
- **Rigor**—Instruction should develop conceptual understanding, procedural skill and fluency, and application.

Grade-level examples of focus, coherence, and rigor are indicated throughout the chapter.

The standards do not give equal emphasis to all content for a particular grade level. Cluster headings can be viewed as the most effective way to communicate the **focus** and **coherence** of the standards. Some clusters of standards require a greater instructional emphasis than others based on the depth of the ideas, the time needed to master those clusters, and their importance to future mathematics or the later demands of preparing for college and careers.

Table K-1 highlights the content emphases at the cluster level for the kindergarten standards. Most of the instructional time should be spent on "Major" clusters and the standards within them, which are indicated throughout the text by a triangle symbol (▲). However, standards in the "Additional/Supporting" clusters should not be neglected; to do so would result in gaps in students' learning, including skills and understandings they may need in later grades. Instruction should reinforce topics in major clusters by using topics in the additional/supporting clusters and including problems and activities that support natural connections between clusters.

Teachers and administrators alike should note that the standards are not topics to be checked off after being covered in isolated units of instruction; rather, they provide content to be developed throughout the school year through rich instructional experiences presented in a coherent manner (adapted from Partnership for Assessment of Readiness for College and Careers [PARCC] 2012).

Table K-1. Kindergarten Cluster-Level Emphases

Counting and Cardinality **K.CC**

Major Clusters

- Know number names and the count sequence. (**K.CC.1–3 ▲**)
- Count to tell the number of objects. (**K.CC.4–5 ▲**)
- Compare numbers. (**K.CC.6–7 ▲**)

Operations and Algebraic Thinking **K.OA**

Major Clusters

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. (**K.OA.1–5 ▲**)

Number and Operations in Base Ten **K.NBT**

Major Clusters

- Work with numbers 11–19 to gain foundations for place value. (**K.NBT.1 ▲**)

Measurement and Data **K.MD**

Additional/Supporting Clusters

- Describe and compare measurable attributes. (**K.MD.1–2**)
- Classify objects and count the number of objects in categories. (**K.MD.3**)

Geometry **K.G**

Additional/Supporting Clusters

- Identify and describe shapes. (**K.G.1–3**)
- Analyze, compare, create, and compose shapes. (**K.G.4–6**)

Explanations of Major and Additional/Supporting Cluster-Level Emphases

Major Clusters (▲) — Areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than others based on the depth of the ideas, the time needed to master them, and their importance to future mathematics or the demands of college and career readiness.

Additional Clusters — Expose students to other subjects; may not connect tightly or explicitly to the major work of the grade.

Supporting Clusters — Designed to support and strengthen areas of major emphasis.

Note of caution: Neglecting material, whether it is found in the major or additional/supporting clusters, will leave gaps in students' skills and understanding and will leave students unprepared for the challenges they face in later grades.

Adapted from Achieve the Core 2012.

Connecting Mathematical Practices and Content

The Standards for Mathematical Practice (MP) are developed throughout each grade and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject. The MP standards represent a picture of what it looks like for students to understand and do mathematics in the classroom and should be integrated into every mathematics lesson for all students.

Although the description of the MP standards remains the same at all grade levels, the way these standards look as students engage with and master new and more advanced mathematical ideas does change. Table K-2 presents examples of how the MP standards may be integrated into tasks appropriate for students in kindergarten. (Refer to the Overview of the Standards Chapters for a description of the MP standards.)

Table K-2. Standards for Mathematical Practice—Explanation and Examples for Kindergarten

Standards for Mathematical Practice	Explanation and Examples
MP.1 Make sense of problems and persevere in solving them.	In kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Real-life experiences should be used to support students' ability to connect mathematics to the world. To help students connect the language of mathematics to everyday life, ask students questions such as "How many students are absent?" or have them gather enough blocks for the students at their table. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?", or they may try another strategy.
MP.2 Reason abstractly and quantitatively.	Younger students begin to recognize that a number represents a specific quantity and connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. For example, a student may write the numeral 11 to represent an amount of objects counted, select the correct number card 17 to follow 16 on a calendar, or build two piles of counters to compare the numbers 5 and 8. In addition, kindergarten students begin to draw pictures, manipulate objects, or use diagrams or charts to express quantitative ideas. Students need to be encouraged to answer questions such as "How do you know?"—which reinforces their reasoning and understanding and helps student develop mathematical language.
MP.3 Construct viable arguments and critique the reasoning of others.	Younger students construct arguments using actions and concrete materials, such as objects, pictures, and drawings. They begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions such as "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking. They begin to develop the ability to reason and analyze situations as they consider questions such as "Are you sure that ________?", "Do you think that would happen all the time?", and "I wonder why ________?"

Table K-2 *(continued)*

Standards for Mathematical Practice	Explanation and Examples
MP.4 Model with mathematics.	In early grades, students begin to represent problem situations in multiple ways—by using numbers, objects, words, or mathematical language, acting out the situation, making a chart or list, drawing pictures, creating equations, and so forth. For example, a student may use cubes or tiles to show the different number pairs for 5, or place three objects on a 10-frame and then determine how many more are needed to "make a ten." Students rely on manipulatives (or other visual and concrete representations) while solving tasks and record an answer with a drawing or equation.
MP.5 Use appropriate tools strategically.	Younger students begin to consider tools available to them when solving a mathematical problem and decide when certain tools might be helpful. For instance, kindergartners may decide to use linking cubes to represent two quantities and then compare the two representations side by side, or later, make math drawings of the quantities. Students decide which tools may be helpful to use depending on the problem or task and explain why they use particular mathematical tools.
MP.6 Attend to precision.	Kindergarten students begin to develop precise communication skills, calculations, and measurements. Students describe their own actions, strategies, and reasoning using grade-level-appropriate vocabulary. Opportunities to work with pictorial representations and concrete objects can help students develop understanding and descriptive vocabulary. For example, students analyze and compare two- and three-dimensional shapes and sort objects based on appearance. While measuring objects iteratively (repetitively), students check to make sure that there are no gaps or overlaps. During tasks involving number sense, students check their work to ensure the accuracy and reasonableness of solutions. Students should be encouraged to answer questions such as, "How do you know your answer is reasonable?"
MP.7 Look for and make use of structure.	Younger students begin to discern a pattern or structure in the number system. For instance, students recognize that $3 + 2 = 5$ and $2 + 3 = 5$. Students use counting strategies, such as counting on, counting all, or taking away, to build fluency with facts to 5. Students notice the written pattern in the "teen" numbers—that the numbers start with 1 (representing 1 ten) and end with the number of additional ones. Teachers might ask, "What do you notice when ________?"
MP.8 Look for and express regularity in repeated reasoning.	In the early grades, students notice repetitive actions in counting, computations, and mathematical tasks. For example, the next number in a counting sequence is 1 more when counting by ones and 10 more when counting by tens (or 1 more group of 10). Students should be encouraged to answer questions such as, "What would happen if ________?" and "There are 8 crayons in the box. Some are red and some are blue. How many of each could there be?" Kindergarten students realize 8 crayons could include 4 of each color ($8 = 4 + 4$), 5 of one color and 3 of another ($8 = 5 + 3$), and so on. For each solution, students repeatedly engage in the process of finding two numbers to join together to equal 8.

Adapted from Arizona Department of Education (ADE) 2010 and North Carolina Department of Public Instruction (NCDPI) 2013b.

Standards-Based Learning at Kindergarten

The following narrative is organized by the domains in the Standards for Mathematical Content. It highlights some necessary foundational skills and provides exemplars to explain the content standards, highlight connections to the various Standards for Mathematical Practice (**MP**), and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. A triangle symbol (▲) indicates standards in the major clusters (see table K-1).

Domain: Counting and Cardinality

A critical area of instruction in kindergarten is representing, relating, and operating on whole numbers, initially with sets of objects.

Counting and Cardinality	K.CC
Know number names and the count sequence.	
1. Count to 100 by ones and by tens.	
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).	

Several learning progressions originate in knowing number names and the count sequence. One of the first major concepts in a student's mathematical development is *cardinality*. Cardinality can be explained as knowing that the number word spoken tells the quantity and that the number on which a person ends when counting represents the entire amount counted. The idea is that numbers mean *amount,* and no matter how you arrange and rearrange the items, the amount is the same. Students can generally say the counting words up to a given number before they can use these numbers to count objects or to tell the number of objects (adapted from the University of Arizona [UA] Progressions Documents for the Common Core Math Standards 2011a and Georgia Department of Education [GaDOE] 2011).

Kindergarten students are introduced to the counting sequence (**K.CC.1–2▲**). When counting orally by ones, students begin to understand that the next number in the sequence is one more. Similarly, when counting by tens, the next number in the sequence is "10 more."

Examples: Counting Sequences for Forward Counting to 100 by Ones	K.CC.1
• The "ones" (1–10) • The "teens" (10, 11, 12, 13, 14, 15, 16, 17, 18, 19) • "Crossing the decade" (15, 16, 17, 18, 19, 20, 21, 22, 23, 24, or, similarly, 26–34, 35–44, and so forth) Students often have trouble with counting forward sequences that cross the decade. Focusing on short counting sequences may be helpful.	

Adapted from Kansas Association of Teachers of Mathematics (KATM) 2012, Kindergarten Flipbook.

Initially, students might think of counting as a string of words, but gradually they transition to using counting as a tool to describe amounts in their world. Counting can be reinforced throughout the school day.

Examples	K.CC.1

- Count the number of chairs of students who are absent.
- Count the number of stairs, shoes, and so on.
- Count groups of 10, such as “fingers in the classroom” (10 fingers per student). (**MP.6, MP.7, MP.8**)

Kindergarten students also count forward—beginning from a given number—instead of starting at 1. Counting forward (or “counting on”) may be confusing for young students, because it conflicts with the initial strategy they learned about counting from the beginning. Activities or games that require students to add on to a previous count to reach a targeted number may encourage development of this concept (adapted from KATM 2012, Kindergarten Flipbook).

Kindergarten students learn to write numbers from 0 to 20 (**K.CC.3▲**) and represent a number of objects with a written numeral in the 0–20 range (using numerals as symbols for quantities). They understand that 0 represents a count of no objects. Students need multiple opportunities to count objects and recognize that a number represents a specific quantity. As this understanding develops, students begin to read and write numerals. The emphasis should first be on quantity and then on connecting quantities to the written symbols.

Example: A Learning Sequence for Understanding Numbers

A specific learning sequence might consist of these steps:

1. Count up to 20 objects in many settings and situations over several weeks.
2. Start to recognize, identify, and read the written numerals, and match the numerals to given sets of objects.
3. Write the numerals to represent counted objects.

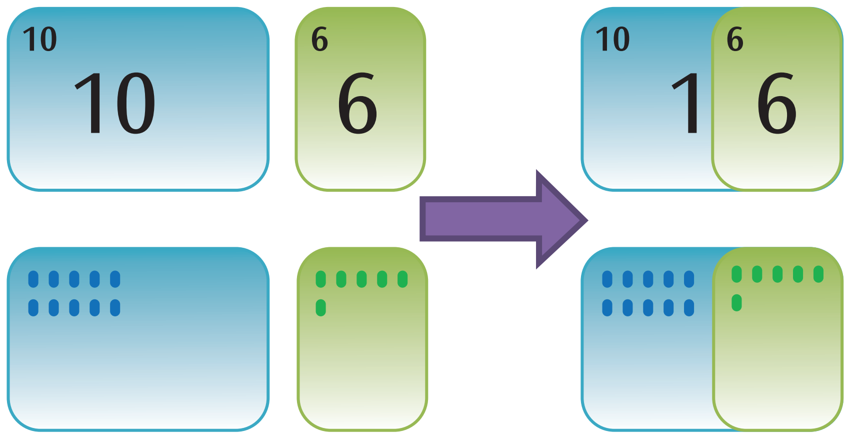
Adapted from ADE 2010.

As students connect quantities and written numerals, they also develop mathematical practices such as reasoning abstractly and quantitatively (**MP.2**). They use precise vocabulary to express how they know that their count is accurate (**MP.6**). They also use the structures and patterns of the number system and apply this understanding to counting (**MP.7, MP.8**) [adapted from ADE 2010].

Common Misconceptions

- Some students might not see zero (0) as a number. Ask students to write 0 and say “zero” to represent the number of items left when all items have been taken away. Avoid using the word *none* to represent this situation.
- Teen numbers can also be confusing for young students. To help avoid confusion, these numbers should be taught as a bundle of 10 ones and some extra ones. This approach supports a foundation for understanding both the place-value concept and symbols that represent each teen number. Layered place-value cards may help students understand the difficult teen numbers; see figure K-1.

Figure K-1. Layering Place-Value Cards to Illustrate Teen Numbers



Adapted from KATM 2012, Kindergarten Flipbook.

Counting and Cardinality	K.CC

Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
    a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
    b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
    c. Understand that each successive number name refers to a quantity that is one larger.
5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

In kindergarten, students develop an understanding of the relationship between numbers and quantities and connect counting to cardinality (**K.CC.4**). Learning to count is a complex mental and physical activity that requires staying connected to the objects that are being counted. Children must understand that the count sequence has meaning when counting objects: that the last count word indicates the amount or the cardinality of the set (Van de Walle 2007). Kindergarten students use their understanding of the relationship between numbers and quantities to count a set of objects and see sets and numerals in relationship to one another, rather than as isolated concepts.

There are numerous opportunities for students to manipulate concrete objects or visual representations (e.g., dot cards, 10-frames) and connect number names with their quantities, which can help students master the concept of counting (adapted from NCDPI 2013b).

As students learn to count a group of objects, they pair each word said with one object (**K.CC.4a▲**). This is usually facilitated by an indicating act (such as touching, pointing to, or moving objects) that keeps each word said paired to only one object (the one-to-one-correspondence principle). Students learn that the last number named tells the number of objects counted (the cardinality principle) and that the number of objects is the same regardless of their arrangement or the order in which they were counted (the order-irrelevance principle). They also understand that each successive number name refers to a quantity that is 1 larger (**K.CC.4.b–c▲**) [adapted from UA Progressions Documents 2011a].

To develop their understanding of the relationship between numbers and quantities, students might count objects, placing one more object in the group at a time.

Example	**K.CC.4**
Using cubes, students count an existing group and then place another cube in the set to continue counting. Students continue placing one more cube in the set at a time and then identify the new total number of cubes. Students see that the counting sequence results in a quantity that increases by one each time another cube is placed in the group. Students may need to recount from one, but the goal is for students to count on from the existing number of cubes—a conceptual start for the grade-one skill of counting to 120, starting at any number less than 120.	

To count accurately, students rely on:

- knowing patterns and arbitrary parts of the number–word sequence;
- assigning one number word to one object (one-to-one correspondence);
- keeping track of objects that have already been counted (adapted from ADE 2010 and GaDOE 2011).

Five Major Principles: Development of Students' Understanding of How to Count and What to Count

1. **One-to-One-Correspondence Principle.** Students assign one, and only one, distinct counting word to each of the items to be counted. To follow this principle, students partition and re-partition the collection of objects to be counted into two categories: those that have been allocated a number name and those that have not. Students model numbers with objects, and each object is assigned a unique number name based on one-to-one correspondence between each object and the number name. If an item is not assigned a number name or is assigned more than one number name, the resulting count will be incorrect; refer to standard **K.CC.4a.**
2. **Standard-Order (of Number Names) Principle.** Students recite a number-name list in a fixed order (e.g., students count "One, two, three" for a collection of three objects). In other words, students can rote-count; refer to standard **K.CC.4a.**
3. **Cardinal Principle.** Students understand that the last number name used for the final object in a collection represents the number of items in that collection. This rule connects counting with "how many"; refer to standard **K.CC.4b.**
4. **Order-Irrelevance Principle.** Students understand that the order in which objects are counted has no effect on the total number of objects and that the quantity of a group of objects remains constant even when the objects are rearranged; refer to standard **K.CC.4b.**
5. **Abstraction Principle.** Students realize that the above four principles of counting apply to any collection of objects, whether tangible (e.g., marbles or blocks) or not (e.g., sounds or actions). They also realize that objects may have similar attributes (e.g., "All of these marbles are yellow") or different attributes (e.g., "These toys are different types and sizes"); refer to standard **K.CC.4.**

Adapted from Thompson 2010.

Students answer questions such as “How many are there?” by counting objects in a set and understanding that the last number stated represents the total amount of objects (cardinality, **K.CC.5▲**). Over time, students realize that the same set counted several different times will be the same amount each time. Counting objects arranged in a line is easiest; with more practice, students learn to count objects in more difficult arrangements, such as rectangular arrays, circles, and scattered configurations.

Scattered arrangements are the most challenging for students, and therefore kindergarten students count only up to 10 objects if arranged this way. Given a number from 1 to 20, kindergarten students also count out that many objects. This is also more difficult for students than simply counting the total number of objects, because as students count, they need to remember the number of objects to be counted out (adapted from UA Progressions Documents 2011a and NCDPI 2013b).

Examples of Counting Strategies	**K.CC.4.a–b**

There are numerous counting strategies that students may use, depending on how objects are arranged. Here are a few examples:

- Move objects as each object is counted.
- Line up objects to count.
- Touch objects in a scattered arrangement as each object is counted.
- Count objects in a scattered arrangement by visually scanning each object without touching.

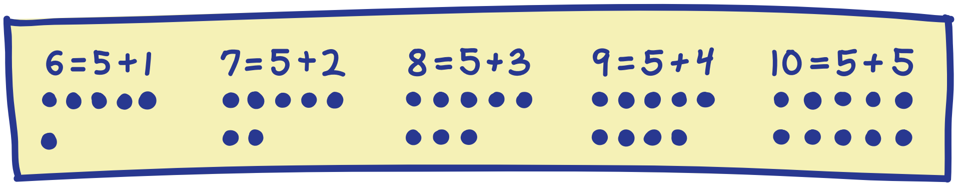
Adapted from KATM 2012, Kindergarten Flipbook.

Focus, Coherence, and Rigor

As students use various counting strategies when they participate in counting activities, they reinforce their understanding of the relationship between numbers and quantities and support mathematical practices such as modeling with mathematics (**MP.4**), the use of precise language (**MP.6**), and repeated reasoning to find a solution (**MP.8**).

Students come to quickly perceive the number of items in small groups—such as recognizing dot arrangements in different patterns without counting the objects. This is known as *perceptual subitizing,* a fundamental skill in the development of students’ understanding of numbers. Perceptual subitizing develops into *conceptual subitizing*—recognizing a collection of objects as a composite of subparts and as a whole (e.g., seeing a five-dot domino and thinking 1 and 4 or seeing a set with two subsets of 2 and saying 4) [adapted from UA Progressions Documents 2011a]. Particularly important is the $5 + n$ pattern, in which one row of 5 circles has 1, 2, 3, 4, or 5 dots below to show 6, 7, 8, 9, and 10; see figure K-2. These rows are separated more than the individual dots to ensure students see the group of 5 and the extra dots.

Figure K-2. The 5+n pattern



Source: UA Progressions Documents 2011a.

Subitizing supports the development of addition and subtraction strategies, such as counting on and composing and decomposing numbers. Students need practice to develop competency in perceptual subitizing.

Example	**K.CC.5**
The teacher might place different amounts of beans on a mat (beginning with amounts of 4 or fewer) and then ask students to say how many beans they see. As students become proficient, dot cards can also be utilized to develop fluency. For example, the teacher can show a large dot card to students, and students then take the number counters they think they need to cover the dots on the card. Then one child places his or her counters on the dots while the rest of the class counts and checks. Eventually, the teacher briefly shows one large dot card and puts it down quickly. Then students try to recognize the number of dots without counting.	

Counting and Cardinality — K.CC

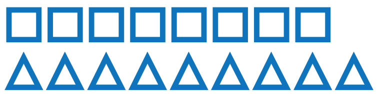
Compare numbers.

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]
7. Compare two numbers between 1 and 10 presented as written numerals.

In kindergarten, students compare the number of objects in one group (with up to 10 objects) to the number of objects in another group (**K.CC.6▲**). Students need a strong sense of the relationship between quantities and numerals to accurately compare groups and answer related questions. They may use matching strategies or counting strategies to determine whether one group is greater than, less than, or equal to the number of objects in another group.

1. Includes groups with up to 10 objects.

Example: More Triangles or More Squares? **K.CC.6▲ (MP.1, MP.2)**

Student 1	Student 2	Student 3
I lined up 1 square with 1 triangle. Since there is 1 extra triangle, there are more triangles than squares. 	I counted the squares and got 8. Then I counted the triangles and got 9. Since 9 is bigger than 8, there are more triangles than squares.	I put them in a pile. I then took away objects. Every time I took a square, I also took a triangle. When I had taken almost all of the shapes away, there was still a triangle left. That means that there are more triangles than squares.

Adapted from KATM 2012, Kindergarten Flipbook.

Matching and Counting Strategies for Comparing Groups of Objects

- **Matching.** Students use one-to-one correspondence, repeatedly matching one object from one set with one object from the other set to determine which set has more objects.
- **Counting.** Students count the objects in each set and then identify which set has more, less, or an equal number of objects.
- **Observation.** Students may use observation to compare two quantities. For example, by looking at two sets of objects, they may be able to tell which set has more or less without counting.
- **Benchmark Numbers.** Introduce the use of 0, 5, and 10 as benchmark numbers to help students further develop their sense of quantity as well as their ability to compare numbers. Benchmarks of 5 and 10 are especially useful with the $5 + n$ patterns.

Adapted from KATM 2012, Kindergarten Flipbook.

An important level of understanding is reached when students can compare two numbers from 1 to 10 represented as written numerals, without counting (**K.CC.7▲**). Students demonstrate their understanding of numbers when they can justify their answers (**MP.3**).

Example **K.CC.7▲**

When a student gives an answer, the teacher may ask a probing question such as “How do you know?” to elicit student thinking and reasoning (**MP.3, MP.8**). Students might justify their answer (e.g., 7 is greater than 5) by demonstrating a one-to-one match, counting again, or using similar approaches that help to explain or verify the answer (adapted from KATM 2012, Kindergarten Flipbook).

Focus, Coherence, and Rigor

Comparing numbers and groups in kindergarten will progress to comparing addition and subtraction situations in grade one. For example, "Which is more?" or "Which is less?" will progress to "How many more?" or "How many less?"

Domain: Operations and Algebraic Thinking

Kindergarten students are introduced to addition and subtraction with small numbers, and they work toward fluency with these operations for numbers within 5.

Operations and Algebraic Thinking	K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Represent addition and subtraction with objects, fingers, mental images, drawings,[2] sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$).
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
5. Fluently add and subtract within 5.

Kindergarten students develop their understanding of addition and subtraction by making sense of word problems (**MP.1, MP.2**). Students experience a variety of addition situations that involve *putting together* and *adding to* and a variety of subtraction situations that involve *taking apart* and *taking from* (**K.OA.1–2▲**). Students use objects (such as two-color counters, clothespins on hangers, connecting cubes, 5-frames, and stickers), fingers, mental images, sounds, drawings, verbal explanations and acting out the situation to represent these operations (**MP. 1, MP.2, MP.4, MP.5**) [adapted from KATM 2012, Kindergarten Flipbook].

Students use both mathematical and non-mathematical language to explain their interpretation of a problem and the solution. Initially, students work with numbers within 5, which helps them move from perceptual subitizing to conceptual subitizing, in which they say the addends and the total (e.g., 2 and 1 make 3). Students will generally use fingers to keep track of addends and parts of addends and should develop rapid visual and kinesthetic recognition of numbers up to 5 on their fingers. Eventually, students will expand their work in addition and subtraction from within 5 to within 10.

2. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Students are introduced to expressions and equations using appropriate symbols, including +, –, and =. Teachers may write expressions (e.g., $3 - 1$) or equations (e.g., $3 - 1 = \square$, or $3 = 1 + 2$) that represent operations and problems with real-world contexts to reinforce students' understanding of these concepts. Teachers should emphasize that an equal sign (=) means "is the same as." Students should see these equations and be encouraged to write them; however, they are not required to write equations. In kindergarten, the use of formal vocabulary for both addition and subtraction (such as *minuend, subtrahend,* and *addend*) is not necessary. For English learners, phonologically identical words (e.g., *sum* and *some, whole* and *hole*) may be challenging; thus it is better to use the word *total* instead of *sum* for all students in kindergarten and grade one. Using the word *partners* instead of *addends* is also a helpful conceptual support for children in these grades. To support English learners, these words should be explicitly taught as they are introduced (adapted from UA Progressions Documents 2011a). For more information, refer to the Universal Access chapter.

Focus, Coherence, and Rigor

When students represent addition and subtraction, this also supports mathematical practices as they use objects or pictures to represent quantities (**K.OA.1▲**), reason quantitatively to make sense of quantities and develop a clear representation of the problem (**MP.2**), mathematize a real-world situation (**MP.4**), and use tools appropriately to model the problem (**MP.5**). Math drawings also facilitate student reflection and discussion and help young students justify answers (**MP.3**).

Word problems with real-life applications provide students with a context to develop their understanding of addition and subtraction (**K.OA.2▲**). Kindergarten students learn that addition is *putting together* and *adding to* and subtraction is *taking apart* and *taking from*. Kindergartners use objects or math drawings (with simple shapes such as circles) to model word problems (adapted from ADE 2010).

The most common types of addition and subtraction problems for kindergarten students are displayed with dark shading in table K-3. Students add and subtract within 10 to solve these types of problems.

Table K-3. Types of Addition and Subtraction Problems (Kindergarten)

Type of Problem	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = \square$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were 5 bunnies. How many bunnies hopped over to the first two? $2 + \square = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were 5 bunnies. How many bunnies were on the grass before? $\square + 3 = 5$
Take from	Five apples were on the table. I ate 2 apples. How many apples are on the table now? $5 - 2 = \square$	Five apples were on the table. I ate some apples. Then there were 3 apples. How many apples did I eat? $5 - \square = 3$	Some apples were on the table. I ate 2 apples. Then there were 3 apples. How many apples were on the table before? $\square - 2 = 3$

	Total Unknown	Addend Unknown	Both Addends Unknown
Put together/ Take apart	Three red apples and 2 green apples are on the table. How many apples are on the table? $3 + 2 = \square$	Five apples are on the table. Three are red, and the rest are green. How many apples are green? $3 + \square = 5$, $5 - 3 = \square$	Grandma has 5 flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	("How many more?" version): Lucy has 2 apples. Julie has 5 apples. How many more apples does Julie have than Lucy?	(Version with *more*): Julie has 3 more apples than Lucy. Lucy has 2 apples. How many apples does Julie have?	(Version with *more*): Julie has 3 more apples than Lucy. Julie has 5 apples. How many apples does Lucy have?
	("How many fewer?" version): Lucy has 2 apples. Julie has 5 apples. How many fewer apples does Lucy have than Julie? $2 + \square = 5$, $5 - 2 = \square$	(Version with *fewer*): Lucy has 3 fewer apples than Julie. Lucy has 2 apples. How many apples does Julie have? $2 + 3 = \square$, $3 + 2 = \square$	(Version with *fewer*): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = \square$, $\square + 3 = 5$

Note: Kindergarten students solve problem types with the darkest shading; students in grades one and two solve problems of all subtypes. Unshaded problems are the most difficult; first-grade students work with these problems but do not master them until grade two (adapted from NGA/CCSSO 2010d and UA Progressions Documents 2011a).

To solve word problems, students learn to apply various computational methods. Kindergarten students generally use Level 1 methods, moving on to Level 2 and Level 3 methods in later grades. The three levels are summarized in table K-4 and explained more thoroughly in appendix C.

Table K-4. Methods Used for Solving Single-Digit Addition and Subtraction Problems

Level 1: Direct Modeling by Counting All or Taking Away
Represent the situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend.

Level 2: Counting On
Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total, but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. The count is tracked and monitored in some way (e.g., with fingers, objects, mental images of objects, body motions, or other count words).

For addition, the count is stopped when the amount of the remaining addend has been counted. The last number word is the total. For subtraction, the count is stopped when the total occurs in the count. The tracking method indicates the difference (seen as the unknown addend).

Level 3: Converting to an Easier Equivalent Problem
Decompose an addend and compose a part with another addend.

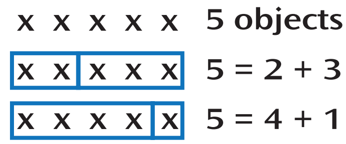
Adapted from UA Progressions Documents 2011a.

Students learn that a set of objects may be broken into two sets in multiple ways. For example, a set of 5 objects may be separated into two sets—3 and 2 or 4 and 1 (**K.OA.3▲**). Thus, when breaking apart a set (decomposing), students develop the understanding that a smaller set of objects exists within that larger set. Students should have numerous experiences with decomposing sets of objects and recording with pictures and numbers, and the teacher should make connections between the drawings and symbols ($5 = 4 + 1$, $5 = 3 + 2$, $5 = 2 + 3$, $5 = 1 + 4$, and $5 = 5 + 0$), showing the total on the left and the two addends on the right. Students can find patterns in all of the decompositions of a given number and eventually summarize these patterns for several numbers. Experience with decomposing also emphasizes that the equal sign (=) means "is the same as."

Students may use objects such as cubes, two-color counters, or square tiles to show different number pairs for a given number. For example, for the number 5, students may split a set of 5 objects into 1 and 4, 2 and 3, and 5 and 0. Students may also use drawings to show different number pairs for a given number (**MP.1, MP.2, MP.4**).

Example: Decomposing 5 K.OA.3▲

Students may draw 5 objects, showing how to decompose in several ways.



They may write equations involving 5 and its decompositions, such as:

5 = 4 + 1
3 + 2 = 5
2 + 3 = 4 + 1

Students can systematically list all the possible number partners for a given number. For example, they may list all number partners for 5 (0 + 5, 1 + 4, 2 + 3, 3 + 2, 4 + 1, and 5 + 0) and describe the pattern in the addends—that is, each number is one less or one more than the previous addend.

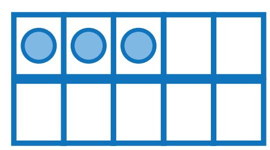
Adapted from KATM 2012, Kindergarten Flipbook.

Working with equations with one number on the left and an operation on the right (e.g., 5 = 2 + 3) to record groups of 5 things decomposed as groups of 2 and 3 things (**K.OA.3▲**) helps students to understand that equations indicate quantities on both sides of the equal sign have the same value (**MP.7**). Understanding the meaning of mathematical symbols allows students to develop precision in their communication about mathematics (**MP.6**). The equation can also be reversed so that an operation is on the left and the number is on the right (e.g., 2 + 3 = 5). Such equations model "add to" situations (adapted from UA Progressions Documents 2011a).

Number pairs that total 10 are foundational for students' ability to work fluently within base-ten numbers and operations. In kindergarten, students find the number that makes 10 when added to the given number for any number from 1 to 9. Students use objects or drawings and record their answers with a drawing or equation (**K.OA.4▲**). Students use different models, such as 10-frames, cubes, and two-color counters to help them visualize these number pairs for 10 (**MP.1, MP.2, MP.4**).

Examples: Tools and Strategies for Making a Ten K.OA.4▲

A student places 3 objects on a 10-frame and then determines how many more are needed to "make a ten." Students may use electronic versions of 10-frames to develop this skill (**MP.5**).



A student snaps 10 cubes together to make a pretend train.

- The student breaks the train into two parts. He or she identifies how many cubes are in each part and records the associated equation (10 = ____ + ____).
- The student breaks the train into two parts. He or she counts how many cubes are in one part and determines how many are in the other part without directly counting that part. Then the student records the associated equation (if the counted part has 4 cubes, the equation would be 10 = 4 + ____).
- The student covers up part of the train, without counting the covered part. He or she counts the cubes that are showing and determines how many are covered up. Then the student records the associated equation (if the counted part has 7 cubes, the equation would be 10 = 7 + ____) [**MP.8**].
- The student tosses 10 two-color counters on the table and records how many of each color are facing up (**MP.8**).

Adapted from KATM 2012, Kindergarten Flipbook.

Later in the year, students solve addition and subtraction equations for numbers within 5 (for example, $2 + 1 = \square$ or $3 - 1 = \square$) while still connecting these equations to situations verbally or with drawings. Experience with decompositions of numbers and with "add to" and "take from" situations enables students to begin to fluently add and subtract within 5 (**K.OA.5▲**).

FLUENCY

In the standards for kindergarten through grade six, there are individual content standards that set expectations for fluency in computation (e.g., "Fluently add and subtract within 5" [**K.OA.5▲**]). Such standards are culminations of progressions of learning that often span several grades and involve conceptual understanding, thoughtful practice, and extra support where necessary.

The word *fluent* is used in the standards to mean "reasonably fast and accurate" and the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade may involve a mixture of simply knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies.

Adapted from UA Progressions Documents 2011a.

Below are several strategies that kindergarten students may use to attain fluency with addition and subtraction within 5:

- Visualizing the small numbers involved
- Counting on (e.g., for 3 + 2, students will say "3," then count on two more, "4, 5," and finish by saying the solution is "5")
- Counting back (e.g., for 4 – 1, students will say "4," then count back one, "3," and state that the solution is "3")
- Counting up to subtract (e.g., for 5 – 3 , students will say "3," then count up until they get to 5, keeping track of how many they counted up, stating that the solution is "2")
- Using doubles (e.g., for 2 + 3, students may say, "I know that 2 + 2 is 4, and 1 more is 5")
- Using the commutative property (e.g., students may say, "I know that 2 + 1 = 3, so 1 + 2 = 3")
- Using fact families (e.g., students may say, "I know that 2 + 3 = 5, so 5 – 3 = 2") [adapted from KATM 2012, Kindergarten Flipbook]

Example: Demonstrating Conceptual Understanding, Application, and Connection to the Mathematical Practices **K.OA.5▲**

Shake and Spill

Students use 5 two-color counters (e.g., red on one side and yellow on the other) and a cup (optional). The students put the counters in the cup, shake it, and spill them onto a table. The students determine how many of each color is showing and record the sum by using drawings or equations. The students should "shake and spill" several times to show different pairs of numbers that sum to 5.

Source: Illustrative Mathematics 2013a.

Domain: Number and Operations in Base Ten

Number and Operations in Base Ten	K.NBT
Work with numbers 11–19 to gain foundations for place value.	
1. Compose and decompose numbers from 11 to 19 into 10 ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of 10 ones and one, two, three, four, five, six, seven, eight, or nine ones.	

Kindergarten teachers help their students lay the foundation for understanding the base-ten system by drawing special attention to the number 10. Students compose and decompose numbers from 11 to 19 into 10 ones and some further ones. Students use objects or drawings and record each composition or decomposition with a drawing or equation (e.g., 16 = 9 + 7) [**K.NBT.1▲**].

Students describe, explore, and explain how the counting numbers from 11 through 19 are composed of 10 ones and some more ones. For example, when focusing on the number 14, students count out 14 objects using one-to-one correspondence and then use those objects to compose one group of 10 ones and 4 additional ones. Students connect the representation to the symbol "14" and recognize the written pattern in these numbers—that the numbers start with 1 (represents 1 ten) and end with the number of additional ones (**MP.1, MP.2, MP.4, MP.5, MP.6, MP.7, MP.8**) [adapted from UA Progressions Documents 2012b].

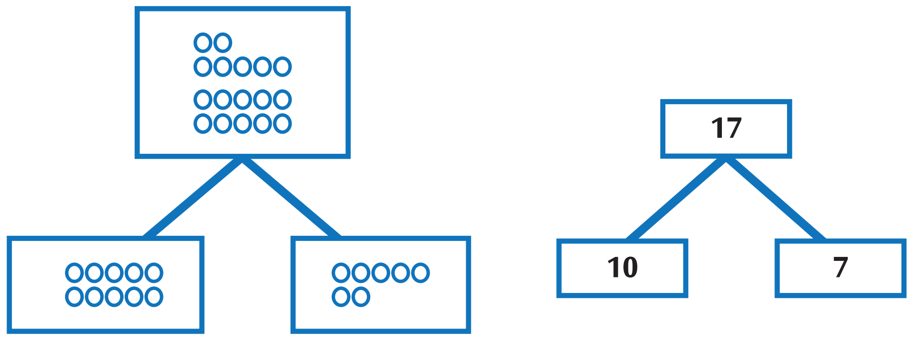
Students may have difficulty understanding that as a singular word, *ten* means "10 things." For many students, understanding that a group of 10 things can be replaced by a single word and that they both represent 10 is confusing. Students learn that this set of numbers (11–19) does not follow a consistent pattern in the verbal counting sequence. For example:

- *Eleven* and *twelve* are special number words.
- *Teen* means 1 ten plus ones.
- The verbal counting sequence for teen numbers is backwards—we say the ones digit before the tens digit. For example, 27 reads tens to ones (twenty-seven), but 17 reads ones to tens (seven-teen). To develop student understanding of written teen numbers, students read numbers as well as describe quantities. For example, for the number 17, students read "seventeen," decompose the number as "1 group of 10 ones and 7 additional ones," and record their understanding as 17 = 10 + 7 or use math drawings. This clarifies the pattern for them. Kindergarten students should see addition and subtraction equations. Student writing of equations in kindergarten is encouraged, but it is not required (adapted from ADE 2010).

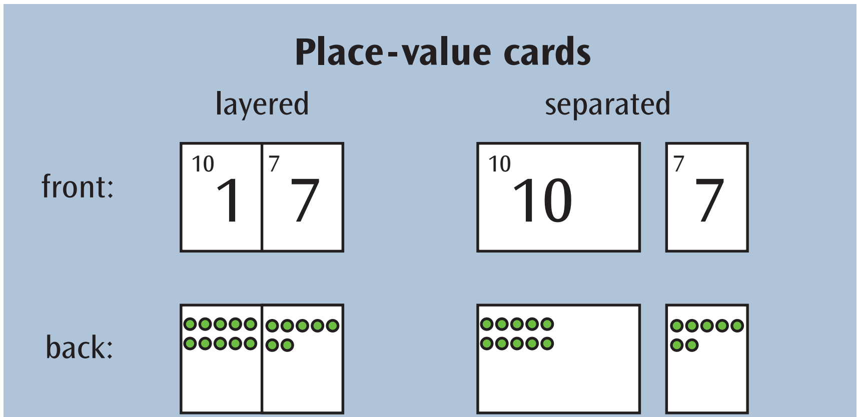
Examples: Understanding Teen Numbers **K.NBT.1▲**

Math drawings and other activities can help students develop place-value understanding of teen numbers.

Using 10-frames and number-bond diagrams



Using layered place-value cards



Children can use layered place-value cards to see the 10 "hiding" inside any teen number. Such decompositions can be connected to numbers represented with objects and math drawings.

Source: UA Progressions Documents 2012b.

Domain: Measurement and Data

Measurement and Data **K.MD**

Describe and compare measurable attributes.

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Students recognize and distinguish measurable attributes (e.g., length, area, volume) from non-measurable attributes (e.g., big or bigger) [**K.MD.1**]. Initially, many students will not be able to differentiate between these two types of attributes. Students will say one object is "bigger" than another without clarifying that it is longer, greater in area or volume, and so forth.

For students to accurately describe attributes such as length and weight, they need multiple opportunities to informally explore these attributes. Teachers encourage students' conversations to extend from describing objects as *big, small, long, tall,* or *high* to naming, discussing, and demonstrating with gestures the appropriate attribute (e.g., length, area, volume, or weight).

For example, a student might describe the measurable attributes of an empty can or milk carton by talking about how tall, wide, and heavy the can is, or how much liquid will fit inside the container. All of these are measurable attributes. By contrast, non-measurable attributes include designs, words, colors, or pictures on the can. As students discuss these situations and compare objects using different attributes, they learn to distinguish, label, and describe several measurable attributes of a single object (**MP.4, MP.5, MP.6, MP.7**).

Students directly compare two objects with a measurable attribute in common, to see which object has "more" or "less of" the attribute and describe the difference (**K.MD.2**). For example, students directly compare the heights of two children and describe one child as taller or shorter. Language plays an important role in this standard, as students describe the similarities and differences of measurable attributes of objects with terms such as *shorter than, taller than, lighter than, the same as,* and so forth (**MP.2, MP.4, MP.6, MP.7**).

When making direct comparisons for length, students must attend to the "starting point" of each object (e.g., the ends need to be lined up at the same point) or students need to compensate when the starting points are not lined up. Students develop an understanding of conservation of length (if an object is moved, its length does not change), an important concept when comparing the lengths of two objects (adapted from ADE 2010 and UA Progressions Documents 2012a).

With practice, students become increasingly competent at direct comparison—comparing the amount of an attribute in two objects without measurement. For example, when comparing the volume of two different boxes, ask students to discuss and justify their answers to these questions: *Which box will hold more? Which box will hold the least? Will the two boxes hold the same amount? How could you find out?* Students can decide to fill one box with dried beans and then pour the beans into the other box to determine the answers to these questions (adapted from KATM 2012, Kindergarten Flipbook).

Table K-5 presents a sample classroom activity that connects the Standards for Mathematical Content and Standards for Mathematical Practice.

Table K-5. Connecting to the Standards for Mathematical Practice—Kindergarten

Standards Addressed	Explanation and Examples
Connections to Standards for Mathematical Practice **MP.2.** Students reason abstractly when they imagine the attributes of given objects and attempt to compare them, even in the absence of physical objects at hand. Students mentally attribute quantities to features of objects when they compare these objects. **MP.3.** Students may be asked to justify why they think a comparison is correct, and if students disagree, they can try to explain their reasoning. **Standards for Mathematical Content** **K.MD.2.** Directly compare two objects with a measurable attribute in common, to see which object has "more of/less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*	**Task: The Comparison Game.** For this game, students have packs of 4 pairs of comparison cards, each pair corresponding to the following comparisons: heavier/lighter, taller/shorter, holds more/holds less, longer/shorter. In addition, each card pair has sample pictures on them that indicate the comparison, and furthermore, the words may be color-coded to aid students who cannot yet read the words on the cards (examples are shown at right). At the front of the room, the teacher shows the students two objects in sequence; the students must raise the appropriate card to compare the *second object to the first.* Several rounds are played with several different objects.  **Classroom Connections.** In alignment with standard **K.MD.2,** the purpose of this task is to give students several opportunities to compare measurable attributes of objects. Teachers can use a variety of objects and a variety of attributes, comparing two different objects and even the same object (e.g., evaluating the width of a cereal box versus its length). These informal comparisons of attributes lead to the development of estimation strategies for measurement and the use of standard units (e.g., "How many smaller unit squares fit into a given rectangle?"). The vocabulary on the cards may be too difficult for students. The teacher may introduce the lesson by holding up real objects, such as a book or a pencil, and state, "This book is heavier than the pencil," and then pass the two objects to the students to hold. After students become familiar with the concepts (e.g., heavier or lighter) through hands-on experiences, the teacher can reinforce the concept by using the cards.

Measurement and Data K.MD

Classify objects and count the number of objects in each category.

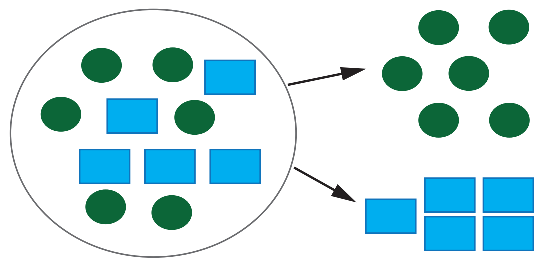
3. Classify objects into given categories; count the number of objects in each category and sort the categories by count.[3]

Kindergarten students connect counting and ordering skills and understandings to help them classify objects or people into given categories, count the number of objects in each category, and sort the categories by count (**K.MD.3**).

Students identify similarities and differences between objects (e.g., size, color, shape) and use these attributes to sort a collection of objects (**MP.2, MP.6, MP.7**).

When the objects are sorted, students count the objects in each set and then order each of the sets by the amount in each set.

Example:



For example, when given a collection of buttons, students separate buttons into different piles based on color. Next, they count the number of buttons in each pile (e.g., blue [5], green [4], orange [3], and purple [4]). Finally, they organize the groups by the quantity in each group—for example, from the smallest group (orange) to the largest group (blue), and groups with the same number (green and purple) are placed together.

Students should be able to explain their thinking. Teachers may use prompts such as these to ask students to explain their thought processes:

- Explain how you sorted the objects.
- Explain how you labeled each set with a category.
- Answer a variety of counting questions (such as "How many ________?").
- Compare the sorted groups using words such as *most, least, same,* and *different* (adapted from KATM 2012, Kindergarten Flipbook).

Focus, Coherence, and Rigor

As kindergartners classify objects, they build a foundation for collecting data and creating and analyzing graphical representations in later grades. Also, as students count the number of objects in each category and order the categories by count, they reinforce important skills and understanding in comparing numbers, which are part of the major work at this grade in the Counting and Cardinality domain (**K.CC.4–7▲**). Students can also reinforce mathematical practices as they make sense of problems by counting and recounting (**MP.1**) and explaining their process and reasoning (**MP.3**).

3. Limit category counts to be less than or equal to 10.

Domain: Geometry

A critical area of instruction in kindergarten is for students to describe shapes and space. Students develop geometric concepts and spatial reasoning from experience with the shapes of objects and the relative positions of objects.

Geometry	K.G

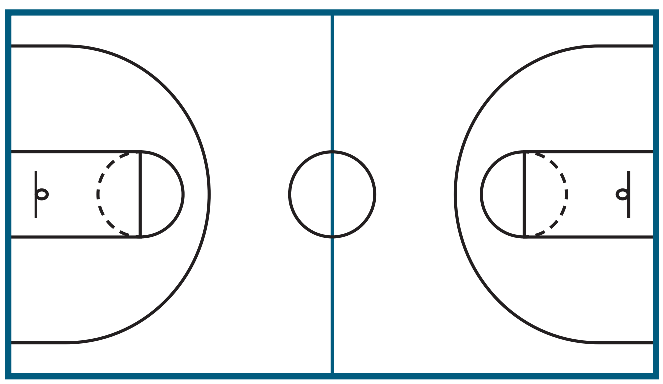
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to.*
2. Correctly name shapes regardless of their orientations or overall size.
3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Students use positional words to describe objects in the environment (**K.G.1**). Examples of positional words include *in* and *out, inside* and *outside, down* and *up, above* and *below, over* and *under, before* and *after, top* and *bottom, front* and *back, right* and *left, on* and *off, begin* and *end,* and *near* and *far.*

Students develop spatial sense by connecting geometric shapes to their everyday lives. Students need opportunities to identify and name two- and three-dimensional shapes in and outside of the classroom and describe relative positions by answering questions such as these:

- Which way is the cafeteria? (*The cafeteria is to the right.*)
- Which shape is near the rectangle? (*The circle is near the rectangle.*)
- Where is the green ball? (*The green ball is on top of the cupboard.*)
- What types of shapes do you see on the floor of the basketball court? (*I see a rectangle and a circle on the basketball court.*)



Students begin to name and describe three-dimensional shapes with mathematical vocabulary, using words such as *sphere, cube, cylinder,* and *cone,* and answer related questions (**MP.6, MP.7**). Examples for standard **K.G.1** include the following:

- Ask students to find rectangles in the classroom and describe the relative positions of the rectangles they see. (*Possible answer:* The rectangle [a poster] is over the sphere [globe]).
- The teacher holds up objects—such as an ice-cream cone, a number cube, or a ball—and asks students to identify each shape.

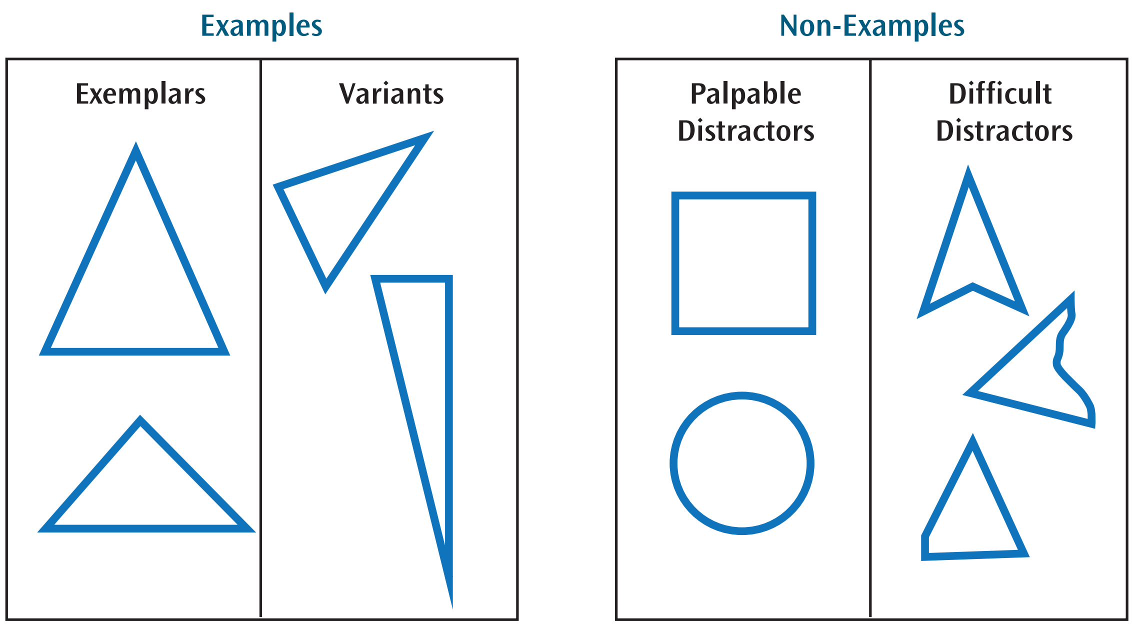
- The teacher places an object next to, behind, above, below, beside, or in front of another object and asks positional questions such as "Where is the object?" (adapted from ADE 2010; KATM 2012, Kindergarten Flipbook; and UA Progressions Documents 2012c).

Kindergarten students work with a variety of shapes that have different sizes. They learn to match two-dimensional shapes even when the shapes have different orientations (**K.G.2**). Students name shapes that occur in everyday situations, such as circles, triangles, and squares, and distinguish them from non-examples of these categories.

Students develop an intuitive image of each shape category. Figure K-3 includes examples and non-examples of triangles, as described below:

- Examples
  - Exemplars—typical visual prototypes of the shape category
  - Variants—other examples of the shape category
- Non-Examples
  - Palpable distractors—non-examples with little or no overall resemblance to the exemplars
  - Difficult distractors—visually similar to examples, but lack at least one defining attribute

Figure K-3. Examples and Non-Examples of Triangles



Adapted from UA Progressions Documents 2012c.

Common Misconceptions

- Most kindergarten students are unable to recognize an "upside-down triangle" as a triangle, because of its orientation. However, students should be exposed to many types of triangles, in many different orientations, to eliminate the misconception that a triangle is always vertex-up and equilateral.
- A square with a vertex pointing downward is often referred to as a "diamond." This needless introduction of a new shape name should be avoided, as it only serves to confuse the fact that such a shape is still a square, though its orientation is atypical.

Below are several strategies to help kindergarten students learn about shapes (**MP.6, MP.7**):

- Students form visual templates or refer to models for shape categories (e.g., children recognize a shape as a rectangle because it looks like a door).
- Students see examples of rectangles that are long and skinny and contrast rectangles with non-rectangles that appear to be similar, but lack an important defining attribute.
- Students see examples of triangles that have sides with three different lengths and then contrast triangles with non-triangles.
- The teacher hands out pairs of paper shapes in different sizes. Each student is given one shape. Then students need to find the partner who has the same shape.
- The teacher brings in a variety of spheres (a tennis ball, basketball, globe, table-tennis ball, and so on) to demonstrate that size does not change the name of a shape (adapted from ADE 2010; KATM 2012, Kindergarten Flipbook; and UA Progressions Documents 2012c).

Students identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid") [**K.G.3**] and differentiate between two-dimensional and three-dimensional shapes (**MP.6, MP.7**). For example:

- Students name a picture of a shape as two-dimensional because it is flat and can be measured in only **two** ways (by its length and width).
- Students name an object as three-dimensional because it is not flat (it is a solid object or shape) and can be measured by length, width, and height (or depth) [adapted from ADE 2010].

Geometry	**K.G**

Analyze, compare, create, and compose shapes.

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

Kindergarten students connect their work with identifying and classifying simple shapes (refer to standards **K.G.1–3**) to help them compare shapes and manipulate two or more shapes to create a new shape. This understanding also builds foundations for students to "reason with shapes and their attributes" in grade one (refer to standards **1.G.1–3**).

Students describe similarities and differences between and among shapes using informal language (**K.G.4**). These experiences help young students begin to understand how three-dimensional shapes are composed of two-dimensional shapes—for example, the base and the top of a cylinder is a circle, the face of a cube is a square, a circle is formed in the shadow of a sphere. In early explorations of geometric properties, students discover how categories of shapes are subsumed within other categories.

Example: Sorting Shapes	**K.G.4**
Students sort a pile of squares and rectangles into two groups. They discuss how the rectangles and squares are alike and how they are different. After students demonstrate an understanding of the differences between squares and rectangles, the teacher hands out either a square or a rectangle cutout to each student. The teacher directs students with the square cutouts to one side of the room and the students with the rectangle cutouts to the opposite side of the room. The rectangle and square cutouts differ in size and color so that students focus on the shape attributes. To avoid the misconception that a square is not a rectangle, students learn informal language such as "A square is a special rectangle that has four sides of equal length."	

Students work with various triangles, rectangles, and hexagons with sides that are not all congruent. Initially, students describe shapes using everyday language and then expand their vocabulary to include geometric terms such as *sides* and *vertices* (or *corners*). Opportunities to work with pictorial representations and concrete objects, as well as technology, will help students develop their understanding and descriptive vocabulary for both two- and three-dimensional shapes (**MP.4, MP.6, MP.7**).

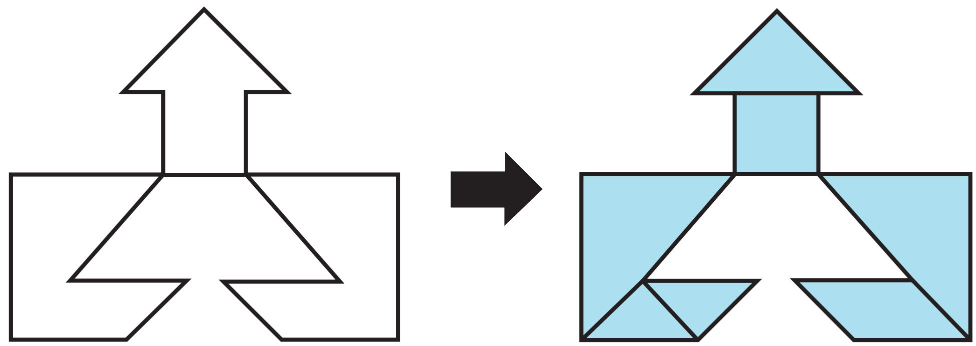
In kindergarten, students model shapes they observe in everyday life by building shapes from various components (e.g., clay, glue, tape, sticks, paper, straws) and by drawing shapes (**K.G.5**). Two-dimensional shapes are flat, and three-dimensional shapes are not flat (and can be "solid"), so students should *draw* or *create* two-dimensional shapes and *build* three-dimensional shapes (**MP.1, MP.4, MP.7**).

Students compose simple shapes to form larger shapes and answer questions such as, "Can you join these two triangles with full sides touching to make a rectangle?" (**K.G.6**). Composing shapes is an important concept in kindergarten. Students move from identifying and classifying simple shapes to manipulating two or more shapes to create a new shape. Students rotate, flip, and arrange puzzle pieces, and they move shapes to make a design or picture. Finally, students manipulate simple shapes to make a new shape (**MP.1, MP.3, MP.4, MP.7**) [adapted from KATM 2012, Kindergarten Flipbook].

Puzzles provide opportunities for students to apply spatial relationships and develop problem-solving skills in an entertaining and meaningful way. Pattern blocks and tangrams are often utilized when students work with two-dimensional shapes.

Example: Exploring Shapes with Tangrams	K.G.6

Students make a schoolhouse using tangrams. The teacher models how to place the pieces and discusses how it is necessary to turn over, rotate, or slide pieces to complete a puzzle.



Each student or pair of students is then provided with a set of tangrams and a simple puzzle, such as the outlined version of the schoolhouse above. Students use their pieces to complete the puzzle.

Adapted from National Council of Teachers of Mathematics Illuminations 2013d.

Composing and decomposing shapes with right angles (squares, rectangles, and right triangles that also make isosceles triangles) provides important foundations for central geometric concepts (such as transformations) in later grades.

Examples of interactive tangram puzzles are available at the National Council of Teachers of Mathematics Web site (http://www.nctm.org/standards/content.aspx?id=25012 [accessed July 31, 2014]).

Essential Learning for the Next Grade

In kindergarten through grade five, the focus is on the addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals, with a balance of concepts, skills, and problem solving. Arithmetic is viewed as an important set of skills and also as a thinking subject that, done thoughtfully, prepares students for algebra. Measurement and geometry develop alongside number and operations and are tied specifically to arithmetic along the way.

In kindergarten through grade two, students focus on addition, subtraction, and measurement using whole numbers. To be prepared for grade-one mathematics, students should be able to demonstrate that they have acquired specific mathematical concepts and procedural skills by the end of kindergarten and have met the fluency expectations. For kindergartners, the expected fluencies are to add and subtract within 5 (**K.OA.5▲**). Addition and subtraction are introduced in kindergarten, and these fluencies and the conceptual understandings that support them are foundational for work in later grades.

It is particularly important for kindergarten students to attain the concepts, skills, and understandings necessary to know the number names and the count sequence (**K.CC.1–3▲**); count to tell the number of objects (**K.CC.4–5▲**); compare numbers (**K.CC.6–7▲**); understand addition as putting together and adding to; and understand subtraction as taking apart and taking from (**K.OA.1–5▲**). Also, working with numbers to gain foundations for place value (**K.NBT.1▲**) is essential to understanding the base-ten number system.

Counting and Cardinality

In kindergarten, students learn to count. Students should connect counting to *cardinality*—knowing that the number word tells the quantity and that the number on which a person ends when counting represents the entire amount counted. Until this concept is developed, counting is merely a routine procedure done when a number is needed, and students will not understand how to apply numbers to solve problems.

By the end of kindergarten, important number concepts and skills for students include counting by ones and tens to 100 (rote counting); continuing a counting sequence when beginning from a number greater than 1 (counting on); counting objects to 20; writing numbers to 20; understanding one-to-one correspondence; identifying a quantity using both numerals and words; representing numbers with numerals (and pictures and words); understanding numbers and the relationships between numbers and quantities; and understanding the concepts of *more* and *less.* Counting to 100 and representing numbers with numerals (0 to 20) will prepare students to read and write numbers to 120 in grade one.

Addition and Subtraction

By the end of kindergarten, students are expected to add and subtract within 10 and solve addition and subtraction word problems. Students are also expected to be fluent with addition and subtraction within 5. Fluency with addition and subtraction will prepare students to add within 100 in grade one. Addition and subtraction constitute a major instructional focus for kindergarten through grade two.

Kindergarten Overview

Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

- Work with numbers 11–19 to gain foundations for place value.

Measurement and Data

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in categories.

Geometry

- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Counting and Cardinality K.CC

Know number names and the count sequence.

1. Count to 100 by ones and by tens.
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
   a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
   d. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
   e. Understand that each successive number name refers to a quantity that is one larger.
5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Compare numbers.

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[4]
7. Compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Represent addition and subtraction with objects, fingers, mental images, drawings,[5] sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
5. Fluently add and subtract within 5.

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4. Includes groups with up to 10 objects.
5. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Number and Operations in Base Ten K.NBT

Work with numbers 11–19 to gain foundations for place value.

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data K.MD

Describe and compare measurable attributes.

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.

2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Classify objects and count the number of objects in each category.

3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[6]

Geometry K.G

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to.*

2. Correctly name shapes regardless of their orientations or overall size.

3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

6. Limit category counts to be less than or equal to 10.