Grade-Three Chapter

of the

Mathematics Framework

for California Public Schools: Kindergarten Through Grade Twelve

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Grade Three

In grade three, students continue to build upon their mathematical foundation as they focus on the operations of multiplication and division and the concept of fractions as numbers. In previous grades, students developed an understanding of place value and used methods based on place value to add and subtract within 1000. They developed fluency with addition and subtraction within 100 and laid a foundation for understanding multiplication based on equal groups and the array model. Students also worked with standard units to measure length and described attributes of geometric shapes (adapted from Charles A. Dana Center 2012).

Critical Areas of Instruction

In grade three, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division, as well as strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with a numerator of 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes (National Governors Association Center for Best Practices, Council of Chief State School Officers [NGA/CCSSO] 2010j). Students also work toward fluency with addition and subtraction within 1000 and multiplication and division within 100. By the end of grade three, students know all products of two one-digit numbers from memory.

Standards for Mathematical Content

The Standards for Mathematical Content emphasize key content, skills, and practices at each grade level and support three major principles:

- **Focus**—Instruction is focused on grade-level standards.
- **Coherence**—Instruction should be attentive to learning across grades and to linking major topics within grades.
- **Rigor**—Instruction should develop conceptual understanding, procedural skill and fluency, and application.

Grade-level examples of focus, coherence, and rigor are indicated throughout the chapter.

The standards do not give equal emphasis to all content for a particular grade level. Cluster headings can be viewed as the most effective way to communicate the **focus** and **coherence** of the standards. Some clusters of standards require a greater instructional emphasis than others based on the depth of the ideas, the time needed to master those clusters, and their importance to future mathematics or the later demands of preparing for college and careers.

Table 3-1 highlights the content emphases at the cluster level for the grade-three standards. The bulk of instructional time should be given to "Major" clusters and the standards within them, which are indicated throughout the text by a triangle symbol (▲). However, standards in the "Additional/Supporting" clusters should not be neglected; to do so would result in gaps in students' learning, including skills and understandings they may need in later grades. Instruction should reinforce topics in major clusters by using topics in the additional/supporting clusters and including problems and activities that support natural connections between clusters.

Teachers and administrators alike should note that the standards are not topics to be checked off after being covered in isolated units of instruction; rather, they provide content to be developed throughout the school year through rich instructional experiences presented in a coherent manner (adapted from Partnership for Assessment of Readiness for College and Careers [PARCC] 2012).

Table 3-1. Grade Three Cluster-Level Emphases

Operations and Algebraic Thinking — 3.OA

Major Clusters

- Represent and solve problems involving multiplication and division. (**3.OA.1–4▲**)
- Understand properties of multiplication and the relationship between multiplication and division. (**3.OA.5–6▲**)
- Multiply and divide within 100. (**3.OA.7▲**)
- Solve problems involving the four operations, and identify and explain patterns in arithmetic. (**3.OA.8–9▲**)

Number and Operations in Base Ten — 3.NBT

Additional/Supporting Clusters

- Use place-value understanding and properties of operations to perform multi-digit arithmetic. (**3.NBT.1–3**)

Number and Operations—Fractions — 3.NF

Major Clusters

- Develop understanding of fractions as numbers. (**3.NF.1–3▲**)

Measurement and Data — 3.MD

Major Clusters

- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. (**3.MD.1–2▲**)
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition. (**3.MD.5–7▲**)

Additional/Supporting Clusters

- Represent and interpret data. (**3.MD.3–4**)
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. (**3.MD.8**)

Geometry — 3.G

Additional/Supporting Clusters

- Reason with shapes and their attributes. (**3.G.1–2**)

Explanations of Major and Additional/Supporting Cluster-Level Emphases

Major Clusters (▲) — Areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than others based on the depth of the ideas, the time needed to master them, and their importance to future mathematics or the demands of college and career readiness.

Additional Clusters — Expose students to other subjects; may not connect tightly or explicitly to the major work of the grade.

Supporting Clusters — Designed to support and strengthen areas of major emphasis.

Note of caution: Neglecting material, whether it is found in the major or additional/supporting clusters, will leave gaps in students' skills and understanding and will leave students unprepared for the challenges they face in later grades.

Adapted from Smarter Balanced Assessment Consortium 2011, 83.

Connecting Mathematical Practices and Content

The Standards for Mathematical Practice (MP) are developed throughout each grade and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject. The MP standards represent a picture of what it looks like for students to understand and do mathematics in the classroom and should be integrated into every mathematics lesson for all students.

Although the description of the MP standards remains the same at all grade levels, the way these standards look as students engage with and master new and more advanced mathematical ideas does change. Table 3-2 presents examples of how the MP standards may be integrated into tasks appropriate for students in grade three. (Refer to the Overview of the Standards Chapters for a description of the MP standards.)

Table 3-2. Standards for Mathematical Practice—Explanation and Examples for Grade Three

Standards for Mathematical Practice	Explanation and Examples
MP.1 Make sense of problems and persevere in solving them.	In third grade, mathematically proficient students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Students may use concrete objects, pictures, or drawings to help them conceptualize and solve problems such as these: "Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase?"; or "Describe another situation where there would be 5 groups of 3 or 5×3." Students may check their thinking by asking themselves, "Does this make sense?" Students listen to other students' strategies and are able to make connections between various methods for a given problem.
MP.2 Reason abstractly and quantitatively.	Students recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. For example, students apply their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. When given $4 \times __ = 40$, they might think: • 4 groups of some number is the same as 40. • 4 times some number is the same as 40. • I know that 4 groups of 10 is 40, so the unknown number is 10. • The missing factor is 10, because 4 times 10 equals 40. To reinforce students' reasoning and understanding, teachers might ask, "How do you know?" or "What is the relationship between the quantities?"
MP.3 Construct viable arguments and critique the reasoning of others.	Students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions that the teacher facilitates by asking questions such as "How did you get that?" and "Why is that true?" Students explain their thinking to others and respond to others' thinking. For example, after investigating patterns on the hundreds chart, students might explain why the pattern makes sense.

Table 3-2 *(continued)*

MP.4 Model with mathematics.	Students represent problem situations in multiple ways using numbers, words (mathematical language), objects, and math drawings. They might also represent a problem by acting it out or by creating charts, lists, graphs, or equations. For example, students use various contexts and a variety of models (e.g., circles, squares, rectangles, fraction bars, and number lines) to represent and develop understanding of fractions. Students use models to represent both equations and story problems and can explain their thinking. They evaluate their results in the context of the situation and reflect on whether the results make sense. Students should be encouraged to answer questions such as "What math drawing or diagram could you make and label to represent the problem?" or "What are some ways to represent the quantities?"
MP.5 Use appropriate tools strategically.	Mathematically proficient students consider the available tools (including drawings or estimation) when solving a mathematical problem and decide when particular tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table and determine whether they have all the possible rectangles. Students should be encouraged to answer questions (e.g., "Why was it helpful to use _____ ?").
MP.6 Attend to precision.	Students develop mathematical communication skills as they use clear and precise language in their discussions with others and in their own reasoning. They are careful to specify units of measure and to state the meaning of the symbols they choose. For instance, when calculating the area of a rectangle they record the answer in square units.
MP.7 Look for and make use of structure.	Students look closely to discover a pattern or structure. For instance, students use properties of operations (e.g., commutative and distributive properties) as strategies to multiply and divide. Teachers might ask, "What do you notice when _____ ?" or "How do you know if something is a pattern?"
MP.8 Look for and express regularity in repeated reasoning.	Students notice repetitive actions in computations and look for "shortcut" methods. For instance, students may use the distributive property as a strategy to work with products of numbers they know to solve products they do not know. For example, to find the product of 7×8, students might decompose 7 into 5 and 2 and then multiply 5×8 and 2×8 to arrive at $40 + 16$, or 56. Third-grade students continually evaluate their work by asking themselves, "Does this make sense?" Students should be encouraged to answer questions such as "What is happening in this situation?" or "What predictions or generalizations can this pattern support?"

Adapted from Arizona Department of Education (ADE) 2010 and North Carolina Department of Public Instruction (NCDPI) 2013b.

Standards-Based Learning at Grade Three

The following narrative is organized by the domains in the Standards for Mathematical Content and highlights some necessary foundational skills from previous grade levels. It also provides exemplars to explain the content standards, highlight connections to Standards for Mathematical Practice (**MP**), and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. A triangle symbol (▲) indicates standards in the major clusters (see table 3-1).

Domain: Operations and Algebraic Thinking

In kindergarten through grade two, students focused on developing an understanding of addition and subtraction. Beginning in grade three, students focus on concepts, skills, and problem solving for multiplication and division. Students develop multiplication strategies, make a shift from additive to multiplicative reasoning, and relate division to multiplication. Third-grade students become fluent with multiplication and division within 100. This work will continue in grades four and five, preparing the way for work with ratios and proportions in grades six and seven (adapted from the University of Arizona Progressions Documents for the Common Core Math Standards [UA Progressions Documents] 2011a and PARCC 2012).

Multiplication and division are new concepts in grade three, and meeting fluency is a major portion of students' work (see **3.OA.7▲**). Reaching fluency will take much of the year for many students. These skills and the understandings that support them are crucial; students will rely on them for years to come as they learn to multiply and divide with multi-digit whole numbers and to add, subtract, multiply, and divide with rational numbers.

There are many patterns to be discovered by exploring the multiples of numbers. Examining and articulating these patterns is an important part of the mathematical work on multiplication and division. Practice—and, if necessary, extra support—should continue all year for those students who need it to attain fluency. This practice can begin with the easier multiplication and division problems while the pattern work is occurring with more difficult numbers (adapted from PARCC 2012). Relating and practicing multiplication and division problems involving the same number (e.g., the 4s) may be helpful.

Operations and Algebraic Thinking — 3.OA

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as* 5×7.
2. Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as* $56 \div 8$.
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$.

A critical area of instruction is to develop student understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models (NGA/CCSSO 2010c). Multiplication and division are new concepts in grade three. Initially,

1. See glossary, table GL-5.

students need opportunities to develop, discuss, and use efficient, accurate, and generalizable methods to compute. The goal is for students to use general written methods for multiplication and division that students can explain and understand (e.g., using visual models or place-value language). The general written methods should be variations of the standard algorithms. Reaching fluency with these operations requires students to use variations of the standard algorithms without visual models, and this could take much of the year for many students.

Students recognize multiplication as finding the total number of objects in a particular number of equal-sized groups (**3.OA.1▲**). Also, students recognize division in two different situations: *partitive division* (also referred to as *fair-share division*), which requires equal sharing (e.g., how many are in each group?); and *quotitive division* (or *measurement division*), which requires determining how many groups (e.g., how many groups can you make?) [**3.OA.2▲**]. These two interpretations of division have important uses later, when students study division of fractions, and both interpretations should be explored as representations of division. In grade three, teachers should use the terms *number of shares* or *number of groups* with students rather than *partitive* or *quotitive*.

Multiplication of Whole Numbers	**3.OA.1▲**
Note that the standards define multiplication of whole numbers $a \times b$ as finding the total number of objects in a groups of b objects. ***Example:*** There are 3 bags of apples on the table. There are 4 apples in each bag. How many apples are there altogether?	
Partitive Division (also known as Fair-Share or Group Size Unknown Division)	**3.OA.2▲**
The number of groups or shares to be made is known, but the number of objects in (or size of) each group or share is unknown. ***Example:*** There are 12 apples on the counter. If you are sharing the apples equally among 3 bags, how many apples will go in each bag?	
Quotitive Division (also known as Measurement or Number of Groups Unknown Division)	**3.OA.2▲**
The number of objects in (or size of) each group or share is known, but the number of groups or shares is unknown. ***Example:*** There are 12 apples on the counter. If you put 3 apples in each bag, how many bags will you fill?	

Students are exposed to related terminology for multiplication (*factor* and *product*) and division (*quotient, dividend, divisor,* and *factor*). They use multiplication and division within 100 to solve word problems (**3.OA.3▲**) in situations involving equal groups, arrays, and measurement quantities. Note that although "repeated addition" can be used in some cases as a strategy for finding whole-number products, repeated addition should not be misconstrued as the meaning of multiplication. The intention of the standards in grade three is to move students beyond additive thinking to multiplicative thinking.

The three most common types of multiplication and division word problems for this grade level are summarized in table 3-3.

Table 3-3. Types of Multiplication and Division Problems (Grade Three)

	Unknown Product	Group Size Unknown[2]	Number of Groups Unknown[3]
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there altogether? **Measurement example** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally and packed into 3 bags, then how many plums will be in each bag? **Measurement example** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed, with 6 plums to a bag, then how many bags are needed? **Measurement example** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays, Area	There are 3 rows of apples with 6 apples in each row. How many apples are there? **Area example** What is the area of a rectangle that measures 3 centimeters by 6 centimeters?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? **Area example** A rectangle has an area of 18 square centimeters. If one side is 3 centimeters long, how long is a side next to it?.	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? **Area example** A rectangle has an area of 18 square centimeters. If one side is 6 centimeters long, how long is a side next to it?
Compare	Grade-three students do not solve multiplicative "compare" problems; these problems are introduced in grade four (with whole-number values) and also appear in grade five (with unit fractions).		
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

Source: NGA/CCSSO 2010d. A nearly identical version of this table appears in the glossary (table GL-5).

In grade three, students focus on equal groups and array problems. Multiplicative-compare problems are introduced in grade four. The more difficult problem types include "Group Size Unknown" ($3 \times ? = 18$ or $18 \div 3 = 6$) or "Number of Groups Unknown" ($? \times 6 = 18$, $18 \div 6 = 3$). To solve problems, students determine the unknown whole number in a multiplication or division equation relating three whole numbers (**3.OA.4▲**). Students use numbers, words, pictures, physical objects, or equations to represent problems, explain their thinking, and show their work (**MP.1**, **MP.2**, **MP.4**, **MP.5**).

2. These problems ask the question, "How many in each group?" The problem type is an example of *partitive* or *fair-share* division.

3. These problems ask the question, "How many groups?" The problem type is an example of *quotitive* or *measurement* division.

Example: Number of Groups Unknown	3.OA.4▲

Molly the zookeeper has 24 bananas to feed the monkeys. Each monkey needs to eat 4 bananas. How many monkeys can Molly feed?

Solution: $? \times 4 = 24$

Students might draw on the remembered product $6 \times 4 = 24$ to say that the related quotient is 6. Alternatively, they might draw on other known products—for example, if $5 \times 4 = 20$ is known, then since $20 + 4 = 24$, one more group of 4 will give the desired factor $(5 + 1 = 6)$. Or, knowing that $3 \times 4 = 12$ and $12 + 12 = 24$, students might reason that the desired factor is $3 + 3 = 6$. Any of these methods (or others) might be supported by a representational drawing that shows the equal groups in the situation.

Operations and Algebraic Thinking 3.OA

Understand properties of multiplication and the relationship between multiplication and division.

5. Apply properties of operations as strategies to multiply and divide.[4] *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)*
6. Understand division as an unknown-factor problem. *For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.*

In grade three, students apply properties of operations as strategies to multiply and divide (**3.OA.5▲**). Third-grade students do not need to use the formal terms for these properties. Students use increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn about the relationship between multiplication and division.

Focus, Coherence, and Rigor

Arrays can be seen as equal-sized groups where objects are arranged by rows and columns, and they form a major transition to understanding multiplication as finding area (connection to **3.MD.7▲**). For example, students can analyze the structure of multiplication and division (**MP.7**) through their work with arrays (**MP.2**) and work toward precisely expressing their understanding of the connections between area and multiplication (**MP.6**).

The distributive property is the basis for the standard multiplication algorithm that students can use to fluently multiply multi-digit whole numbers in grade five. Third-grade students are introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they do not know (**MP.2**, **MP.7**).

4. Students need not use formal terms for these properties.

Example: Using the Distributive Property	**3.OA.5▲**
Students can use the distributive property to discover new products of whole numbers (such as 7×8) based on products they can find more easily.	
Strategy 1: By creating an array, I can find how many total stars there are in 7 columns of 8 stars. ★	**Strategy 2:** By creating an array, I can find how many total stars there are in 8 rows of 7 stars. ★
I see that I can arrange the 7 columns into a group of 5 columns and a group of 2 columns. ★ ★ ★ ★ ★ | ★ ★ ★ ★ ★ ★ ★ | ★ ★ ★ ★ ★ ★ ★ | ★ ★ ★ ★ ★ ★ ★ | ★ ★ ★ ★ ★ ★ ★ | ★ ★ ★ ★ ★ ★ ★ | ★ ★ ★ ★ ★ ★ ★ | ★ ★ ★ ★ ★ ★ ★ | ★ ★	I see that I can arrange the 8 rows of stars into 2 groups of 4 rows. ★ ——————— ★
I know that the 5×8 array gives me 40 and the 2×8 array gives me 16. So altogether I have $5 \times 8 + 2 \times 8 = 40 + 16 = 56$ stars.	I know that each new 4×7 array gives me 28 stars, so altogether I have $4 \times 7 + 4 \times 7 = 28 + 28 = 56$ stars.

Adapted from ADE 2010.

The connection between multiplication and division should be introduced early in the year. Students understand division as an unknown-factor problem (**3.OA.6▲**). *For example, find $15 \div 3$ by finding the number that makes 15 when multiplied by 3.* Multiplication and division are inverse operations, and students use this inverse relationship to compute and check results.

Operations and Algebraic Thinking 3.OA

Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of grade 3, know from memory all products of two one-digit numbers.

Students in grade three use various strategies to fluently multiply and divide within 100 (**3.OA.7▲**). The following are some general strategies that can be used to help students know from memory all products of two one-digit numbers.

Strategies for Learning Multiplication Facts **3.OA.7▲**

Patterns

- Multiplication by zeros and ones
- Doubles (twos facts), doubling twice (fours), doubling three times (eights)
- Tens facts (relating to place value, 5×10 is 5 tens, or 50)
- Fives facts (knowing the fives facts are half of the tens facts)

General Strategies

- Use skip-counting (counting groups of specific numbers and knowing how many groups have been counted). For example, students count by twos, keeping track of how many groups (to multiply) and when they reach the known product (to divide). Students gradually abbreviate the "count by" list and are able to start within it.
- Decompose into known facts (e.g., 6×7 is 6×6 plus one more group of 6).
- Use "turn-around facts" (based on the commutative property—for example, knowing that 2×7 is the same as 7×2 reduces the total number of facts to memorize).

Other Strategies

- Know square numbers (e.g., 6×6).
- Use arithmetic patterns to multiply. Nines facts include several patterns. For example, using the fact that $9 = 10 - 1$, students can use the tens multiplication facts to help solve a nines multiplication problem.
  $9 \times 4 = 9$ fours = 10 fours – 1 four = $40 - 4 = 36$
  Students may also see this as:
  $4 \times 9 = 4$ nines = 4 tens – 4 ones = $40 - 4 = 36$

Strategies for Learning Division Facts

- Turn the division problem into an unknown-factor problem. Students can state a division problem as an unknown-factor problem (e.g., $24 \div 4 = ?$ becomes $4 \times ? = 24$). Knowing the related multiplication facts can help a student obtain the answer and vice versa, which is why studying multiplication and division involving a particular number can be helpful.
- Use related facts (e.g., $6 \times 4 = 24$; $24 \div 6 = 4$; $24 \div 4 = 6$; $4 \times 6 = 24$).

Adapted from ADE 2010.

Multiplication and division are new concepts in grade three, and reaching fluency with these operations within 100 represents a major portion of students' work. By the end of grade three, students also know all products of two one-digit numbers from memory (**3.OA.7▲**). Organizing practice to focus most heavily on products and unknown factors that are understood but not yet fluent in students can speed learning and support fluency with multiplication and division facts. Practice and extra support should continue all year for those who need it to attain fluency.

FLUENCY

California's Common Core State Standards for Mathematics (K–6) set expectations for fluency in computation (e.g., "*Fluently* multiply and divide within 100 . . ." [**3.OA.7▲**]). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding, thoughtful practice, and extra support where necessary. The word *fluent* is used in the standards to mean "reasonably fast and accurate" and possessing the ability to use certain facts and procedures with enough facility that using such knowledge does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade may involve a mixture of knowing some answers, knowing some answers from patterns, and knowing some answers through the use of strategies.

Adapted from UA Progressions Documents 2011a.

Students in third grade begin to take steps toward formal algebraic language by using a letter for the unknown quantity in expressions or equations when solving one- and two-step word problems (**3.OA.8▲**).

Operations and Algebraic Thinking — 3.OA

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[5]
9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Students do not formally solve algebraic equations at this grade level. Students know to perform operations in the conventional order when there are not parentheses to specify a particular order (order of operations). Students use estimation during problem solving and then revisit their estimates to check for reasonableness.

5. This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Example 1: Chicken Coop **3.OA.8▲**

There are 5 nests in a chicken coop and 2 eggs in each nest. If the farmer wants 25 eggs, how many more eggs does she need?

Solution: Students might create a picture representation of this situation using a tape diagram:

2	2	2	2	2	m
25					

Students might solve this by seeing that when they add up the 5 nests with 2 eggs, they have 10 eggs. Thus, to make 25 eggs the farmer would need $25-10=15$ more eggs. A simple equation that represents this situation could be $5\times 2+m=25$, where m is the number of additional eggs the farmer needs.

Example 2: Soccer Club **3.OA.8▲**

The soccer club is going on a trip to the water park. The cost of attending the trip is \$63, which includes \$13 for lunch and the price of 2 wristbands (one for the morning and one for the afternoon). Both wristbands are the same price. Find the price of one of the wristbands, and write an equation that represents this situation.

Solution: Students might solve the problem by seeing that the total cost of the two tickets must be $\$63-\$13=\$50$.

w	w	\$13
\$63		

Therefore, the cost of one wristband must be $\$50\div 2=\$25$. Equations that represent this situation are $w+w+13=63$ and $63=w+w+13$.

Adapted from Kansas Association of Teachers of Mathematics (KATM) 2012, 3rd Grade Flipbook, and NCDPI 2013b.

In grade three, students identify arithmetic patterns and explain them using properties of operations (**3.OA.9▲**). Students can investigate addition and multiplication tables in search of patterns (**MP.7**) and explain or discuss why these patterns make sense mathematically and how they are related to properties of operations (e.g., why is the multiplication table symmetric about its diagonal from the upper left to the lower right?) [**MP.3**].

Domain: Number and Operations in Base Ten

Use place-value understanding and properties of operations to perform multi-digit arithmetic.[6]

1. Use place-value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., $9\times 80, 5\times 60$) using strategies based on place value and properties of operations.

6. A range of algorithms may be used.

In grade three, students are introduced to the concept of rounding whole numbers to the nearest 10 or 100 (**3.NBT.1**), an important prerequisite for working with estimation problems. Students can use a number line or a hundreds chart as tools to support their work with rounding. They learn when and why to round numbers and extend their understanding of place value to include whole numbers with four digits.

Third-grade students continue to add and subtract within 1000 and achieve fluency with strategies and algorithms that are based on place value, properties of operations, and/or the relationship between addition and subtraction (**3.NBT.2**). They use addition and subtraction methods developed in grade two, where they began to add and subtract within 1000 without the expectation of full fluency and used at least one method that generalizes readily to larger numbers—so this is a relatively small and incremental expectation for third-graders. Such methods continue to be the focus in grade three, and thus the extension at grade four to generalize these methods to larger numbers (up to 1,000,000) should also be relatively easy and rapid.

Students in grade three also multiply one-digit whole numbers by multiples of 10 (**3.NBT.3**) in the range 10–90, using strategies based on place value and properties of operations (e.g., "I know $5 \times 90 = 450$ because $5 \times 9 = 45$, and so 5×90 should be 10 times as much"). Students also interpret 2×40 as 2 groups of 4 tens or 8 groups of ten. They understand that 5×60 is 5 groups of 6 tens or 30 tens, and they know 30 tens are 300. After developing this understanding, students begin to recognize the patterns in multiplying by multiples of 10 (ADE 2010). The ability to multiply one-digit numbers by multiples of 10 can support later student learning of standard algorithms for multiplication of multi-digit numbers.

Domain: Number and Operations—Fractions

In grade three, students develop an understanding of fractions as numbers. They begin with unit fractions by building on the idea of partitioning a whole into equal parts. Student proficiency with fractions is essential for success in more advanced mathematics such as percentages, ratios and proportions, and algebra.

Number and Operations—Fractions[7]	**3.NF**
Develop understanding of fractions as numbers.	
1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.	
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.	
a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.	
b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.	

7. Grade-three expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

In grades one and two, students partitioned circles and rectangles into two, three, and four equal shares and used fraction language (e.g., *halves, thirds, half of, a third of*). In grade three, students begin to enlarge their concept of *number* by developing an understanding of fractions as numbers (adapted from PARCC 2012).

Grade-three students understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts and the fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$ (**3.NF.1▲**).

Focus, Coherence, and Rigor

When working with fractions, teachers should emphasize two main ideas:

- Specifying the whole
- Explaining what is meant by "equal parts"

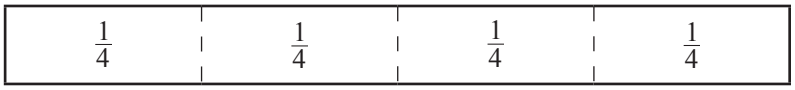
Student understanding of fractions hinges on understanding these ideas.

To understand fractions, students build on the idea of *partitioning* (dividing) a whole into equal parts. Students begin their study of fractions with *unit fractions* (fractions with the numerator 1), which are formed by partitioning a whole into equal parts (the number of equal parts becomes the denominator). One of those parts is a unit fraction. An important goal is for students to see unit fractions as the basic building blocks of all fractions, in the same sense that the number 1 is the basic building block of whole numbers. Students make the connection that, just as every whole number is obtained by combining a sufficient number of 1s, every fraction is obtained by combining a sufficient number of unit fractions (adapted from UA Progressions Documents 2013a). They explore fractions first, using concrete models such as fraction bars and geometric shapes, and this culminates in understanding fractions on the number line.

Examples **3.NF.1▲**

Teacher: Show fourths by folding the piece of paper into equal parts.

Student: I know that when the number on the bottom is *4*, I need to make four equal parts. By folding the paper in half once and then again, I get four parts, and each part is equal. Each part is worth $\frac{1}{4}$.

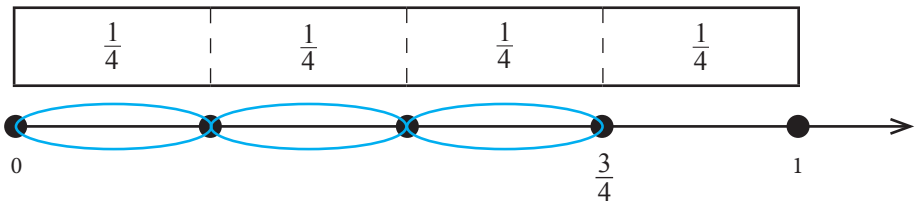


Teacher: Shade $\frac{3}{4}$ using the fraction bar you created.

Student: My fraction bar shows fourths. The *3* tells me I need three of them, so I'll shade them. I could have shaded any three of them and I would still have $\frac{3}{4}$.

Teacher: Explain how you know your mark is in the right place. **3.NF.2b▲**

Student (Solution): When I use my fraction strip as a measuring tool, it shows me how to divide the unit interval into four equal parts (since the denominator is *4*). Then I start from the mark that has *0* and measure off three pieces of $\frac{1}{4}$ each. I circled the pieces to show that I marked three of them. This is how I know I have marked $\frac{3}{4}$.

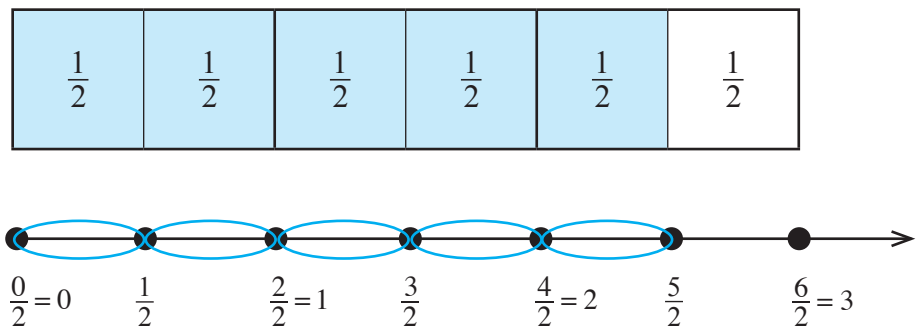


Third-grade students need opportunities to place fractions on a number line and understand fractions as a related component of the ever-expanding number system. The number line reinforces the analogy between fractions and whole numbers. Just as 5 is the point on the number line reached by marking off 5 times the length of the unit interval from 0, so is $\frac{5}{3}$ the point obtained by marking off 5 times the length of a different interval as the basic unit of length, namely the interval from 0 to $\frac{1}{3}$.

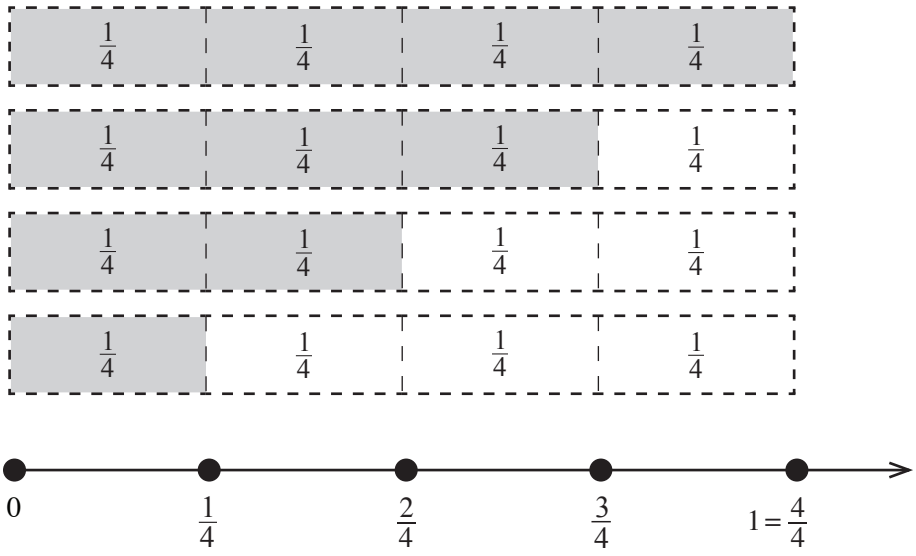
Fractions Greater Than 1

California's Common Core State Standards for Mathematics do not designate fractions greater than 1 as "improper fractions." Fractions greater than 1, such as $\frac{5}{2}$, are simply numbers in themselves and are constructed in the same way as other fractions.

Thus, to construct $\frac{5}{2}$ we might use a fraction strip as a measuring tool to mark off lengths of $\frac{1}{2}$. Then we count five of those halves to get $\frac{5}{2}$.



Students recognize that when examining fractions with common denominators, the wholes have been divided into the same number of equal parts, so the fraction with the larger numerator has the larger number of equal parts. Students develop an understanding of the numerator and denominator as they label each fractional part based on how far it is from 0 to the endpoint (**MP.7**).



Number and Operations—Fractions[8] 3.NF

Develop understanding of fractions as numbers.

3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
   b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form* $3 = \frac{3}{1}$*; recognize that* $\frac{6}{1} = 6$*; locate* $\frac{4}{4}$ *and 1 at the same point of a number line diagram.*
   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Students develop an understanding of fractions as they use visual models and a number line to represent, explain, and compare unit fractions, equivalent fractions (e.g., $\frac{1}{2} = \frac{2}{4}$), whole numbers as fractions (e.g., $3 = \frac{3}{1}$), and fractions with the same numerator (e.g., $\frac{4}{3}$ and $\frac{4}{6}$) or the same denominator (e.g., $\frac{4}{8}$ and $\frac{5}{8}$) [**3.NF.2–3▲**].

Students develop an understanding of order in terms of position on a number line. Given two fractions—thus two points on the number line—students understand that the one to the left is said to be smaller and the one to the right is said to be larger (adapted from UA Progressions Documents 2013a).

Students learn that when comparing fractions, they need to look at the size of the parts and the number of the parts. For example, $\frac{1}{8}$ is smaller than $\frac{1}{2}$ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole of the same size is cut into 2 pieces.

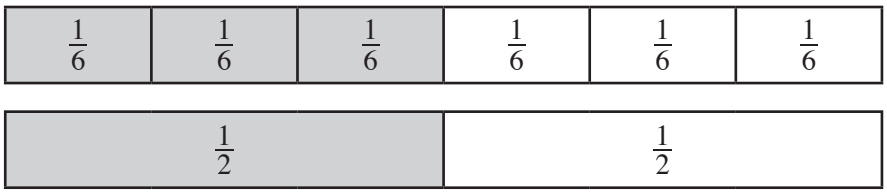
To compare fractions that have the same numerator but different denominators, students understand that each fraction has the same number of equal parts but the size of the parts is different. Students can infer that the same number of smaller pieces is less than the same number of bigger pieces (adapted from ADE 2010 and KATM 2012, 3rd Grade Flipbook).

Students develop an understanding of equivalent fractions as they compare fractions using a variety of visual fraction models and justify their conclusions (**MP.3**). Through opportunities to compare fraction models with the same whole divided into different numbers of pieces, students identify fractions that show the same amount or name the same number, learning that they are equal (or *equivalent*).

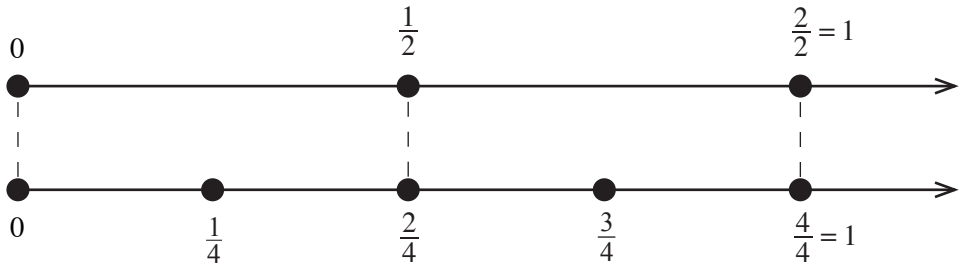
8. Grade-three expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Using Models to Understand Basic Fraction Equivalence

Fraction bars



Number line



Adapted from UA Progressions Documents 2013a.

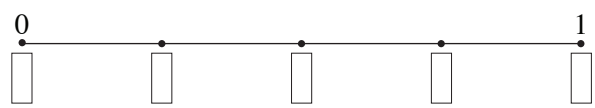
Important Concepts Related to Understanding Fractions

- Fractional parts must be the same size.
- The number of equal parts tells how many make a whole.
- As the number of equal pieces in the whole increases, the size of the fractional pieces decreases.
- The size of the fractional part is relative to the whole.
- When a shape is divided into equal parts, the denominator represents the number of equal parts in the whole and the numerator of a fraction is the count of the demarcated congruent or equal parts in a whole (e.g., $\frac{3}{4}$ means that there are 3 one-fourths or 3 out of 4 equal parts).
- Common benchmark numbers such as 0, $\frac{1}{2}$, $\frac{3}{4}$, and 1 can be used to determine if an unknown fraction is greater or smaller than a benchmark fraction.

Adapted from ADE 2010 and KATM 2012, 3rd Grade Flipbook.

Illustrative Mathematics offers a Fractions Progression Module (http://www.illustrativemathematics.org/pages/fractions_progression [Illustrative Mathematics 2013k]) that provides an overview of fractions. Table 3-4 presents a sample classroom activity that connects the Standards for Mathematical Content and Standards for Mathematical Practice.

Table 3-4. Connecting to the Standards for Mathematical Practice—Grade Three

Standards Addressed	Explanation and Examples
Connections to Standards for Mathematical Practice **MP.2.** Students reason quantitatively as they determine why a placement was correct or incorrect and assign a fractional value to a distance. **MP.4.** Students use the number line model for fractions. Although this is not an application of mathematics to a real-world situation in the true sense of *modeling*, it is an appropriate use of modeling for the grade level. **MP.8.** Students see repeated reasoning in dividing up the number line into equal parts (of varied sizes) and form the basis for how they would place fifths, tenths, and other fractions. **Standards for Mathematical Content** **3.NF.1.** Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$. **3.NF.2.** Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line. b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line. **3.NF.3.** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. b. Recognize and generate simple equivalent fractions (e.g., $\frac{1}{2}=\frac{2}{4}$, $\frac{4}{6}=\frac{2}{3}$). Explain why the fractions are equivalent, e.g., by using a visual fraction model.	**Task: The Human Fraction Number Line Activity.** In this activity, the teacher posts a long sheet of paper on a wall of the classroom to act as a number line, with 0 marked at one end and 1 marked at the other. Gathered around the wall, groups of students are given cards with different-sized fractions indicated on them—for example, $\frac{0}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$—and are asked to locate themselves along the number line according to the fractions assigned to them. Depending on the size of the class and the length of the number line, fractions with denominators 2, 3, 4, 6, and 8 may be used. The teacher can ask students to explain to each other why their placements are correct or incorrect, emphasizing that the students with cards marked in fourths, say, have divided the number line into four equal parts. Furthermore, a student with the card $\frac{a}{b}$ is standing in the correct place if he or she represents a lengths of size $\frac{1}{b}$ from 0 on the number line. As a follow-up activity, teachers can give students several unit number lines that are marked off into equal parts but that are unlabeled. Students are required to fill in the labels on the number lines. An example is shown here:  **Classroom Connections.** There are several big ideas included in this activity. One is that when talking about fractions as points on a number line, the whole is represented by the *length* or amount of distance from 0 to 1. By requiring students to physically line up in the correct places on the number line, the idea of partitioning this distance into equal parts is emphasized. In addition, other students can physically mark off the placement of fractions by starting from 0 and walking the required number of lengths $\frac{1}{b}$ from 0; for example, with students placed at the locations for sixths, another student can start at 0 and walk off a distance of $\frac{5}{6}$. As an extension, teachers can have students mark off equivalent fraction distances, such as $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$, and can discuss why those fractions represent the same amount.

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[9] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[10]

Students have experience telling and writing time from analog and digital clocks to the hour and half hour in grade one and in five-minute intervals in grade two. In grade three, students write time to the nearest minute and measure time intervals in minutes. Students solve word problems involving addition and subtraction of time intervals in minutes and represent these problems on a number line (**3.MD.1▲**).

Students begin to understand the concept of continuous measurement quantities, and they add, subtract, multiply, or divide to solve one-step word problems involving such quantities. Multiple opportunities to weigh classroom objects and fill containers will help students develop a basic understanding of the size and weight of a liter, a gram, and a kilogram (**3.MD.2▲**).

Focus, Coherence, and Rigor

Students' understanding and work with measuring and estimating continuous measurement quantities, such as liquid volume and mass (**3.MD.2▲**), are an important context for the fraction arithmetic they will experience in later grade levels.

Measurement and Data 3.MD

Represent and interpret data.

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*
4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

9. Excludes compound units such as cm^3 and finding the geometric volume of a container.

10. Excludes multiplicative comparison problems (problems involving notions of "times as much"; see glossary, table GL-5).

In grade three, the most important development in data representation for categorical data is that students draw picture graphs in which each picture represents more than one object, and they draw bar graphs in which the scale uses multiples, so the height of a given bar in tick marks must be multiplied by the scale factor to yield the number of objects in the given category. These developments connect with the emphasis on multiplication in this grade (adapted from UA Progressions Documents 2011b).

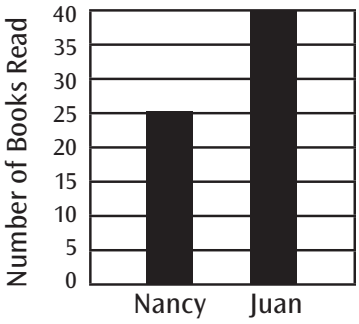
Students draw a scaled pictograph and a scaled bar graph to represent a data set and solve word problems (**3.MD.3**).

Examples **3.MD.3**

Students might draw or reference a pictograph with symbols that represent multiple units.

Number of Books Read	
Nancy	
Juan	
Note: represents 5 books.	

Students might draw or reference bar graphs to solve related problems.



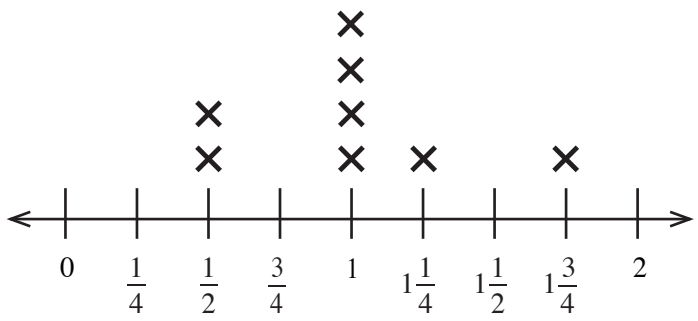
Adapted from KATM 2012, 3rd Grade Flipbook.

Focus, Coherence, and Rigor

Pictographs and scaled bar graphs offer a visually appealing context and support major work in the cluster "Represent and solve problems involving multiplication and division" as students solve multiplication and division word problems (**3.OA.3▲**).

Students use their knowledge of fractions and number lines to work with measurement data involving fractional measurement values. They generate data by measuring lengths using rulers marked with halves and fourths of an inch and create a line plot to display their findings (**3.MD.4**) [adapted from UA Progressions Documents 2011b].

For example, students might use a line plot to display data.



Adapted from NCDPI 2013b.

A critical area of instruction at grade three is for students to develop an understanding of the structure of rectangular arrays and of area measurement.

Measurement and Data	**3.MD**

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
   a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
   b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
7. Relate area to the operations of multiplication and addition.
   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
   b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
   c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b+c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
   d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

Students recognize area as an attribute of plane figures, and they develop an understanding of concepts of area measurement (**3.MD.5▲**). They discover that a square with a side length of 1 unit, called "a unit square," is said to have one square unit of area and can be used to measure area.

Students measure areas by counting unit squares (square centimeters, square meters, square inches, square feet, and improvised units) [**3.MD.6▲**]. Students develop an understanding of using square units to measure area by using different-sized square units, filling in an area with the same-sized square units, and then counting the number of square units.

Students relate the concept of area to the operations of multiplication and addition and show that the area of a rectangle can be found by multiplying the side lengths (**3.MD.7▲**). Students make sense of these quantities as they learn to interpret measurement of rectangular regions as a multiplicative relationship of the number of square units in a row and the number of rows. Students should understand and explain why multiplying the side lengths of a rectangle yields the same measurement of area as counting the number of tiles (with the same unit length) that fill the rectangle's interior. For example, students might explain that one length tells the number of unit squares in a row and the other length tells how many rows there are (adapted from UA Progressions Documents 2012a).

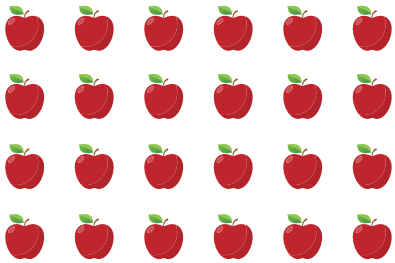
Students need opportunities to tile a rectangle with square units and then multiply the side lengths to show that they both give the area. For example, to find the area, a student could count the squares or multiply $4 \times 3 = 12$.

1	2	3	4
5	6	7	8
9	10	11	12

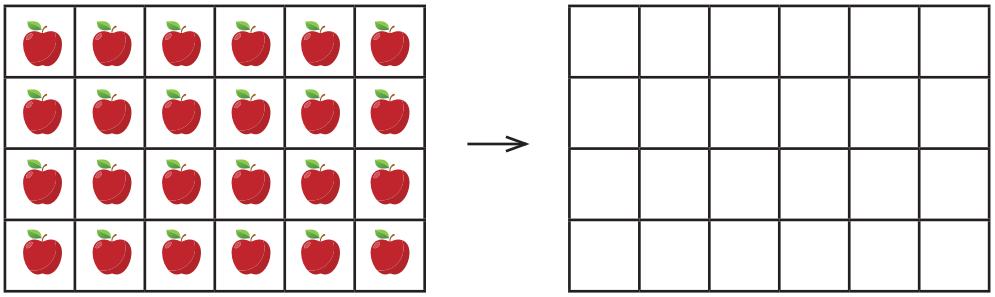
The transition from counting unit squares to multiplying side lengths to find area can be aided when students see the progression from multiplication as equal groups to multiplication as a total number of objects in an array, and then see the area of a rectangle as an array of unit squares. An example is presented below.

Students see multiplication as counting objects in equal groups—for example, 4×6 as 4 groups of 6 apples:

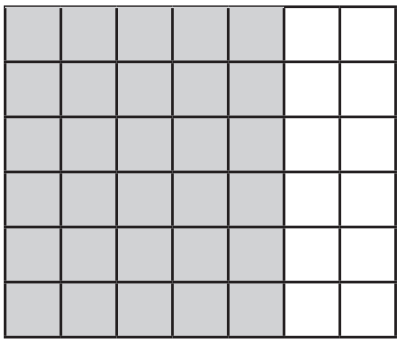
They see the objects arranged in arrays, as in a 4×6 array of the same apples:



They eventually see that finding area by counting unit squares is like counting an array of objects, where the objects are unit squares.



Students use area models to represent the distributive property in mathematical reasoning. For example, the area of a 6×7 figure can be determined by finding the area of a 6×5 figure and a 6×2 figure and adding the two products.

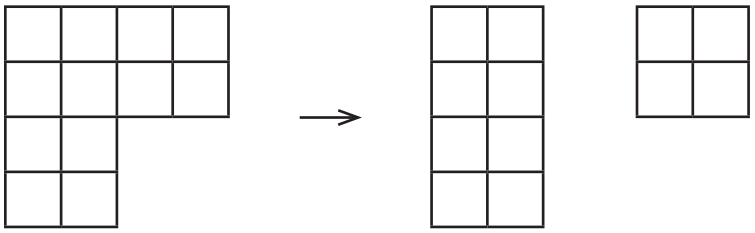


6×5 6×2

Students recognize area as additive and find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts.

Example **3.MD.7d▲**

The standards mention *rectilinear figures*. A *rectilinear figure* is a polygon whose every angle is a right angle. Such figures can be decomposed into rectangles to find their areas.



Area of rectilinear figure = ? 4×2 2×2

By breaking the figure into two pieces, it becomes easier to see that the area of the figure is $8 + 4 = 12$ square units.

Adapted from NCDPI 2013b.

Students apply these techniques and understandings to solve real-world problems.

Focus, Coherence, and Rigor

The use of area models (**3.MD.7▲**) also supports multiplicative reasoning, a major focus in grade three in the domain "Operations and Algebraic Thinking" (**3.OA.1–9▲**). Students must begin work with multiplication and division at or near the start of the school year to allow time for understanding and to develop fluency with these skills. Because area models for products are an important part of this process (**3.MD.7▲**), work on concepts of area (**3.MD.5–6▲**) should begin at or near the start of the year as well (adapted from PARCC 2012).

Measurement and Data 3.MD

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

8. Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

In grade three, students solve real-world and mathematical problems involving perimeters of polygons (**3.MD.8**). Students can develop an understanding of the concept of *perimeter* as they walk around the perimeter of a room, use rubber bands to represent the perimeter of a plane figure with whole-number side lengths on a geoboard, or trace around a shape on an interactive whiteboard. They find the perimeter of objects, use addition to find perimeters, and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles. They explain their reasoning to others. Given a perimeter and a length or width, students use objects or pictures to find the unknown length or width. They justify and communicate their solutions using words, diagrams, pictures, and numbers (adapted from ADE 2010).

Domain: Geometry

Reason with shapes and their attributes.

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as $1/4$ of the area of the shape.*

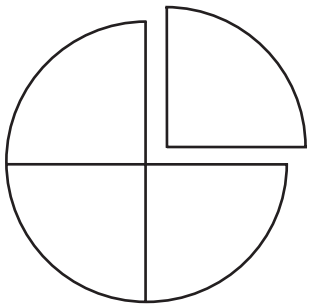
A critical area of instruction at grade three is for students to describe and analyze two-dimensional

shapes. Students compare common geometric shapes (e.g., rectangles and quadrilaterals) based on common attributes, such as four sides (**3.G.1**). In earlier grades, students informally reasoned about particular shapes through sorting and classifying based on geometric attributes. Students also built and drew shapes given the number of faces, number of angles, and number of sides. In grade three, students describe properties of two-dimensional shapes in more precise ways, referring to properties that are shared rather than the appearance of individual shapes. For example, students could start by identifying shapes with right angles, explain and discuss why the remaining shapes do not fit this category, and determine common characteristics of the remaining shapes.

Students relate their work with fractions to geometry as they partition shapes into parts with equal areas and represent each part as a unit fraction of the whole (**3.G.2**).

Example **3.G.2**

The figure below was partitioned (divided) into four equal parts. Each part is $\frac{1}{4}$ of the total area of the figure.



Adapted from NCDPI 2013b.

Focus, Coherence, and Rigor

As students partition shapes into parts with equal areas (**3.G.2**), they also reinforce concepts of area measurement and fractions that are part of the major work at the grade in the clusters "Geometric measurement: understand concepts of area and relate area to multiplication and to addition" (**3.MD.5–7▲**) and "Develop understanding of fractions as numbers" (**3.NF▲**).

Adapted from PARCC 2012.

Essential Learning for the Next Grade

In kindergarten through grade five, the focus is on the addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals, with a balance of concepts, procedural skills, and problem solving. Arithmetic is viewed as an important set of skills and also as a thinking subject that, when done thoughtfully, prepares students for algebra. Measurement and geometry develop alongside number and operations and are tied specifically to arithmetic along the way. Multiplication and division of whole numbers and fractions are an instructional focus in grades three through five.

To be prepared for grade-four mathematics, students should be able to demonstrate they have acquired certain mathematical concepts and procedural skills by the end of grade three and have met the fluency expectations for the grade. For third-graders, the expected fluencies are to add and subtract within 1000 using strategies and algorithms (**3.NBT.2**), multiply and divide within 100 using various strategies, and know all products of two one-digit numbers from memory (**3.OA.7▲**). These fluencies and the conceptual understandings that support them are foundational for work in later grades.

Of particular importance for grade four are concepts, skills, and understandings needed to represent and solve problems involving multiplication and division (**3.OA.1–4▲**); understand properties of multiplication and the relationship between multiplication and division (**3.OA.5–6▲**); multiply and divide within 100 (**3.OA.7▲**); solve problems involving the four operations and identify and explain patterns in arithmetic (**3.OA.8–9▲**); develop understanding of fractions as numbers (**3.NF.1–3▲**); solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects (**3.MD.1–2▲**); and geometric measurement—concepts of area and relating area to multiplication and to addition (**3.MD.5–7▲**).

Multiplication and Division

By the end of grade three, students develop both conceptual understanding and procedural skills of multiplication and division. Students are expected to fluently multiply and divide within 100 and to know from memory all of the products of two one-digit numbers (**3.OA.7▲**). Fluency in multiplication and division within 100 includes understanding and being able to apply strategies such as using mental math, understanding division as an unknown-factor problem, applying the properties of operations, and identifying arithmetic patterns. Students also need to understand the relationship between multiplication and division and apply that understanding by using inverse operations to verify the reasonableness of their answers. Students with a firm grasp of grade-three multiplication and division can apply their knowledge to interpret, solve, and even compose simple word problems, including problems involving equal groups, arrays, and measurement quantities. Fluency in multiplication and division ensures that when students know from memory all of the products of two one-digit numbers, they have an understanding of the two operations—and have not merely learned to produce answers through rote memorization.

Fractions

In grade three, students are formally introduced to fractions as numbers, thus broadening their understanding of the number system. Students must understand that fractions are composed of unit fractions; this is essential for their ongoing work with the number system. Students must be able to place fractions on a number line and use the number line as a tool to compare fractions and recognize equivalent fractions. They should be able to use other visual models to compare fractions. Students also must be able to express whole numbers as fractions and place them on a number line. It is essential for students to understand that the denominator determines the number of equally sized pieces that make up a whole and the numerator determines how many pieces of the whole are being referred to in the fraction.

Addition and Subtraction

By the end of grade three, students are expected to fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction (**3.NBT.2**). This fluency is both the culmination of work at previous grade levels and preparation for solving multi-step word problems using all four operations beginning in grade four. Students should be able to use more than one strategy to add or subtract and should also be able to relate the strategies they use to a written method.

Grade 3 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten

- Use place-value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions

- Develop understanding of fractions as numbers.

Measurement and Data

- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Represent and interpret data.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 3.OA

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as* 5×7.

2. Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as* $56 \div 8$.

3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[11]

4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 \times ? = 48,\ 5 = \square \div 3,\ 6 \times 6 = ?$.

Understand properties of multiplication and the relationship between multiplication and division.

5. Apply properties of operations as strategies to multiply and divide.[12] *Examples: If* $6 \times 4 = 24$ *is known, then* $4 \times 6 = 24$ *is also known. (Commutative property of multiplication.)* $3 \times 5 \times 2$ *can be found by* $3 \times 5 = 15$, *then* $15 \times 2 = 30$, *or by* $5 \times 2 = 10$, *then* $3 \times 10 = 30$. *(Associative property of multiplication.) Knowing that* $8 \times 5 = 40$ *and* $8 \times 2 = 16$, *one can find* 8×7 *as* $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. *(Distributive property.)*

6. Understand division as an unknown-factor problem. *For example, find* $32 \div 18$ *by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[13]

9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

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11. See glossary, table GL-5.

12. Students need not use formal terms for these properties.

13. This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Number and Operations in Base Ten — 3.NBT

Use place-value understanding and properties of operations to perform multi-digit arithmetic.[14]

1. Use place-value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.

Number and Operations—Fractions[15] — 3.NF

Develop understanding of fractions as numbers.

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
   a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.
   b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
   b. Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.*
   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

14. A range of algorithms may be used.

15. Grade-three expectations in this domain are limited to fractions with denominations 2, 3, 4, 6, and 8.

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[16] Add, subtract, multiply, or divide to solve one-step word involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[17]

Represent and interpret data.

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
   a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
   b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

7. Relate area to the operations of multiplication and addition.
   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
   b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
   c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b+c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
   d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

8. Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

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16. Excludes compound units such as cm^3 and finding the geometric volume of a container.

17. Excludes multiplicative comparison problems (problems involving notions of "times as much"; see glossary, table GL-5).

Geometry 3.G

Reason with shapes and their attributes.

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as* $1/4$ *of the area of the shape.*