
Criteria for Evaluating Mathematics Instructional Materials

Adopted by the State Board of Education
March 9, 2005

Instructional materials adopted by the state help teachers present the content set forth in the *Mathematics Content Standards for California Public Schools* (referred to in this publication as the *California Mathematics Standards*). To accomplish this purpose, this chapter establishes criteria for evaluating the instructional materials for the six-year adoption cycle beginning with the primary adoption in 2007. These criteria serve as evaluation guidelines for the statewide adoption of mathematics instructional materials for kindergarten through grade eight, as defined in *Education Code* Section 60010.

The weight of textbooks is not a part of the criteria for evaluating instructional materials; however, beginning with the 2007 kindergarten through grade eight mathematics primary adoption, maximum weight standards for elementary and secondary school textbooks will apply to adopted materials (pursuant to Assembly Bill 2532, Chapter 1096, Statutes of 2002). Textbook publishers will be required to provide local school districts with options for lighter-weight materials, such as split volumes, electronic editions, or classroom sets, for textbooks that do not comply with the maximum standards approved by the California State Board of Education. Those weight standards are three pounds for kindergarten through grade four, four pounds for grades five through eight, and five pounds for grades nine through twelve. More information about implementing the weight standards will be included in the *Invitation to Submit*, which contains guidelines for publishers planning to submit materials for possible adoption.

The *California Mathematics Standards* are rigorous. Instructional materials play a central role in helping students to achieve the standards. Organized by grade level from kindergarten through grade seven, the mathematics content standards are presented in five strands: Number Sense; Algebra and Functions; Measurement and Geometry; Statistics, Data Analysis, and Probability; and Mathematical Reasoning. However, publishers are not required to adhere to this organization of strands as long as they address all the individual standards. In addition, the order of the mathematics standards does not imply a required organization for the materials within a grade level or discipline. Instructional materials may contain groups of related standards and address them simultaneously for coherence and utility.

The standards for grades eight through twelve are organized differently from those for kindergarten through grade seven. Mathematics in the higher grades is not organized by strand because the mathematics studied in grades eight through twelve falls naturally under the disciplines algebra, geometry, and so forth. Local

educational agencies and teachers have flexibility in planning courses of study because the standards for grades eight through twelve do not mandate that a particular discipline be initiated and completed in a single grade. Most schools teach the disciplines in traditional courses; others teach the material in an integrated program.¹ The content of each discipline must be covered, and students are expected to achieve the standards regardless of the sequence of the disciplines. Thus the content covered in an integrated program that combines Algebra I, geometry, and Algebra II must be the same content that is covered in traditional courses for those disciplines.

The acquisition of mathematical reasoning is intrinsically entwined in the learning, the doing, and the understanding of a particular portion of the mathematics content. Therefore, the standards under the Mathematical Reasoning strand are to be addressed not in isolation but by and through the presentation of content listed under the other strands.

Instructional materials adopted by the California State Board of Education, on the whole, should provide programs that will be effective for all students—those who have mastered most of the content taught in the earlier grades and those who may have significant gaps in learning. In addition, instructional materials must specifically address the needs of teachers who instruct a diverse student population. Publishers should select research-based pedagogical approaches that give teachers effective alternatives in teaching mathematics. Guidance on evaluating educational research can be found in the report *Identifying and Implementing Educational Practices Supported by Rigorous Evidence: A User Friendly Guide* (U.S. Department of Education 2003).²

The criteria require that instructional materials address the learning needs of students and that programs be submitted in one of three categories: **basic grade level**, **intervention**, and **algebra readiness**. Districts will decide, on the basis of individual assessment data, whether each student uses one or a combination of basic grade-level, intervention, or algebra readiness materials. Regardless of the type of program, instructional materials must provide all students with access to the concepts, skills, and problem-solving tools described in the *California Mathematics Standards*.

Student Access

The *Mathematics Framework* outlines strategies for successful diagnostic teaching by recommending instructional assistance designed to help all students, including struggling students, learn the key concepts in mathematics well so that they develop a foundation on which to build further mathematical understand-

¹ If a publisher submits an integrated program for grade eight, the entire program series must be submitted (e.g., to evaluate a three-year integrated Algebra I/geometry/Algebra II series, materials for all three years of the program would be reviewed to determine alignment with the Algebra I standards).

² This report appears on the following Web site: <http://www.ed.gov/rschstat/research/pubs/rigorousetid/index.html>.

ing. (The strategies for and categories of students are defined in Chapter 6, “Universal Access,” in this framework.) The descriptions that follow explain to publishers the correlation between the learning needs of students (defined in Chapter 6) and the three types of programs appropriate to use with students.

1. Benchmark Students

Benchmark students are making good progress toward achieving the standards but may be experiencing temporary or minor difficulties. Students’ needs must be addressed quickly, often by reteaching concepts in a different way. These students typically participate in a **basic grade-level program**.

2. Strategic Students

Some students may be a year below grade level. The regular classroom teacher can bring them to grade level, but their difficulties must be examined systematically and fully. These students may also need to take two periods of mathematics a day to master the content of the standards. Generally, these students participate in a **basic grade-level program with additional support**.

3. Intensive Students

Some students are at serious risk of not meeting the standards, as indicated by the students’ repeated low performance on valid and reliable measures. Generally, these students are performing two or more years below grade level. Intensive intervention and extended instructional time will be required. These students require an **intervention program** that focuses on foundational skills and concepts essential for students to succeed in a basic grade-level mathematics program. The program should not serve as a fixed-term course since the intent is for students to accelerate their achievement so that they can succeed in the basic grade-level program.

Three Types of Programs

Three types of programs will be considered for adoption: basic grade level (kindergarten through grade eight), intervention (grades four through seven), and algebra readiness (grade eight). All three types of programs must be stand-alone products and will be reviewed separately:

1. Basic Grade-Level Program (Kindergarten Through Grade Eight)

The basic grade-level program is the comprehensive mathematics curriculum for students in kindergarten through grade eight. It provides the foundation for instruction and is intended to ensure that all students master the *California Mathematics Standards*. This comprehensive curriculum should provide instructional content for at least 50 minutes per day. In addition to the basic material provided to cover the standards for students who are achieving at or near grade level (benchmark level), there should be instruction to support the learning of students who are at the strategic level. Instruction suitable for advanced learners should be included in a basic grade-level program as well.

2. Intervention Program (Grades Four Through Seven)

Students who have significant gaps in their knowledge of mathematics may benefit from a mathematics intervention program. These programs contain materials that teachers can use to support instruction in the six topical volumes described in Appendix E, “Mathematics Intervention and Algebra Readiness Instructional Materials.”

3. Algebra Readiness Program (Grade Eight or Above)

Although those students who have mastered the content described in the *California Mathematics Standards* for kindergarten through grade seven will be ready to take Algebra I in grade eight, other students will not have the necessary preparation and will not be ready for algebra. Those who are not prepared to take an algebra course in grade eight will require specialized instructional materials that focus on the prerequisite standards described in Appendix E. Algebra readiness instructional materials must be designed to serve students for a full year of instruction; schools, however, may use the materials in different instructional settings. Algebra readiness programs prepare students for success in Algebra I.

Valid and reliable diagnostic assessments embedded in the curriculum must be provided to inform the teacher of students’ strengths and weaknesses. Instruction that is based on the requisite standards, coupled with the information obtained from the assessments, should prepare students for algebra.

Evaluation Criteria

The criteria for the evaluation of mathematics instructional resources for kindergarten through grade eight are divided into five categories (these categories apply to all three types of programs).

1. **Mathematics content/alignment with the standards.** The content as specified in the *California Mathematics Standards* and presented in accord with the guidance provided in the *Mathematics Framework for California Public Schools*
2. **Program organization.** The sequence and organization of the mathematics program that provide structure for what students should learn each year in basic programs and for the duration of algebra readiness and mathematics intervention programs
3. **Assessment.** The strategies presented in the instructional materials for measuring what students know and are able to do
4. **Universal access.** Instructional materials that address the needs of special student populations, including students eligible for special education, students whose achievement is either significantly below or above that typical of their class or grade level, and students with special needs related to English-language proficiency
5. **Instructional planning and support.** The instructional planning and support information and materials, typically including a separate teacher’s edition to help teachers in implementing the mathematics program

Materials that fail to meet the criteria for mathematics content/alignment with the standards will not be considered satisfactory for adoption. Only those programs that meet all criteria in category 1 and that have strengths in each of categories 2 through 5 will be deemed worthy of adoption. Unless otherwise noted, the following information in categories 1 through 5 applies to all three types of programs:

Category 1. Mathematics Content/Alignment with the Standards

Mathematics materials should support teaching to the *California Mathematics Standards* in accord with the guidance provided in the *Mathematics Framework for California Public Schools*. Instructional materials suitable for adoption must satisfy the following criteria:

1. The mathematics content is correct, factually accurate, and written with precision. Mathematical terms are defined and used appropriately.
2. The materials in basic instructional programs support comprehensive teaching of the *California Mathematics Standards* at each grade level as detailed, discussed, and prioritized in Chapter 2, “The California Mathematics Content Standards,” and Chapter 3, “Grade-Level Considerations.” The materials in algebra readiness and mathematics intervention programs support comprehensive teaching, as detailed in Appendix E. So that the mathematics all students should know and be able to do is made clear, the only standards that may be referenced in any program are the California academic content standards developed under *Education Code* Section 60605, and the instructional design must reflect current and confirmed research. The materials must not conflict with the content in the *California Mathematics Standards* or the *Mathematics Framework*.
3. The attention given to each standard in the basic program is in accord with its level of emphasis in Chapter 3. Appendix E contains guidance for instructional materials for algebra readiness and mathematics intervention.
4. Mathematical topics are presented at broadly different levels of rigor and are written to bring students to the level of proficient or advanced performance (materials for advanced learners are not required for intervention programs).
5. For the basic program a substantial majority of the content relates directly to the *California Mathematics Standards* for each grade level, although standards from earlier grades may be reinforced, and a foundation for the mastery of later standards must be built at each grade level. For the intervention and algebra readiness programs, a substantial majority of the content relates directly to the subset of *California Mathematics Standards* selected for those programs (see Appendix E) and to the foundational concepts and skills necessary for student proficiency in the standards.
6. All content must be clearly relevant to mathematics. Unrelated topics not directly focused on the *California Mathematics Standards* and the *Mathematics Framework* are kept to a minimum.

7. A checklist of *California Mathematics Standards* is included in the teacher's guide, together with citations for page numbers for the standards or lists of other references that demonstrate alignment with the *California Mathematics Standards*, and, to the extent possible, the *Mathematics Framework*. Material referenced to show alignment with a standard in the Mathematical Reasoning strand should also be aligned with one or more standards outside that strand.
8. Concepts and procedures are explained and accompanied with examples to reinforce the lessons. **All formulas and theorems appropriate to the grade must be proven; informal or heuristic proofs are acceptable if a complete proof is provided in a later lesson or grade within the program.** Students are provided with sufficient material so that they can develop a complete understanding of the mathematical concepts and reasoning skills outlined in the *California Mathematics Standards*.
9. Many mathematical problems are provided to help develop automatic use of procedures and to foster the development of mathematical understanding. Strategies for solving various classes of problems are also provided. The types of problems are those that:
 - Help students develop and understand a concept.
 - Provide practice in learning a skill.
 - Provide practice with mental calculations.
 - Provide practice with written calculations.
 - Involve routine single-step calculations.
 - Involve multistep procedures.
 - Require a mathematical proof.
 - Are mathematically interesting and challenging.
 - Provide opportunities for mathematical, logical reasoning.
 - Are applications of previously learned mathematics.
10. Applications of the mathematics must be clearly marked as such and must not be equated with the mathematics itself or dictate the scope and sequence of the mathematics program.
11. Materials drawn from other subject-matter areas are scholarly and consistent with the content at the appropriate grade level in the currently adopted California curriculum framework for that subject-matter area.
12. Intervention programs are designed to accelerate the progress of students in mathematics in the shortest possible time so that they can begin to make progress, using the basic grade-level programs. To serve this purpose, intervention programs must provide targeted and explicit instruction on the subset of mathematics standards indicated in Appendix E and be free of unrelated or unnecessary content. (See "A Mathematics Intervention Program [Grades Four Through Seven]" in Appendix E for further explanation.)
13. Algebra readiness programs must target the specific subset of 16 mathematics standards listed in "Algebra Readiness (Grade Eight or Above)" in Appendix E by addressing the foundational concepts and skills from earlier grades

and by breaking each of the 16 standards into component concepts and skills. At a minimum, materials must address foundational skills and concepts that develop fluency with operations on whole numbers; representing fractions, mixed numbers, decimals, and percentages; operations on positive fractions; use of symbols to express verbal information; writing and solving simple linear equations; plotting points; interpreting ordered pairs from a graph; interpreting lengths of horizontal and vertical line segments on a coordinate plane; and graphing and interpreting relationships of the form $y = mx$. The foundational skills and concepts must be addressed repeatedly, building in depth and complexity and providing perspective and distributed practice.

Category 2. Program Organization

The sequence and organization of the mathematics program provide structure to what students should learn each year and allow teachers to convey the mathematics content efficiently and effectively. The program content is organized and presented in agreement with the guidance provided in the *Mathematics Framework*. To be considered suitable for adoption, instructional materials in mathematics must provide these essential components:

1. For the basic program, materials for each grade are developed in a logical order and increase in depth and complexity during each school year and from grade to grade. Materials for each grade are organized around the key topics presented in Chapter 3. For the intervention programs, materials must be organized around the six volumes and the subset of standards specified in Appendix E. No specific order of topics within these volumes is required, and volumes may be split into smaller units for publication. For the algebra readiness programs, no specific order of topics is required; however, materials must be organized so that foundational skills and concepts can be assessed and taught, as needed, before students are taught the subset of 16 standards for grade seven and Algebra I specified in Appendix E.
2. Concepts are developed in a logical order and increase in depth and complexity. The scope and sequence of the materials are presented as follows:
 - New concepts are introduced at a reasonable pace, and sufficient instructional and practice materials on all the important topics are provided.
 - The order in which topics are presented is mathematically and pedagogically sound.
 - Prerequisite skills and ideas are presented before the more complex topics that depend on them are introduced.
 - Mathematical content and instructional activities are sequenced to prevent common student misconceptions.
 - The connections between related topics are taught after each topic has been separately introduced to prevent confusing the two topics, and the organization of the material supports the understanding of these connections.
 - Student materials contain guidance to help build understanding of a topic, including references to earlier sections of the instructional program and summative reviews.

- Repetition and review are used to develop automaticity or to prepare for further learning.
 - Computational and procedural skills, conceptual understanding, and problem solving are interconnected and are included throughout the program.
 - Mathematical discussions are brought to closure. Examples of a lesson that has been brought to closure follow: The solutions to problems are complete, and the concepts that students should have learned have been summarized; new concepts and definitions introduced during the lesson have been discussed and emphasized; or the demonstration of a new theorem has been completed. If the demonstration has been postponed, the students know when it will be completed and which concepts will be involved.
3. Materials for teachers and students contain an overview of the chapters that clearly identifies the mathematical concepts. Materials must include tables of contents, indexes, and glossaries containing important mathematical terms.
 4. Materials must provide for individual study, in addition to classroom instruction, and for practice and tutoring outside the classroom.
 5. Those support materials provided, such as electronic learning resources or manipulatives, are an integral part of the instructional program and are clearly aligned with the content in the *California Mathematics Standards* and the *Mathematics Framework*.
 6. For grades four through eight, the relevant grade-level standards must be explicitly stated in both the teachers' and the students' editions.

Category 3. Assessment

Instructional materials, following the guidance provided in Chapter 5, "Assessment," in this publication, should contain strategies and tools for continually measuring student achievement. To be considered suitable for adoption, instructional materials in mathematics must provide these essential components:

1. In the basic and algebra readiness programs, guidance is given for the teacher in assessing each student's level of achievement at the beginning of the school year. This initial assessment should be comprehensive and help the teacher determine whether the student needs additional materials and resources to achieve the grade-level standards or intervention materials that reteach concepts and skills that should have been mastered previously. In the algebra readiness and the intervention programs, the entry-level assessments should identify which students need the program and their strengths and weaknesses.
2. Questions are provided to monitor students' comprehension during instruction.
3. Sufficient background information for teachers about the content is included.
4. Assessments, such as lesson quizzes and chapter and unit tests, which have valid content and measure individual student progress both at regular intervals

and at strategic points of instruction, are provided. The assessments are based on research and are designed to:

- Measure each student's skills and knowledge.
 - Monitor students' progress toward meeting the standards.
 - Provide summative evaluations of individual student achievement.
 - Identify students who are not making reasonable progress.
 - Monitor students' conceptual understanding, use of basic skills and procedures, and problem-solving ability.
 - Monitor students' reasoning, from informal explanations to formal proofs.
 - Provide multiple methods of assessing what students know and are able to do.
 - Help the teacher determine the effectiveness of instruction.
 - Help the teacher keep parents and students informed about students' progress.
5. Suggestions are given on how to use assessment data to guide decisions about instructional practices and how to modify an instructional program so that all students continually progress toward meeting or exceeding the standards.
 6. In the intervention program frequent diagnostic assessments are provided to tailor instruction to the standards with which students are having difficulty. The program should include an initial assessment to determine students' placement in a program (e.g., referenced to the six volumes or sections within each volume to be used or both), diagnostic assessments to identify areas of strengths and weaknesses, formative assessments to demonstrate students' progress toward meeting identified benchmarks, and a summative assessment to determine whether a student has mastered the materials. For example, Grade 2 Number Sense Standard 2.2 (Find the sum or difference of two whole numbers up to three digits long) involves both addition and subtraction and covers a range of component concepts and skills. Assessments on this standard should identify whether students have difficulty because they do not understand place value, lack a knowledge of basic facts, or make errors in regrouping (carrying or borrowing) or in keeping the digits in place-value columns, and so forth. In this and other cases, diagnostic assessment should be informed by the types of errors students are apt to make in each content area. Teachers' editions should help educators select and use assessment tools that provide student data to help educators meet the instructional needs of students. All materials should include information and strategies for making the lessons accessible to all categories of special needs students.
 7. In the algebra readiness program, extensive diagnostic assessments on the foundational concepts and skills from earlier grades are provided, as indicated in Appendix E. Those assessments can be used to guide instruction. For the 16 specified standards that make up the program, extensive diagnostic assessments must also be provided to guide instruction on the standards and on the component concepts and skills.

Category 4. Universal Access

Students with special needs must have access to the same academic standards-based curriculum provided to all students, as set forth in Chapter 6. Instructional materials must conform to the policies of the California State Board of Education and to other applicable state and federal requirements regarding diverse populations and students with special needs. To be considered suitable for adoption, instructional materials in mathematics must provide these essential components:

1. Comprehensive guidance and strategies, based on current and confirmed research, to adapt the curriculum to meet students' identified special needs and to provide effective, efficient instruction for all students. Strategies may include:
 - Suggestions that describe specific ways to address the learning needs of benchmark, strategic, or intensive students (as defined in Chapter 6)
 - Suggestions for reinforcing or expanding the curriculum to meet the needs of benchmark, strategic, and intensive students (as defined in Chapter 6) and for grouping students within or across grade levels to accommodate a wide range of achievement levels
 - Additional instructional time and practice, especially in key standards, including specialized teaching methods or materials and accommodations for students with special needs
 - Special help for students who are below grade level, including clearer explanations, with ample opportunities for review and practice or other assistance to help accelerate students' performance to grade level
2. Suggestions on how to help strategic or intensive students learn the key concepts in mathematics in the basic program and have access to grade-level content
3. In the basic program, teachers' and students' editions that include alternatives for advanced students that allow students to accelerate beyond their grade-level content (acceleration) or to study the content in the *California Mathematics Standards* in greater depth or complexity (enrichment)
4. Instructional materials designed to help meet the needs of students whose reading, writing, listening, and speaking skills fall below grade level. (Materials should help students understand and use appropriate academic language in mathematics.)
5. Evidence of adherence to the "Design Principles for Perceptual Alternatives," pursuant to *Education Code* Section 60061.8, as described in Appendix F, "Design Principles"

Category 5. Instructional Planning and Support

Instructional materials must contain a clear road map for teachers to follow when they are planning instruction. The materials should be designed to help teachers implement a mathematics program that ensures opportunities for all students to learn the essential skills and knowledge called for in the *California*

Mathematics Standards and the *Mathematics Framework*. To be considered suitable for adoption, instructional materials in mathematics must provide these essential components:

1. A teacher's edition that includes ample and useful annotations and suggestions on how to present the content in the students' edition and in the ancillary materials
2. A checklist of lessons in the teacher's edition cross-referencing the standards covered and detailing the time necessary for instruction
3. Lesson plans, including suggestions for organizing resources in the classroom and for pacing lessons (Pacing plans should allow for instruction at a reasonable pace and give particular attention to topics emphasized in the framework.)
4. Clear, grade-appropriate explanations of mathematics concepts in a form that teachers can easily adapt for classroom presentation
5. Strategies to anticipate, identify, address, and correct common student errors and misconceptions
6. A system for accelerating or decelerating the rate at which new material is introduced to students in a manner suited to their ability to assimilate new material
7. Different kinds of lessons and alternative ways in which to explain concepts, offering teachers choice and flexibility in implementing the program
8. Prioritization of critical components of lessons (Learning objectives and instruction and the relationship of lessons to standards or skills within standards are explicit.)
9. Review and practice problems, as described in Chapter 4, "Instructional Strategies," which are distributed in the program to enhance students' understanding and to promote generalization and transfer of skills and knowledge
10. Materials designed to help teachers identify the reason that students may find a particular type of problem more challenging than another (e.g., identify component skills not mastered) and to point to specific remedies
11. Standards-based goals that are explicitly and clearly associated with instruction and assessment
12. All parts of the program so that teachers have little or no need to identify, gather, or develop supplementary materials (For example, blackline masters are designed to minimize dark areas on a page so that toner is not wasted during photocopying. Answer keys are provided for all workbooks and other related student activities.)
13. All required manipulatives or suggestions for inexpensive alternatives (Manipulatives must be designed to promote student learning consistent with the content in the *California Mathematics Standards*, and clear instructions must be provided for their efficient use.)

14. For a basic program a teacher's edition that explains the role of the specific grade-level mathematics in the context of the overall mathematics curriculum for kindergarten through grade twelve
15. A teacher's edition that contains full, adult-level explanations and examples of the more advanced mathematics concepts in the lessons so that teachers can improve their knowledge of the subject, as necessary
16. Charts showing the cost of staff development services and the time necessary for preparing teachers to fully implement the mathematics program
17. Technical support and suggestions for appropriate use of resources associated with a unit: audiovisual, multimedia, and information technology
18. Homework activities that extend and reinforce classroom instruction and provide additional practice of skills and development of concepts that have been taught (These activities are optional for kindergarten through grade two.)
19. Strategies for informing parents or guardians about the mathematics program and suggestions for ways in which they can help support student achievement
20. In intervention and algebra readiness programs, suggestions for how to use the materials in different instructional settings (A key feature of these programs is the close link between diagnostic assessment and guidance on instruction. In particular, intervention programs must be guided by diagnostic assessments so that, in the shortest possible time, students can begin to make progress, using the basic grade-level programs. Algebra readiness programs must provide diagnostic assessments on the foundational concepts and skills and must contain lessons that can be implemented in the classroom, as needed, to rebuild the missing foundational content.) Strategies from Appendix E follow:
 - Instructional materials for algebra readiness focus on a subset of standards. The specialized focus of this program is students' mastery of arithmetic. It provides the opportunity for coverage of the specified standards in depth and support for a variety of instructional strategies, including various ways to explain a concept and to develop students' conceptual understanding.
 - Instructional materials prioritize the concepts and skills to be taught so that the teacher can make optimal use of time and resources.
 - Instructional materials provide an adequate sample of the range of examples that illustrate each concept.
 - Instructional materials include extensive diagnostic components to guide instruction. Diagnosis may often apply to the many smaller, embedded concepts and skills and not to a whole standard.
 - Instructional materials reflect the interests and the ages of the students.
 - Instructional materials provide assistance in the specialized vocabulary of mathematics and in the academic language of instruction, including instructional strategies in the teacher's edition for approaches appropriate for English learners.