Instructional Materials to Support the California Common Core State Standards for Mathematics Chapter

of the

Mathematics Framework

for California Public Schools: Kindergarten Through Grade Twelve

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Instructional Materials to Support the California Common Core State Standards for Mathematics

Ithough instructional resources have changed over the years from slate boards and chalk to interactive whiteboards, one thing remains true: high-quality instructional resources help teachers to teach and students to learn. Instructional resources are an important component in the implementation of the California Common Core State Standards for Mathematics (CA CCSSM). They should be selected with great care and with the instructional needs of all students in mind.

Instructional resources for mathematics include a variety of instructional materials—tools such as connectable cubes, rulers, protractors, graph paper, calculators, and objects to count; and technology such as interactive whiteboards and student-response devices. The term *instructional materials* is broad-ly defined to include textbooks, technology-based materials, other educational materials, and tests. This chapter provides guidance on the selection of instructional materials, including the state adoption of instructional materials, guidance for local districts on the adoption of instructional materials for students in grades nine through twelve, the social content review process, supplemental instructional materials, and accessible instructional materials.

State Adoption of Instructional Materials

The California State Board of Education (SBE) adopts instructional materials for use by students in kindergarten through grade eight. Under current state law, local educational agencies (LEAs)—school districts, charter schools, and county offices of education—are not required to purchase state-adopted instructional materials. LEAs have the authority and the responsibility to conduct their own evaluation of instructional materials and to adopt the materials that best meet the needs of their students. Additionally, there is no state-level adoption of instructional materials for use by students in grades nine through twelve; LEAs have the sole responsibility and authority to adopt instructional materials for those students.

The primary source of guidance for the selection of instructional materials is the *Criteria for Evaluating Mathematics Instructional Materials for Kindergarten Through Grade Eight (Criteria)*, adopted by the SBE on January 16, 2013 (see next page). The *Criteria* document provides a comprehensive description of effective instructional programs that are aligned with the CA CCSSM and support the principles of focus, coherence, and rigor. The *Criteria* document was the basis for the 2014 Primary Adoption of Mathematics Instructional Materials and is a useful tool for LEAs that conduct their own evaluations of instructional materials.

Criteria for Evaluating Mathematics Instructional Materials for Kindergarten Through Grade Eight

Adopted by the California State Board of Education on January 16, 2013

Instructional materials that are adopted by the state help teachers to present and students to learn the content set forth in the *Common Core State Standards for Mathematics with California Additions* (Standards)¹; this refers to the content standards and the standards for mathematical practice, as revised pursuant to California *Education Code* Section 60605.11 (added by Senate Bill 1200, Statutes of 2012). To accomplish this purpose, this document establishes criteria for evaluating instructional materials for the eight-year adoption cycle beginning with the primary adoption in 2013–14. These criteria serve as evaluation guidelines for the statewide adoption of mathematics instructional materials for kindergarten through grade eight, as called for in *Education Code* Section 60207.

The Standards require focus, coherence, and rigor, with content and mathematical practice standards intertwined throughout. The Standards are organized by grade level in kindergarten through grade eight and by conceptual categories for higher mathematics. For this adoption, the standards for higher mathematics are organized into model courses and are assigned to a first course in a traditional or an integrated sequence of courses. There are a number of supportive and advisory documents that are available for publishers and producers of instructional materials that define the depth of instruction necessary to support the focus, coherence, and rigor of the standards. These documents include the *Progressions Documents for Common Core Math Standards* (http://ime.math.arizona.edu/progressions/); the PARCC *Model Content Frameworks* (available at http://www.parcconline.org/); Smarter Balanced Test Specifications (available at http://www.smarterbalanced.org/); the Illustrative Mathematics project (http://illustrativemathematics.org/); and California's mathematics framework. Overall, the Standards do not dictate a singular approach to instructional resources—to the contrary, they provide opportunities to raise student achievement through innovations.

It is the intent of the State Board of Education that these criteria be seen as neutral on the format of instructional materials in terms of digital, interactive online, and other types of curriculum materials.

I. Focus, Coherence, and Rigor in the Common Core State Standards for Mathematics

With the advent of the Common Core, a decade's worth of recommendations for greater focus and coherence finally have a chance to bear fruit. Focus and coherence are the two major evidence-based design principles of the Standards. These principles are meant to fuel greater achievement in a rigorous curriculum, in which students acquire conceptual understanding, procedural skill and fluency, and the ability to apply mathematics to solve problems. Thus, the implications of the standards for mathematics education could be summarized briefly as follows:

Focus: Place strong emphasis where the Standards focus.

Coherence: Think across grades, and link to major topics in each grade.

^{1.} As of 2014, the Standards are now called the California Common Core State Standards for Mathematics (CA CCSSM).

Rigor: In major topics, pursue with equal intensity:

- conceptual understanding;
- procedural skill and fluency;
- applications.

Focus

Focus requires that we significantly narrow the scope of content in each grade so that students more deeply experience that which remains.

The overwhelming focus of the Standards in early grades is arithmetic, along with the components of measurement that support it. That includes the **concepts** underlying arithmetic, the **skills** of arithmetic computation, and the ability to **apply** arithmetic to solve problems and put arithmetic to engaging uses. Arithmetic in the K–5 standards is an important life skill, as well as a thinking subject and a rehearsal for algebra in the middle grades.

Focus remains important through the middle and high school grades in order to prepare students for college and careers; surveys suggest that postsecondary instructors value greater mastery of prerequisites over shallow exposure to a wide array of topics with dubious relevance to postsecondary work.

Both of the assessment consortia have made the focus, coherence, and rigor of the Standards central to their assessment designs.² Choosing materials that also embody the Standards will be essential for giving teachers and students the tools they need to build a strong mathematical foundation and succeed on standards-aligned assessments.

Coherence

Coherence is about making math make sense. Mathematics is not a list of disconnected tricks or mnemonics. It is an elegant subject in which powerful knowledge results from reasoning with a small number of principles such as place value and properties of operations.³ The standards define progressions of learning that leverage these principles as they build knowledge over the grades.⁴

When people talk about coherence, they often talk about making connections between topics. The most important connections are vertical: the links from one grade to the next that allow students to progress in their mathematical education. That is why it is critical to think across grades and examine the progressions in the standards to see how major content develops over time.

Connections at a single grade level can be used to improve focus, by tightly linking secondary topics to the major work of the grade. For example, in grade three, bar graphs are not "just another topic to cover." Rather, the standard about bar graphs asks students to use information presented in bar graphs

4. For more information on progressions in the Standards, visit <u>http://ime.math.arizona.edu/progressions/</u> (accessed September 3, 2015).

^{2 .} See the Smarter Balanced content specifications and item development specifications, as well as the PARCC Model Content Framework and item development ITN. Complete information about the consortia can be found at <u>http://www.smarterbalanced.</u> org/ and <u>http://www.parcconline.org/</u>.

^{3.} For some remarks by Phil Daro on this theme, see the video at <u>https://vimeo.com/45730600</u> (accessed September 3, 2015).

to solve word problems using the four operations of arithmetic. Instead of allowing bar graphs to detract from the focus on arithmetic, the Standards are showing how bar graphs can be positioned in support of the major work of the grade. In this way coherence can support focus.

Materials cannot match the contours of the Standards by approaching each individual content standard as a separate event. Nor can materials align with the Standards by approaching each individual **grade** as a separate event: "The standards were not so much assembled out of topics as woven out of progressions. Maintaining these progressions in the implementation of the standards will be important for helping all students learn mathematics at a higher level . . . For example, the properties of operations, learned first for simple whole numbers, then in later grades extended to fractions, play a central role in understanding operations with negative numbers, expressions with letters, and later still the study of polynomials. As the application of the properties is extended over the grades, an understanding of how the properties of operations work together should deepen and develop into one of the most fundamental insights into algebra. The natural distribution of prior knowledge in classrooms should not prompt abandoning instruction in grade-level content, but should prompt explicit attention to connecting grade-level content to content from prior learning. To do this, instruction should reflect the progressions on which the CCSSM [Common Core State Standards for Mathematics] are built."⁵

Rigor

To help students meet the expectations of the Standards, educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skill and fluency, and applications. The word *understand* is used in the Standards to set explicit expectations for conceptual understanding; the word *fluently* is used to set explicit expectations for fluency; and the phrase *real-world problems* and the star (\star) symbol are used to set expectations and flag opportunities for applications and modeling (which is a standard for mathematical practice as well as a content category in high school). Real-world problems and standards that support modeling are also opportunities to provide activities related to careers and the work world.

To date, curricula have not always been balanced in their approach to these three aspects of rigor. Some curricula stress fluency in computation without acknowledging the role of conceptual understanding in attaining fluency. Some stress conceptual understanding without acknowledging that fluency requires separate classroom work of a different nature. Some stress pure mathematics without first acknowledging that applications can be highly motivating for students and, moreover, that a mathematical education should prepare students for more than just their next mathematics course. At another extreme, some curricula focus on applications without acknowledging that math does not teach itself.

The Standards do not take sides in these ways, but rather they set high expectations for all three components of rigor in the major work of each grade. Of course, that makes it necessary that we first follow through on the focus in the Standards—otherwise we are asking teachers and students to do more with less.

^{5.} See "Appendix: The Structure of the Standards" in *K*–8 *Publishers' Criteria for the Common Core State Standards for Mathematics*, p. 21 (<u>http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Summer%202012_FINAL.pdf</u> [accessed September 3, 2015]).

II. Criteria for Materials and Tools Aligned with the Standards

Three Types of Programs

Three types of programs will be considered for adoption: basic grade-level for kindergarten through grade eight, Algebra I, and Integrated Mathematics I (hereafter referred to as Mathematics I). All three types of programs must stand alone and will be reviewed separately. Publishers may submit programs for one grade or any combination of grades. In addition, publishers may include intervention and acceleration components to support students.

Basic Grade-Level Program

The basic grade-level program is the comprehensive curriculum in mathematics for students in kindergarten through grade eight. It provides the foundation for instruction and is intended to ensure that all students master the *Common Core State Standards for Mathematics with California Additions*.

Common Core Algebra I and Common Core Mathematics I

When students have mastered the content described in the *Common Core State Standards for Mathematics with California Additions* for kindergarten through grade eight, they will be ready to complete Common Core Algebra I or Common Core Mathematics I. The course content will be consistent with its high school counterpart and will articulate with the subsequent courses in the sequence.

Criteria for Materials and Tools Aligned with the Standards

The criteria for the evaluation of mathematics instructional resources for kindergarten through grade eight are organized into six categories:

- 1. Mathematics Content/Alignment with the Standards. Content as specified in the *Common Core State Standards for Mathematics with California Additions*, including the Standards for Mathematical Practices, and sequence and organization of the mathematics program that provide structure for what students should learn at each grade level.
- 2. **Program Organization.** Instructional materials support instruction and learning of the standards and include such features as lists of the standards, chapter overviews, and glossaries.
- 3. Assessment. Strategies presented in the instructional materials for measuring what students know and are able to do.
- 4. Universal Access. Access to the standards-based curriculum for all students, including English learners, advanced learners, students below grade level in mathematical skills, and students with disabilities.
- 5. **Instructional Planning.** Information and materials that contain a clear road map for teachers to follow when planning instruction.
- 6. **Teacher Support.** Materials designed to help teachers provide effective standards-based mathematics instruction.

Materials that fail to meet the criteria for category 1 (Mathematics Content/Alignment with the Standards) will not be considered suitable for adoption. The criteria for category 1 must be met in the core materials or via the primary means of instruction, rather than in ancillary components. In addition, programs must have strengths in each of categories 2 through 6 to be suitable for adoption.

Category 1: Mathematics Content/Alignment with the Standards

Mathematics materials should support teaching to the *Common Core State Standards for Mathematics with California Additions*. Instructional materials suitable for adoption must satisfy the following criteria:

- 1. The mathematics content is correct, factually accurate, and written with precision. Mathematical terms are defined and used appropriately. Where the standards provide a definition, materials use that as their primary definition to develop student understanding.
- 2. The materials in basic instructional programs support comprehensive teaching of the *Common Core State Standards for Mathematics with California Additions* and include the standards for mathematical practice at each grade level or course. The standards for mathematical practice must be taught in the context of the content standards at each grade level or course. The principles of instruction must reflect current and confirmed research. The materials must be aligned with and support the design of the *Common Core State Standards for Mathematics with California Additions* and address the grade-level content standards and standards for mathematical practice in their entirety.
- 3. In any single grade in the kindergarten-through-grade-eight sequence, students and teachers using the materials as designed spend the large majority of their time on the major work of each grade. The major work (major clusters) of each grade is identified in the Content Emphases by Cluster documents for K–8.⁶ In addition, major work should especially predominate in the first half of the year (e.g., in grade 3 this is necessary so that students have sufficient time to build understanding and fluency with multiplication). Note that an important subset of the major work in grades K–8 is the progression that leads toward Algebra I and Mathematics I (see table IM-1 on the next page). Materials give especially careful treatment to these clusters and their interconnections. Digital or online materials that allow navigation or have no fixed pacing plan are explicitly designed to ensure that students' time on task meets this criterion.

^{6.} For cluster-level emphases in grades K–8, see h<u>ttp://www.achievethecore.org/downloads/</u> Math%20Shifts%20and%20Major%20Work%20of%20Grade.pdf (accessed September 4, 2015).

Table IM-1. Progression to Algebra I and Mathematics I in Kindergarten Through Grade Eight

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Adapted from Achieve the Core 2012.

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*Indicates a cluster that is well thought of as part of a student's progress to algebra, but that is currently not designated as Major by one or both of the assessment consortia (PARCC and Smarter Balanced) in their draft materials. Apart from the one exception marked by an asterisk, the clusters listed here are a subset of those designated as Major in both of the assessment consortia's draft documents.

- 4. Focus: In aligned materials there are no chapter tests, unit tests, or other assessment components that make students or teachers responsible for any topics before the grade in which they are introduced in the Standards. (One way to meet this criterion is for materials to omit these topics entirely prior to the indicated grades.) If the materials address topics outside of the *Common Core State Standards for Mathematics with California Additions*, the publisher will provide a mathematical and pedagogical justification.
- 5. Focus and Coherence Through Supporting Work: Supporting clusters do not detract from focus, but rather enhance focus and coherence simultaneously by engaging students in the major clusters of the grade. For example, materials for K–5 generally treat data displays as an occasion for solving grade-level word problems using the four operations.⁷
- 6. Rigor and Balance: Materials and tools reflect the balances in the Standards and help students meet the Standards' rigorous expectations, by all of the following:
 - a. Developing students' conceptual understanding of key mathematical concepts, where called for in specific content standards or cluster headings, including connecting conceptual understanding to procedural skills. Materials amply feature high-quality conceptual problems and questions that can serve as fertile conversation starters in a classroom if students are unable to answer them. In addition, group discussion suggestions include facilitation strategies and protocols. In the materials, conceptual understanding is not a generalized imperative applied with a broad brush, but is attended to most thoroughly in those places in the content standards where explicit expectations are set for understanding or interpreting. (Conceptual understanding of key mathematical concepts is thus distinct from applications or fluency work, and these three aspects of rigor must be balanced as indicated in the Standards.)
 - b. Giving attention throughout the year to individual standards that set an expectation of fluency. The Standards are explicit where fluency is expected. In grades K–6, materials should help students make steady progress throughout the year toward fluent (accurate and reasonably fast) computation, including knowing single-digit products and sums from memory (see, for example, standards 2.0A.2 and 3.0A.7). The word *fluently* in particular as used in the Standards refers to fluency with a written or mental method, not a method using manipulatives or concrete representations. Progress toward these goals is interwoven with developing conceptual understanding of the operations in question.⁸

Manipulatives and concrete representations such as diagrams that enhance conceptual understanding are closely connected to the written and symbolic methods to which they refer (see, for example, standard **1.NBT**). As well, purely procedural problems and exercises are present. These include cases in which opportunistic strategies are valuable—for example, the sum

^{7.} For more information about this example, see Table 1 in the *Progression* for K–3 Categorical Data and 2–5 Measurement Data (<u>https://commoncoretools.files.wordpress.com/2011/06/ccss_progression_md_k5_2011_06_20.pdf</u>). More generally, the PARCC *Model Content Frameworks* give examples in each grade of how to improve focus and coherence by linking supporting topics to the major work.

^{8.} For more about how students develop fluency in tandem with understanding, see the *Progressions* for Operations and Algebraic Thinking (<u>https://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf</u>) and for Number and Operations in Base Ten (<u>https://commoncoretools.files.wordpress.com/2011/04/ccss_progression_nbt_2011_04_073.pdf</u>).

698 + 240 or the system x + y = 1, 2x + 2y = 3 — as well as an ample number of generic cases so that students can learn and practice efficient algorithms (e.g., the sum 8767 + 2286). Methods and algorithms are general and based on principles of mathematics, not mnemonics or tricks.⁹ Materials do not make fluency a generalized imperative to be applied with a broad brush, but attend most thoroughly to those places in the content standards where explicit expectations are set for fluency. In higher grades, algebra is the language of much of mathematics. Like learning any language, we learn by using it. Sufficient practice with algebraic operations is provided so as to make realistic the attainment of the Standards as a whole; for example, fluency in algebra can help students get past the need to manage computational details so that they can observe structure (MP.7) and express regularity in repeated reasoning (MP.8).

c. Allowing teachers and students using the materials as designed to spend sufficient time working with engaging applications, without losing focus on the major work of each grade. Materials in grades K–8 include an ample number of single-step and multi-step contextual problems that develop the mathematics of the grade, afford opportunities for practice, and engage students in problem solving. Materials for grades 6–8 also include problems in which students must make their own assumptions or simplifications in order to model a situation mathematically. Applications take the form of problems to be worked on individually, as well as classroom activities centered on application scenarios. Materials attend thoroughly to those places in the content standards where expectations for multi-step and real-world problems are explicit. Applications in the materials draw only on content knowledge and skills specified in the content standards, with particular stress on applying major work, and a preference for the more fundamental techniques from additional and supporting work. Modeling builds slowly across K–8, and applications are relatively simple in early grades. Problems and activities are grade-level appropriate, with a sensible tradeoff between the sophistication of the problem and the difficulty or newness of the content knowledge the student is expected to bring to bear.¹⁰

Additional aspects of the Rigor and Balance Criterion:

- (1) *The three aspects of rigor are not always separate in materials.* (Conceptual understanding needs to underpin fluency work; fluency can be practiced in the context of applications; and applications can build conceptual understanding.)
- (2) *Nor are the three aspects of rigor always together in materials*. (Fluency requires dedicated practice to that end. Rich applications cannot always be shoehorned into the mathematical topic of the day. And conceptual understanding will not come along for free unless explicitly taught.)
- (3) Digital and online materials with no fixed lesson flow or pacing plan are not designed for superficial browsing, but rather instantiate the Rigor and Balance criterion and promote depth and mastery.

^{10.} See *Common Core State Standards for Mathematics* (CCSSM, 84) at <u>http://www.corestandards.org/the-standards</u> (accessed September 4, 2015). Also note that modeling is a mathematical practice in every grade, but in high school it is also a content category (CCSSM, 72–73); therefore, modeling is generally enhanced in high school materials, with more elements of the modeling cycle (CCSSM, 72).

- 7. Consistent Progressions: Materials are consistent with the progressions in the Standards, by (all of the following):
 - a. Basing content progressions on the grade-by-grade progressions in the Standards.

Progressions in materials match closely with those in the Standards. This does not require the table of contents in a book to be a replica of the content standards; but the match between the Standards and what students are to learn should be close in each grade. Discrepancies are clearly aimed at helping students meet the Standards as written, rather than effectively rewriting the standards. Comprehensive materials do not introduce gaps in learning by omitting content that is specified in the Standards.

The basic model for grade-to-grade progression involves students making tangible progress during each given grade, as opposed to substantially reviewing and then marginally extending from previous grades. Remediation may be necessary, particularly during transition years, and resources for remediation may be provided, but review is clearly identified as such to the teacher, and teachers and students can see what their specific responsibility is for the current year.

Digital and online materials that allow students and/or teachers to navigate content across grade levels promote the Standards' coherence by tracking the structure and progressions in the Standards. For example, such materials might link problems and concepts so that teachers and students can browse a progression.

b. Giving all students extensive work with grade-level problems.

Differentiation is sometimes necessary, but materials often manage unfinished learning from earlier grades inside grade-level work, rather than setting aside grade-level work to re-teach earlier content. Unfinished learning from earlier grades is normal and prevalent; it should not be ignored nor used as an excuse for cancelling grade-level work and retreating to below-grade work. (For example, the development of fluency with division using the standard algorithm in grade six is the occasion to surface and deal with unfinished learning about place value; this is more productive than setting aside division and backing up.) Likewise, students who are "ready for more" can be provided with problems that take grade-level work in deeper directions, not just exposed to later-grades' topics.

c. Relating grade-level concepts explicitly to prior knowledge from earlier grades.

The materials are designed so that prior knowledge becomes reorganized and extended to accommodate the new knowledge. Grade-level problems in the materials often involve application of knowledge learned in earlier grades. Although students may well have learned this earlier content, they have not learned how it extends to new mathematical situations and applications. They learn basic ideas of place value, for example, and then extend them across the decimal point to tenths and beyond. They learn properties of operations with whole numbers and then extend them to fractions, variables, and expressions. The materials make these extensions of prior knowledge explicit. Note that cluster headings in the Standards sometimes signal key moments where reorganizing and extending previous knowledge is important in order to accommodate new knowledge (e.g., see the cluster headings that use the phrase "Apply and extend previous understanding").

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- 8. Coherent Connections: Materials foster coherence through connections at a single grade, where appropriate and where required by the Standards, by (all of the following):
 - a. Including learning objectives that are visibly shaped by CCSSM cluster headings, with meaningful consequences for the associated problems and activities. While some clusters are simply the sum of their individual standards (e.g., Grade 8, Expressions and Equations, Cluster C: Analyze and solve linear equations and pairs of simultaneous linear equations), many are not (e.g., Grade 8, Expressions and Equations, Cluster B: Understand the connection between proportional relationships, lines, and linear equations). In the latter cases, cluster headings function like topic sentences in a paragraph in that they state the point of, and lend additional meaning to, the individual content standards that follow. Cluster headings can also signal multi-grade progressions by using phrases such as "Apply and extend previous understandings of [X] to do [Y]." Hence an important criterion for coherence is that some or many of the learning objectives in the materials are visibly shaped by CCSSM cluster headings, with meaningful consequences for the associated problems and activities. Materials do not simply treat the Standards as a sum of individual content standards and individual practice standards.
 - b. Including problems and activities that serve to connect two or more clusters in a domain, or two or more domains in a grade, in cases where these connections are natural and important. If instruction only operates at the individual standard level, or even at the individual cluster level, then some important connections will be missed. For example, robust work in standard 4.NBT should sometimes or often synthesize across the clusters listed in that domain; robust work in grade four should sometimes or often involve students applying their developing computation NBT skills in the context of solving word problems detailed in OA. Materials do not invent connections not explicit in the standards without first attending thoroughly to the connections that are required explicitly in the Standards (e.g., standard 3.MD.7 connects area to multiplication, to addition, and to properties of operations; standard A-REI.11 connects functions to equations in a graphical context; proportion connects to percentage, similar triangles, and unit rates). Not everything in the standards is naturally well connected or needs to be connected (e.g., Order of Operations has essentially nothing to do with the properties of operations, and connecting these two things in a lesson or unit title is actively misleading). Instead, connections in materials are mathematically natural and important (e.g., base-ten computation in the context of word problems with the four operations), reflecting plausible, direct implications of what is written in the Standards without creating additional requirements. Instructional materials include problems and activities that connect to real-world and career settings, where appropriate.
- 9. Practice-to-Content Connections: Materials meaningfully connect content standards and practice standards. The National Governors Association Center for Best Practices, Council of Chief State School Officers (NGA/CCSSO) states, "Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction" (NGA/CCSSO 2010c, 8). Over the course of any given year of instruction, each mathematical practice standard is meaningfully present in the form of activities or problems that stimulate students to develop the habits of mind described in the practice standards. These

practices are well grounded in the content standards. Materials are accompanied by an analysis, aimed at evaluators, of how the authors have approached each practice standard in relation to content within each applicable grade or grade band. Materials do not treat the practice standards as static across grades or grade bands, but instead tailor the connections to the content of the grade and to grade-level-appropriate student thinking. Materials also include teacher-directed materials that explain the role of the practice standards in the classroom and in students' mathematical development.

- 10. Focus and Coherence via Practice Standards: Materials promote focus and coherence by connecting practice standards with content that is emphasized in the Standards. Content and practice standards are not connected mechanistically or randomly, but instead support focus and coherence. Examples: Materials connect looking for and making use of structure (MP.7) with structural themes emphasized in the Standards such as properties of operations, place-value decompositions of numbers, numerators and denominators of fractions, numerical and algebraic expressions, and so forth; materials connect looking for and expressing regularity in repeated reasoning (MP.8) with major topics by using regularity in repetitive reasoning as a tool with which to explore major topics. (In K–5, materials might use regularity in repetitive reasoning to shed light on, for example, the 10 × 10 addition table, the 10 × 10 multiplication table, the properties of operations, the relationship between addition and subtraction or multiplication and division, and the place-value system; in 6–8, materials might use regularity in repetitive reasoning to shed light on proportional relationships and linear functions; in high school, materials might use regularity in repetitive reasoning to shed light on formal algebra as well as functions, particularly recursive definitions of functions.)
- 11. Careful Attention to Each Practice Standard: Materials attend to the full meaning of each practice standard. For example, standard MP.1 does not say "Solve problems" or "Make sense of problems and solve them." It says, "Make sense of problems and persevere in solving them." Thus, students using the materials as designed build their perseverance in grade-level-appropriate ways by occasionally solving problems that require them to persevere to a solution beyond the point when they would like to give up. Standard MP.5 does not say "Use tools" or "Use appropriate tools." It says, "Use appropriate tools strategically." Thus, materials include problems that reward students' strategic decisions about how to use tools or about whether to use them at all. Standard MP.8 does not say "Extend patterns" or "Engage in repetitive reasoning." It says, "Look for and express regularity in repeated reasoning." Thus, it is not enough for students to extend patterns or perform repeated calculations. Those repeated calculations must lead to an insight (e.g., "When I add a multiple of 3 to another multiple of 3, then I get a multiple of 3"). The analysis for evaluators explains how the full meaning of each practice standard has been attended to in the materials.
- 12. Emphasis on Mathematical Reasoning: Materials support the Standards' emphasis on mathematical reasoning, by all of the following:
 - a. Prompting students to construct viable arguments and critique the arguments of others concerning key grade-level mathematics that is detailed in the content standards (see standard MP.3). Materials provide sufficient opportunities for students to reason mathematically in independent thinking and express reasoning through classroom discussion and written work. Reasoning is not confined to optional or avoidable sections of the materials but is inevitable
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when using the materials as designed. Materials do not approach reasoning as a generalized imperative, but instead create opportunities for students to reason about key mathematics detailed in the content standards for the grade. Materials thus attend first and most thoroughly to those places in the content standards setting explicit expectations for explaining, justifying, showing, or proving. Students are asked to critique given arguments, for example, by explaining under what conditions, if any, a mathematical statement is valid. Materials develop students' capacity for mathematical reasoning in a grade-level-appropriate way, with a reasonable progression of sophistication from early grades up through high school.¹¹ Teachers and students using the materials as designed spend classroom time communicating reasoning (by constructing viable arguments and explanations and critiquing those of others concerning key grade-level mathematics)—recognizing that learning mathematics also involves time spent working on applications and practicing procedures. Materials provide examples of student explanations and arguments (e.g., fictitious student characters might be portrayed).

- b. Engaging students in problem solving as a form of argument. Materials attend thoroughly to those places in the content standards that explicitly set expectations for multi-step problems; multi-step problems are not scarce in the materials. Some or many of these problems require students to devise a strategy autonomously. Sometimes the goal is the final answer alone (see standard MP.1); sometimes the goal is to show work and lay out the solution as a sequence of well-justified steps. In the latter case, the solution to a problem takes the form of a cogent argument that can be verified and critiqued, instead of a jumble of disconnected steps with a scribbled answer indicated by drawing a circle around it (see standard MP.6). Problems and activities of this nature are grade-level-appropriate, with a reasonable progression of sophistication from early grades up through high school.
- c. Explicitly attending to the specialized language of mathematics. Mathematical reasoning involves specialized language. Therefore, materials and tools address the development of mathematical and academic language associated with the Standards. The language of argument, problem solving, and mathematical explanations are taught rather than assumed. Correspondences between language and multiple mathematical representations including diagrams, tables, graphs, and symbolic expressions are identified in material designed for language development. Note that variety in formats and types of representations—graphs, drawings, images, and tables in addition to text—can relieve some of the language demands that English learners face when they have to show understanding in math.
- d. Materials help English learners access challenging mathematics, learn content, and develop grade-level language. For example, materials might include annotations to help with comprehension of words, sentences, and paragraphs, and give examples of the use of words in other situations. Modifications to language do not sacrifice the mathematics, nor do they put off necessary language development.

^{11.} As students progress through the grades, their production and comprehension of mathematical arguments evolves from informal and concrete toward more formal and abstract. In early grades, students employ imprecise expressions which, with practice over time, become more precise and viable arguments in later grades. Indeed, the use of imprecise language is part of the process in learning how to make more precise arguments in mathematics. Ultimately, conversation about arguments helps students transform assumptions into explicit and precise claims.

Category 2: Program Organization

The organization and features of the instructional materials support instruction and learning of the Standards. Teacher and student materials include such features as lists of the standards, chapter overviews, and glossaries. Instructional materials must have strengths in these areas to be considered suitable for adoption.

- 1. A list of *Common Core State Standards for Mathematics with California Additions* is included in the teacher's guide together with page-number citations or other references that demonstrate alignment with the content standards and standards for mathematical practice. All standards must be listed in their entirety with their cluster heading included.
- 2. Materials drawn from other subject-matter areas are consistent with the currently adopted California standards at the appropriate grade level, including the *California Career Technical Education Model Curriculum Standards* where applicable.
- 3. Intervention components, if included, are designed to support students' progress in mathematics and develop fluency. Intervention materials should provide targeted instruction on standards from previous grade levels and develop student learning of the standards for mathematical practice.
- 4. Middle school acceleration components, if included, are designed to support students' progress beyond grade-level standards in mathematics. Acceleration materials should provide instruction targeted toward readiness for higher mathematics at the middle school level.
- 5. Teacher and student materials contain an overview of the chapters, clearly identify the mathematical concepts, and include tables of contents, indexes, and glossaries that contain important mathematical terms.
- 6. Support materials are an integral part of the instructional program and are clearly aligned with the *Common Core State Standards for Mathematics with California Additions*.
- 7. The grade-level content standards and the standards for mathematical practice demonstrating alignment with student lessons shall be explicitly stated in the student editions.

Category 3: Assessment

Instructional materials should contain strategies and tools for continually measuring student achievement. Formative assessment is a systematic process to continuously gather evidence and provide feedback about learning while instruction is under way. Formative assessments can take multiple forms and occur over varied durations of time. They are to be used to gather information about student learning and to address student misunderstandings. Formative assessments are to provide guidance for the teacher in determining whether the student needs additional materials or resources to achieve grade-level standards and conceptual understanding. Instructional materials in mathematics must have strengths in these areas to be considered suitable for adoption:

- Not every form of assessment is appropriate for every student or every topic area, so a variety of assessment types need to be provided for formative assessment. Some of these could include (but are not limited to) graphic organizers, student observation, student interviews, journals and learning logs, exit ticket activities, mathematics portfolios, self- and peer evaluations, short tests and quizzes, and performance tasks.
- 2. *Summative assessment* is the assessment of learning at a particular time point and is meant to summarize a learner's skills and knowledge at a given point in time. Summative assessments frequently come in the form of chapter or unit tests, weekly quizzes, end-of-term tests, or diagnostic tests.
- 3. All assessments should have content validity and measure individual student progress both at regular intervals and at strategic points of instruction. The assessments should be designed to:
 - monitor student progress toward meeting the content and mathematical practice standards;
 - assess all three aspects of rigor—conceptual understanding, procedural skill and fluency, and applications;
 - provide summative evaluations of individual student achievement;
 - provide multiple methods of assessing what students know and are able to do, such as selected response, constructed response, real-world problems, performance tasks, and open-ended questions;
 - assist the teacher in keeping parents and students informed about student progress.
- 4. Intervention aspects of mathematics programs should include initial assessments to identify areas of strengths and weaknesses, formative assessments to demonstrate student progress toward meeting grade-level standards, and a summative assessment to determine student preparedness for grade-level work.
- 5. Suggestions on how to use assessment data to guide decisions about instructional practices and how to modify instruction so that all students are consistently progressing toward meeting or exceeding the standards should be included.
- 6. Assessments that ask for variety in what students produce, answers and solutions, arguments and explanations, diagrams, mathematical models.
- 7. Assessment tools for grades six through eight help to determine student readiness for Common Core Algebra I and Common Core Mathematics I.
- 8. Middle school acceleration aspects of mathematics programs include an initial assessment to identify areas of strengths and weaknesses, formative assessments to demonstrate student progress toward exceeding grade-level standards, and a summative assessment to determine student preparedness for above-grade-level work.

Category 4: Universal Access

Students with special needs must be provided access to the same standards-based curriculum that is provided to all students, including both the content standards and the standards for mathematical practice. Instructional materials should provide access to the standards-based curriculum for all students, including English learners, advanced learners, students below grade level in mathematical skills, and students with disabilities. Instructional materials in mathematics must have strengths in these areas to be considered suitable for adoption:

- 1. Comprehensive guidance and differentiation strategies, based on current and confirmed research, to adapt the curriculum to meet students' identified special needs and to provide effective, efficient instruction for all students. Strategies may include:
 - working with students' misconceptions to strengthen their conceptual understanding;
 - intervention strategies that describe specific ways to address the learning needs of students using rich problems that engage them in the mathematics reviewed and stress conceptual development of topics rather than focusing only on procedural skills;
 - suggestions for reinforcing or expanding the curriculum;
 - additional instructional time and additional practice, including specialized teaching methods or materials and accommodations for students with special needs;
 - help for students who are below grade level, including more explicit explanations with ample and different opportunities for review and practice of both content and mathematical practices standards, or other assistance that will help to accelerate student performance to grade level;
 - technology that may be used to aid in the implementation of these strategies.
- 2. Strategies for English learners that are consistent with the English Language Development Standards adopted under *Education Code* Section 60811. Materials incorporate strategies for English learners in both lessons and teachers' editions, as appropriate, at every grade level and course level.
- 3. Materials incorporate instructional strategies to address the needs of students with disabilities in both lessons and teachers' editions, as appropriate, at every grade level and course level, pursuant to *Education Code* section 60204(b)(2).
- 4. Teacher and student editions include thoughtful and well-conceived alternatives for advanced students and that allow students to accelerate beyond their grade-level content (acceleration) or to study the content in the *Common Core State Standards for Mathematics with California Additions* in greater depth or complexity (enrichment).
- 5. Materials should help students understand and use appropriate academic language and participate in discussions about mathematical concepts and reasoning. Materials should include content that is relevant to English learners, advanced learners, students below grade level in mathematical skills, and students with disabilities.
- 6. Materials help English learners access challenging mathematics, learn content, and develop gradelevel language. For example, materials might include annotations to help with comprehension

of words, sentences and paragraphs, and give examples of the use of words in other situations. Modifications to language do not sacrifice the mathematics, nor do they put off necessary language development.

- 7. Materials are consistent with the strategies found in Response to Instruction and Intervention (<u>http://www.cde.ca.gov/ci/cr/ri/</u>).
- 8. The visual design of the materials does not distract from the mathematics, but instead serves to support students in engaging thoughtfully with the subject.

Category 5: Instructional Planning

Instructional materials must contain a clear road map for teachers to follow when planning instruction. Instructional materials in mathematics must have strengths in these areas to be considered suitable for adoption:

- 1. A teacher's edition with ample and useful annotations and suggestions on how to present the content in the student edition and in the ancillary materials, including modifications for English learners, advanced learners, students below grade level in mathematical skills, and students with disabilities.
- 2. A list of program lessons in the teacher's edition, cross-referencing the standards covered and providing an estimated instructional time for each lesson, chapter, and unit.
- 3. Unit and lesson plans, including suggestions for organizing resources in the classroom and ideas for pacing lessons.
- 4. A curriculum guide for the academic instructional year.
- 5. All components of the program are user friendly and, in the case of electronic materials, platform neutral.
- 6. Answer keys for all workbooks and other related student activities.
- 7. Concrete models, including manipulatives, support instruction of the *Common Core State Standards for Mathematics with California Additions* and include clear instructions for teachers and students.
- 8. A teacher's edition that explains the role of the specific grade-level mathematics in the context of the overall mathematics curriculum for kindergarten through grade twelve.
- 9. Technical support and suggestions for appropriate use of audiovisual, multimedia, and information technology resources.
- 10. Homework activities, if included, that extend and reinforce classroom instruction and provide additional practice of mathematical content, practices, and applications that have been taught.
- 11. Strategies for informing parents or guardians about the mathematics program and suggestions for how they can help support student progress and achievement.

Category 6: Teacher Support

Instructional materials should be designed to help teachers provide mathematics instruction that ensures opportunities for all students to learn the essential skills and knowledge specified in the *Common Core State Standards for Mathematics with California Additions*. Instructional materials in mathematics must have strengths in these areas to be considered suitable for adoption:

- 1. Clear, grade-appropriate explanations of mathematics concepts that teachers can easily adapt for instruction of all students, including English learners, advanced learners, students below grade level in mathematical skills, and students with disabilities.
- 2. Strategies to identify, address, and correct common student errors and misconceptions.
- 3. Suggestions for accelerating or decelerating the rate at which new material is introduced to students.
- 4. Different kinds of lessons and multiple ways in which to explain concepts, offering teachers choice and flexibility.
- 5. Materials designed to help teachers identify the reason(s) that students may find a particular type of problem(s) more challenging than another (e.g., identify skills not mastered) and point to specific remedies.
- 6. Learning objectives that are explicitly and clearly associated with instruction and assessment.
- 7. A teacher's edition that contains full, adult-level explanations and examples of the more advanced mathematics concepts in the lessons so that teachers can improve their own knowledge of the subject, as necessary.
- 8. Explanations of the instructional approaches of the programs and identification of the researchbased strategies.
- 9. Explanations of the mathematically appropriate use of manipulatives or other visual and concrete representations.

Guidance for Instructional Materials for Grades Nine through Twelve

The *Criteria* document (above) is intended to guide publishers in the development of instructional materials for students in kindergarten through grade eight. It also provides guidance for selection of instructional materials for students in grades nine through twelve. The six categories in the *Criteria* document are an appropriate lens through which to view any instructional materials a district or school is considering purchasing. Additional guidance for evaluating instructional materials for grades nine through twelve is provided in the *High School Publishers' Criteria for the Common Core State Standards for Mathematics* (http://www.corestandards.org/assets/Math_Publishers_Criteria_HS_Spring%202013_FINAL.pdf [Preceding link no longer valid] [NGA/CCSSO 2013]).

The major points from the NGA/CCSSO's criteria are presented here. For the complete NGA/CCSSO criteria and in-depth explanations of the major points, see the *High School Publishers' Criteria for the Common Core State Standards for Mathematics* (NGA/CCSSO 2013).

Focus, Coherence, and Rigor

Focus: Place strong emphasis where the Standards focus.

Coherence: Think across grades, and link to major topics in each grade.

Rigor: Pursue with equal intensity:

- conceptual understanding;
- procedural skill and fluency;
- applications.

Focus

In high school, *focus* is important in order to prepare students for college and careers. A college-ready high school curriculum that includes all of the standards without a (+) symbol should devote the majority of students' time to building the particular knowledge and skills that are most important as prerequisites for a wide range of college majors, postsecondary programs, and careers.

Coherence

Coherence is about making math make sense. Taking advantage of coherence can reduce clutter in the curriculum. For example, if students can see that both the distance formula and the trigonometric identity $\sin^2(t) + \cos^2(t) = 1$ are manifestations of the Pythagorean Theorem, they have an understanding that helps them reconstruct these formulas and not just memorize them temporarily.

Rigor

To help students meet the expectations of the standards, educators need to pursue, with equal intensity, three aspects of *rigor*: (1) conceptual understanding, (2) procedural skill and fluency, and (3) applications. The word *rigor* isn't a code word for just one of these three aspects; rather, it means equal intensity in all three. The word *understand* is used in the standards to set explicit expectations for conceptual understanding, and the phrase *real-world problems* and the star (\star) symbol are used to set expectations and flag opportunities for applications and modeling.

Criteria for Materials and Tools Aligned with the Standards

The following criteria were adapted from the *High School Publishers' Criteria for the Common Core State Standards for Mathematics* (NGA/CCSSO 2013).

1. Focus on Widely Applicable Prerequisites: In any single course, students using the materials as designed spend the majority of their time developing knowledge and skills that are widely applicable as prerequisites for postsecondary education.

- 2. Rigor and Balance: Materials and tools reflect the balances in the standards and help students meet the standards' rigorous expectations, by (all of the following, in the case of comprehensive materials; at least one of the following for supplemental or targeted resources):
 - a. Developing students' conceptual understanding of key mathematical concepts, especially where called for in specific content standards or cluster headings.
 - b. Giving attention throughout the year to procedural skill and fluency.
 - c. Allowing teachers and students using the materials as designed to spend sufficient time working with engaging applications and modeling.

Additional aspects of the Rigor and Balance Criterion:

- 1) *The three aspects of rigor are not always separate in materials.* (Conceptual understanding and fluency go hand in hand; fluency can be practiced in the context of applications; and brief applications can build conceptual understanding.)
- 2) *Nor are the three aspects of rigor always together in materials*. (Fluency requires dedicated practice to that end. Rich applications cannot always be shoehorned into the mathematical topic of the day. And conceptual understanding will not always come along for free unless explicitly taught.)
- 3) Digital and online materials with no fixed lesson flow or pacing plan are not designed for superficial browsing, but rather should be designed to instantiate the Rigor and Balance criterion.
- 3. Consistent Content: Materials are consistent with the content in the standards, by (all of the following):
 - a. Basing courses on the content specified in the standards.
 - b. Giving all students extensive work with course-level problems.
 - c. Relating course-level concepts explicitly to prior knowledge from earlier grades and courses.
- 4. Coherent Connections: Materials foster coherence through connections in a single course, where appropriate and where required by the standards, by (all of the following):
 - a. Including learning objectives that are visibly shaped by CA CCSSM cluster and domain headings.
 - b. Including problems and activities that serve to connect two or more clusters in a domain, two or more domains in a category, or two or more categories, in cases where these connections are natural and important.
 - c. Preserving the focus, coherence, and rigor of the standards even when targeting specific objectives.
- 5. Practice-Content Connections: Materials meaningfully connect content standards and practice standards.
- 6. Focus and Coherence via Practice Standards: Materials promote focus and coherence by connecting practice standards with content that is emphasized in the standards.

- 7. Careful Attention to Each Practice Standard: Materials attend to the full meaning of each practice standard.
- 8. Emphasis on Mathematical Reasoning: Materials support the standards' emphasis on mathematical reasoning, by (all of the following):
 - a. Prompting students to construct viable arguments and critique the arguments of others concerning key course-level mathematics that is detailed in the content standards (see standard MP.3).
 - b. Engaging students in problem solving as a form of argument.
 - c. Explicitly attending to the specialized language of mathematics.

Indicators of Quality in Instructional Materials and Tools for Mathematics

In addition to the major points listed above, the NGA/CCSSO criteria suggest indictors of quality that instructional resources and tools should exhibit. The overarching indicators are listed below without their full explanations. For more detailed information, see the *High School Publishers' Criteria for the Common Core State Standards for Mathematics* (NGA/CCSSO 2013).

Quality Indicators (adapted from NGA/CCSSO 2013):

- Problems in the materials are worth doing.
- There is variety in the pacing and grain size of content coverage.
- There is variety in what students produce.
- Lessons are thoughtfully structured and support the teacher in leading the class through the learning paths at hand, with active participation by all students in their own learning and in the learning of their classmates.
- There are separate teacher materials that support and reward teacher study.
- The use of manipulatives follows best practices (see, for example, National Research Council 2001).
- The visual design is not distracting, chaotic, or aimed at adult purchasers, but instead serves only to support young students in engaging thoughtfully with the subject.
- Materials are carefully reviewed in an effort to ensure:
 - ✓ Freedom from mathematical errors
 - ✓ Age appropriateness
 - ✓ Freedom from bias—for example, problem contexts that use culture-specific background knowledge do not assume readers from all cultures have that knowledge; simple explanations, illustrations, or hints scaffold comprehension
 - ✓ Freedom from unnecessary language complexity
- Support for English learners is thoughtful and helps those learners to meet the same standards as all other students.

The process of selecting instructional materials at the district or school level usually begins with the appointment of a committee of educators, including teachers and curriculum specialists, who determine what instructional materials are needed, develop evaluation criteria and rubrics for reviewing materials, and establish a review process that involves teachers and content-area experts on review committees. After the review committee develops a list of instructional materials that are being considered for adoption, the next step is to pilot the instructional materials. An effective piloting process helps determine if the materials provide teachers with the resources necessary to implement an instructional program based on the CA CCSSM. One resource on piloting is the SBE policy document "Guidelines for Piloting Textbooks and Instructional Materials," which is available through the California Department of Education (CDE) Web site (http://www.cde.ca.gov/); enter "Guidelines for Piloting Textbooks" in the Search box to access a link to the document.

Selection of instructional materials at the local level is a time-consuming but very important process. Poor instructional materials that are not fully aligned with the principles of focus, coherence, and rigor and the CA CCSSM waste precious instructional time. High-quality instructional materials support effective instruction and student learning.

Social Content Review

To ensure that instructional materials reflect California's multi-cultural society, avoid stereotyping, and contribute to a positive learning environment, instructional materials used in California public schools must comply with the state laws and regulations that involve social content. Instructional materials must conform to *Education Code* sections 60040–60045, as well as the SBE's *Standards for Evaluating Instructional Materials for Social Content* (available through the CDE Web site at http://www.cde.ca.gov/ci/cr/cf/lc.asp). Instructional materials that are adopted by the SBE meet the social content requirements. The CDE conducts social content reviews of a range of instructional materials and maintains a searchable database of the materials that meet these social content requirements; the database is available at http://www.cde.ca.gov/ci/cr/cf/ap2/search.aspx.

If an LEA intends to purchase instructional materials that have not been adopted by the state or are not included on the list of instructional materials that meet the social content requirements maintained by the CDE, then the LEA must complete its own social content review. Information about the review process is posted on the CDE's Social Content Review Web page at <u>http://www.cde.ca.gov/ci/cr/cf/lc.asp</u>.

Supplemental Instructional Materials

The SBE traditionally adopts only basic instructional materials programs,¹² but has occasionally adopted supplemental instructional materials. LEAs adopt supplemental materials for local use more frequently. Supplemental instructional materials are defined in California *Education Code* section 60010(l) and are generally designed to serve a specific purpose, such as providing more complete coverage of a topic or subject, meeting the instructional needs of groups of students, and providing current, relevant technology to support interactive learning.

^{12.} These programs are designed for use by students and their teachers as a principal learning resource and meet, in organization and content, the basic requirements of a full course of study (generally, one school year in length).

With the adoption of the CA CCSSM, there was a demand from educators for instructional materials to help schools transition from the previously adopted mathematics standards to the CA CCSSM. In response to this demand for CA CCSSM—aligned instructional materials, the CDE conducted a supplemental instructional materials review (SIMR). The SIMR was a two-phase review of supplemental instructional materials that bridge the gap between the CA CCSSM and programs being used by LEAs that were aligned with the previously adopted mathematics standards. At the recommendation of the CDE, the SBE approved seven mathematics supplemental instructional programs in November 2012 and an additional four programs in July 2013.

Open Educational Resources

Open educational resources (OERs) are online instructional materials and resources that are available to teachers, students, and parents free of charge. OERs include a range of offerings, from full courses to quizzes, classroom activities, tasks, and games. Students may create OERs to fulfill an assignment. Teachers may work together to develop curriculum, lesson plans, or projects and assignments and make them available for others as OERs. OERs offer the promise of more engaging and more relevant instructional content, variety, and up-to-the-minute information. However, they should be subjected to the same type of evaluation as other instructional materials used in schools and reviewed to determine (a) if they are aligned with the content that students are expected to learn, and (b) whether they are at an appropriate level for intended students. Furthermore, OERs need to be reviewed with the social content requirements in mind to ensure that students are not inadvertently exposed to name brands, corporate logos, or materials that demean or stereotype people.

The California Learning Resource Network (CLRN) reviews supplemental electronic learning resources by applying review criteria and using a process approved by the SBE. A complete explanation of the process is provided in the document titled *California Learning Resource Network (CLRN) Supplemental Electronic Learning Resources Review Criteria and Process* (Link removed Mar 29, 2017 [CLRN 2000]). This document was produced before the CA CCSSM were adopted and refers to the previously adopted California standards, but it still serves as a general resource for guiding selection of supplemental electronic resources. Below is a short checklist to consider when reviewing electronic instructional materials.

Minimum Requirements

- 1. The resource addresses standards as evidenced in the CLRN standards match, provides for a systematic approach to the teaching of the standard(s), and contains no material contrary to any of the other California student content standards.
- 2. Instructional activities (sequences) are linked to the stated objectives for this electronic learning resource (ELR).
- 3. Reading and/or vocabulary levels are commensurate with the skill levels of intended learners.
- 4. The ELR exhibits correct spelling, punctuation, and grammar, unless it is a primary source document.

- 5. Content is current, accurate, and scholarly; this includes material taken from other subject areas.
- 6. The presentation of instructional content must be enhanced and clarified by the use of technology through approaches that may include access to real-world situations (graphics, video, audio); multi-sensory representations (auditory, graphic, text); independent opportunities for skill mastery; collaborative activities and communication; access to concepts through hypertext, interactivity, or customization features; use of the tools of scholarship (research, experimentation, problem solving); and simulated laboratory situations.
- 7. The resource is user friendly, as evidenced by the use of features such as effective help functions, clear instructions, a consistent interface, and intuitive navigational links.
- 8. Documentation and instruction on how to install and operate the ELR are provided and are clear and easy to use.
- 9. The model lesson or unit plan demonstrates effective use of the ELR in an instructional setting.

The NGA/CCSSO criteria provide the following guidance on the selection of digital and online instructional materials:

Digital materials offer substantial promise for conveying mathematics in new and vivid ways and customizing learning. In a digital or online format, diving deeper and reaching back and forth across the grades is easy and often useful. That can enhance focus and coherence. But if such capabilities are poorly designed, focus and coherence could also be diminished. In a setting of dynamic content navigation, the navigation experience must preserve the coherence of Standards clusters and progressions while allowing flexibility and user control: Users can readily see where they are with respect to the structure of the curriculum and its basis in the Standards' domains, clusters and standards.

Digital materials that are smaller than a course can be useful. The smallest granularity for which they can be properly evaluated is a cluster of standards. These criteria can be adapted for clusters of standards or progressions within a cluster, but might not make sense for isolated standards. (NGA/CCSSO 2013)

Three OER Web sites that support instruction and learning of the CA CCSSM and offer high-quality resources for use in the classroom and for professional learning are listed below:

- Illustrative Mathematics (<u>https://www.illustrativemathematics.org/</u>). An initiative of the Institute for Mathematics and Education, Illustrative Mathematics provides tasks, videos, lesson plans, and curriculum modules for teachers; mathematics content for teachers and instructional leaders; and a forum for educators to share information and expertise.
- Inside Mathematics (<u>http://www.insidemathematics.org/</u>). This site features classroom examples, tools for instruction, and problems designed for schoolwide participation.
- The Mathematics Assessment Project (<u>http://map.mathshell.org/index.php</u>). This site provides tools for both formative and summative assessment, including tasks for middle and high school students and lessons for middle and high school teachers.

Accessible Instructional Materials

The CDE's Clearinghouse for Specialized Media and Translations (CSMT) provides instructional resources in accessible and meaningful formats to students with disabilities, including students who have hearing or vision impairments, severe orthopedic impairments, or other print disabilities. The CSMT produces accessible versions of textbooks, workbooks, literature books, and assessment books. Specialized instructional materials include braille, large print, audio recordings, digital talking books, electronic files, and American Sign Language video books. Local assistance funds finance the conversion and production of these specialized materials. The distribution of various specialized media to public schools provides general education curricula to students with disabilities. Information about accessible instructional materials and other resources, including what is available and how to order, is posted on the CSMT's Media Ordering Guide page (https://webopac.klas.com/camclive/weborder/index.jsf).