

MATHEMATICS FRAMEWORK

for California Public Schools

Kindergarten Through Grade Twelve



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SUMMARY



What Is the Mathematics Framework?

The purpose of the 2023 *Mathematics Framework for California Public Schools* (*Mathematics Framework*) is to support the implementation of the California Common Core State Standards for Mathematics (CA CCSSM). Adopted in 2010 and updated in 2013, the standards put forth the state’s vision for mathematics, kindergarten through grade twelve (K-12), mapping out what students need to know and be able to do in math by the end of each grade level. The framework offers guidance for enacting the standards through curriculum and instructional approaches grounded in research and reflecting best practices across the globe. The goal is to ensure equity and excellence in math learning so that all California students become powerful users of mathematics to better understand and positively impact the world—in their careers, in college, and in civic life.

Why a New Framework?

Given the status of mathematics access and achievement, California and the United States need to develop new approaches for deepening math learning, addressing achievement gaps, and modernizing instruction to meet the demands of a fast-changing world. The US has long rated below the international average in mathematics on the Programme for International Student Assessment (PISA), and scores have declined since PISA began in 2000. Currently, American fifteen-year-olds score lower than those in 30 other education systems. And California students score below the US average in mathematics with greatly inequitable achievement outcomes for historically underserved students of color and those from low-income families.

To address the urgency to improve mathematics learning, the framework provides new guidance for curriculum and instruction that builds on a body of research on effective mathematics teaching and learning and the successful approaches used in nations that produce high and equitable achievement in math. Helping students develop the thinking skills and content knowledge necessary to master the standards is critical to improving academic performance and creating conditions for later success. The 2023 *Mathematics Framework* includes strategies to support educators to integrate California's Standards for Mathematical Practice in ways that challenge, engage, and support all students in deep and relevant math content learning. Drawing on the experiences of educators who have worked for years to develop successful strategies for teaching California's rigorous standards, the framework's strategies and practices aim to ensure that all students develop deep skills and a love of mathematics and that many more choose to pursue a science, technology, engineering, or math (STEM) major in college or pursue other careers that benefit from quantitative knowledge and reasoning. Strong mathematical knowledge will better enable all of California's students to thrive as citizens in a complex world.

What's in the Mathematics Framework?

The 2023 *Mathematics Framework* includes guidance that will help educators structure the teaching of the state's rigorous standards around "Big Ideas" that integrate rather than isolate transitional kindergarten through grade twelve (TK–12) math concepts—a best practice in high-performing countries. This approach encourages teachers to think about how the Big Ideas in mathematics connect within and across grade levels in developmental progressions. It emphasizes developing the habits of mind and interaction described in the Standards for Mathematical Practice—for example, persevering in problem-solving, explaining one's thinking, and constructing arguments—that students need to become powerful users of mathematics and better interpret and understand their world.

The framework's instructional approach connects learning to the "real world" through authentic examples and the use of data, an approach that not only addresses "Why do I need to learn this?" but sparks student curiosity and encourages inquiry, problem-solving, and deep learning. The framework also emphasizes students' need to have opportunities to explicitly connect their conceptual understanding with facts and procedures in ways that make sense to them so that they can use known facts, including those drawn from memory, to determine unknown facts and develop fluency.

The framework draws on research that illustrates how all students can become productive mathematics learners and continue their progress through advanced coursework that can prepare them for STEM and other quantitative careers, supporting both high achievers and those who may need additional assistance. The framework encourages schools to enable students to reach higher levels of mathematics learning through expert teaching focused on deep understanding, differentiated instruction, personalized learning, strategic grouping, additional course-taking opportunities, and acceleration into more advanced courses at multiple junctures when students are ready. It discusses the conditions research has found to be associated with successful acceleration, and it provides guidance about multiple

strategies, in addition to acceleration, for ensuring that students have the opportunity to reach advanced mathematics courses, such as calculus and statistics, during high school.

The framework draws on research that describes how to help students develop a growth mindset in mathematics, which bolsters perseverance, confidence, and achievement. It also includes guidance to help educators make mathematics instruction culturally relevant and empowering, thus allowing students to see themselves in the curriculum and in math-related careers. The pedagogical strategies it highlights have been found to promote higher and more equitable engagement and achievement for students who have been historically underrepresented in STEM careers, including girls and African American, Native American, Latinx, and Pacific Islander students. The framework supports instruction for linguistically and culturally diverse English learners who are developing mathematical proficiency informed by the California English Language Development Standards (CA ELD Standards) and the California Department of Education advice for integrating the CA ELD Standards into mathematics teaching. It incorporates explicit guidance for planning assessment and instructional materials adoptions that are responsive to English learner needs.

In response to the explosion of data use in our daily lives and across fields, the framework highlights the opportunities data presents for bringing mathematics alive for students. It discusses ways to integrate data use into math instruction throughout the grades and to encourage students to become proficient at understanding and using data—including helping students use data to make decisions in their roles as global citizens and identify misleading data so their decisions are sound.

Aligning with State Board of Education guidelines and the CA CCSSM, the framework details how students can progress through two high school pathways—Traditional and Integrated—and clarifies the autonomy of local schools and districts in designing courses and pathways that best meet the needs of local families. The guidance helps educators integrate and align math concepts taught at the middle and high school levels, ensure that all high school math pathways are open to all students, and expand high school math course options to encourage more students to go beyond minimum course-taking requirements to pursue areas of mathematics that are interesting and useful to them in their current pursuits and aspirations.

The content of each of the framework chapters is described below.

Chapter 1—Mathematics for All: Purpose, Understanding, and Connection

Chapter 1 introduces and sets up the *Mathematics Framework*, which lays out the curricular and instructional approaches that evidence suggests will afford all students the opportunities they need to learn meaningful and rigorous mathematics, meet state mathematics standards, access pathways to high-level mathematics courses, and achieve success. The CA CCSSM were adopted in 2010 and modified in 2013. The standards map out what California students need to know and be able to do, grade by grade, in mathematics. Built upon principles of *focus*, *coherence*, and *rigor*, they hold the promise of enabling all California students to become powerful users

of mathematics in order to better understand and positively impact the world. To fulfill that promise and reach the goal of deep, active learning of mathematics for all, the framework is centered around the investigation of Big Ideas in mathematics, connected to authentic, real-world contexts and taught in multidimensional ways that meet varied learning needs. While this approach to mathematics education may be a tall order, research shows that it is the means to both provide access for all students and teach mathematics effectively. Chapter 1 also provides an overview of how subsequent chapters are organized.

Chapter 2—Teaching for Equity and Engagement

Chapter 2 focuses on how teachers can create equitable and engaging mathematics classroom environments, TK-12, that support all students and improve mathematics access and outcomes. The chapter describes dimensions of the system that contribute to access and equity and expands on the five components of instructional design, which are introduced in chapter 1, that encourage equitable outcomes and active student engagement:

- teaching Big Ideas,
- using open tasks,
- teaching for social justice,
- inviting students' questions and conjectures, and
- prioritizing reasoning and justification.

Instruction that incorporates these components can enable a diverse group of students to see themselves as mathematically capable individuals whose curiosity and love of mathematics learning will be sustained throughout their schooling.

Chapter 3—Number Sense

By way of an in-depth discussion of number sense, chapter 3 illustrates how the progression of mathematical concepts occurs across TK-12 as elementary, middle, and high school teachers use investigations and connections to teach the mathematical Big Ideas of each grade level. The chapter shows that number sense can itself be described as a progression of Big Ideas, which include, for example:

- in transitional kindergarten through grade two, organizing and counting with numbers;
- in grades three through five, extending flexibility with numbers;
- in grades six through eight, number line understanding; and
- in grades nine through twelve, seeing parallels between numbers and functions.

The chapter emphasizes the growth of number fluency—the ability to use strategies that are flexible, efficient, and accurate—and highlights the value of math talks and games, which encourage students' mental problem-solving and communication as well as playful exploration and skill practice.

Chapter 4—Exploring, Discovering, and Reasoning With and About Mathematics

Chapter 4 goes deeply into California’s Standards for Mathematical Practice, providing an important background for discussing how elementary, middle, and high school teachers teach the Big Ideas of mathematics (see chapters 6, 7, and 8). The Standards for Mathematical Practice embed the habits of mind and habits of interaction—for example, persevering in problem-solving, explaining one’s thinking, and constructing arguments—that form the basis of mathematics learning. Using three interrelated Standards for Mathematical Practice for illustration, the chapter demonstrates how key mathematical practices, integrated with each other, can help teachers across grade levels create powerful mathematics experiences centered on exploring, discovery, and reasoning—thus enabling students to develop and deepen those skills, in relation to progressions in mathematics content, as they move through the grades.

Chapter 5—Mathematical Foundations for Data Science

This chapter is a first step in helping educators identify how the CA CCSSM can support K-12 students in developing foundational knowledge and skills for using data as a tool for mathematical problem-solving. Like number sense, which is discussed in chapter 3, foundations of data science are embedded in mathematics at all grade levels. Making sense of data, identifying misleading uses of data, and using data to make decisions are all important skills for students in their roles as global citizens. Developing these abilities requires that students generate questions and work with data beginning in kindergarten (or before). This work should continue and increase in depth and complexity throughout their school careers. Across all grade levels, students should be encouraged and supported in understanding and describing tendencies and variability in data and data distributions; considering data collection, sampling, and random processes; and comparing distributions and identifying associations between variables. Students who wish to focus extra attention on data science should have an opportunity to pursue advanced courses in high school.

Chapter 6—Mathematics: Investigating and Connecting, Transitional Kindergarten Through Grade Five

This chapter discusses how the approach to mathematics teaching unfolds throughout elementary school. Mathematics in transitional kindergarten through grade five is a vibrant, interactive, student-centered endeavor of investigating and connecting the Big Ideas of mathematics. In these grades, children experience enormous growth in maturity, reasoning, and conceptual understanding. They develop an understanding of concepts that include place value, arithmetic operations, fractions, geometric shapes and properties, and measurement. Building on this deep understanding, they also develop fluency, including the ability to retrieve and use facts and procedures

as they undertake more complex topics. The chapter examines how teachers can use meaningful mathematics activities that nourish curiosity and develop reasoning skills while simultaneously connecting content and mathematical practices within and across grade levels. Students who have gained an understanding of elementary mathematics and enter sixth grade viewing themselves as mathematically capable are positioned for success in middle school and beyond.

Chapter 7—Mathematics: Investigating and Connecting, Grades Six Through Eight

This chapter discusses how the approach to mathematics teaching unfolds throughout middle school. Building on the foundational understanding of mathematics concepts developed in transitional kindergarten through grade five, middle school's major topics include proportional reasoning, rational numbers, and measurement in geometrical and data science scenarios—all developed through activities that are situated in intriguing, authentic contexts and require students to build connections among ideas. The critical element of success continues to be piquing students' curiosity and interest through engagement with meaningful and relevant mathematics activities and experiences. As the chapter discusses, students' middle school experiences are pivotal in shaping their attitudes toward mathematics and self-perceptions as mathematics learners. Those experiences, combined with the guidance they receive, determine whether or not students get on a pathway to high-level mathematics, crucially affecting their mathematics futures in high school and beyond.

Chapter 8—Mathematics: Investigating and Connecting, High School

This chapter discusses how the approach to mathematics teaching unfolds throughout high school. California's high school mathematics content standards are organized into conceptual categories:

- number and quantity;
- algebra;
- functions;
- modeling;
- geometry; and
- statistics and probability.

This chapter provides an overview of the traditional and integrated pathways schools may offer and describes the mathematical content that students learn in the courses associated with each pathway. It also describes the range of third- and fourth-year high school mathematics course options from which students may choose to reflect their interests and future aspirations and clarifies the expectations for coursework leading to different higher education options, including STEM careers.

Examples are included of how course content can support the Standards for Mathematical Practice. The pathways integrate content with mathematics practices—that is, the investigation/connection approach of the framework enables students to develop a disposition toward reasoning and communication in mathematics, knowledge of mathematical ideas and skills, and the ability to think both critically and creatively in solving problems. As this chapter discusses, teaching this way requires careful consideration of many issues in addition to learning goals, including motivation, coherence, students’ and teachers’ cultural and linguistic assets, access and equity, context, and sustainability.

Chapter 9—Structuring School Experiences for Equity and Engagement

Chapter 9 describes methods of teaching and coursework designs that can enable all students to be appropriately challenged and many more to reach advanced mathematics—without requiring that all students work on the same mathematics or be placed in inflexible course sequences that make it difficult for them to move into or between STEM or non-STEM pathways if they so choose. As with appropriate course sequencing (described in chapter 8), the goal is to expand access to rigorous mathematics for all students, allowing each student to experience the joy and excitement of well-taught mathematics in ways that stimulate their learning and engagement.

Chapter 10—Supporting Educators in Offering Equitable and Engaging Mathematics Instruction

Chapter 10 discusses how to plan and design structured, ongoing, high-quality programs of professional learning that support teachers throughout their careers. Teaching for equity and engagement is as rewarding as it is complex. Teachers who focus on Big Ideas and connections and teach mathematics by way of carefully designed, intriguing investigations see their students come alive through exploration and discovery. But especially since most teachers did not learn mathematics this way, they need support to rethink mathematics teaching and acquire skills and strategies that result in the changes in practice vital to improving student learning.

Chapter 11—Technology and Distance Learning in the Teaching of Mathematics

This chapter provides guidance to help educators enrich students’ learning of the mathematics content and practice standards by using technology. Today, a host of technologies have the potential to support rich and deep mathematical learning for all students. Introducing students to technology is important in itself, given its increasingly integral role in our lives. At the same time, this chapter emphasizes that using technology in teaching mathematics should support instructional objectives and student learning of mathematics, deeply understood. Based on principles for technology use in mathematics learning, the chapter recommends adopting technology in combination with changes to teaching practices that make the

technology an integral and sustained component of the instruction—accompanied by high-quality, ongoing professional learning for teachers. The chapter also discusses distance learning, features of effective distance learning, and tips for success.

Chapter 12—Mathematics Assessment in the Twenty-First Century

This chapter encourages educators, administrators, and policymakers to focus on assessment that engages students in continuous improvement efforts by using mastery-based approaches—notably, by assessing with rubrics and using self, peer, and teacher feedback. Such an approach reflects the goal of achieving conceptual understanding, problem-solving capacity, and procedural fluency. It also promises to maximize the amount of learning each child is capable of while minimizing the sociocultural effects of narrow testing. The chapter discusses California’s evolving comprehensive assessment system, with its two primary forms of mathematics assessments—formative and summative—and how they relate to mathematics instruction and learning. In California as nationwide, mathematics assessment is in transition, shifting from rote tests of fact-based skills to multidimensional measures of procedural skills, problem-solving capacity, and evidence-based reasoning. The shift reflects a growing alignment between how mathematics is being taught and how it is being tested—in turn reflecting shifting classroom, school, district, and state priorities.

Chapter 13—Instructional Materials to Support Equitable and Engaging Learning of the California Common Core State Standards for Mathematics

Chapter 13 is directed to the developers and publishers of instructional materials and conveys information needed to participate in the State Board of Education instructional materials adoption process. It explains the need for developers and publishers to provide the content (including coverage, depth, and delivery) to address standards while at the same time maintaining coherence as learning progresses across grade levels. Materials need to reflect grade-level shifts in Big Ideas, and activities need to continue to be relevant to student lives. Resources that bridge content and context and provide guidance to teachers—while also allowing the flexibility necessary for supporting all students—will be critical for implementing the framework. Notably, developers and publishers should consider the lens of equity when developing lessons and units to serve California’s diverse student population. The chapter also provides guidance for local districts on the adoption of instructional materials for students in grades nine through twelve, the social content review process, supplemental instructional materials, and accessible instructional materials.

Appendix A—Mathematical Progressions within the High School Pathways and Key Mathematical Ideas to Promote Student Success in Introductory University Courses in Quantitative Fields

This chapter presents key mathematical ideas to promote student success in introductory university courses in quantitative fields.

Appendix B—Works Cited

This appendix lists all references cited in the framework, by chapter.

Appendix C—Vignettes

This appendix contains all vignettes referenced in the framework, by chapter, that are longer than two pages. Hyperlinks in the chapters give readers easy access to each vignette.

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