Technology in the Teaching of Mathematics Chapter

of the

Mathematics Framework

for California Public Schools: Kindergarten Through Grade Twelve

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Technology in the Teaching of Mathematics

The field of mathematics education has changed greatly because of technology. Educational technology can facilitate simple computation and the visualization of mathematics situations and relationships, allowing students to better comprehend mathematical concepts in practice. Technology can be a tool for students to model mathematical relationships in real-world situations. Technology is also an integral part of the Common Core State Standards Initiative and its emphasis on preparing students for college and twenty-first-century careers.

Technology pervades modern society. In such an environment, the question is not whether educational technology will be used in the classroom, but how best to use it (Cheung and Slavin 2011). Current-generation students are digital natives, and the generation of teachers who will enter the profession over the next few decades will likewise be the product of a culture in which technology is a constant presence and where the use of technology in education is a fundamental assumption. Training and supporting teachers in the use of technology are essential to the effective use of technology in the classroom.

Educational technology is a broad category that includes both a wide range of electronic devices and the applications that deliver content and support learning. Technology is an essential tool for learning mathematics in the twenty-first century, but it is only a tool; it cannot replace conceptual understanding, computational fluency, or problem-solving skills. Technological tools include both content-specific technologies (e.g., computer programs and computational devices) and content-neutral technologies, such as communication and collaboration tools (National Council of Teachers of Mathematics [NCTM] 2011a). According to guidelines adopted by the state of Massachusetts to help construct and evaluate curriculum, "Technology changes the mathematics to be learned, as well as when and how it is learned . . . Some mathematics becomes more important because technology requires it, some becomes less important because technology replaces it, and some becomes possible because technology allows it" (Massachusetts Department of Elementary and Secondary Education 2011).[1]

Research completed over the past decade has confirmed the potential benefits of educational technology applied to the teaching and learning of mathematics. When used effectively, educational technology can enhance student understanding of mathematical concepts, bolster student engagement, and strengthen problem-solving skills. Most of the recent meta-analyses of research studies in this area, however, note that these benefits depend on how educational technology is implemented, whether it is integrated with instruction, and the degree to which teachers are trained and interested in its use (Guerrero, Walker, and Dugdale 2004; Kahveci and Imamoglu 2007; Goos and Bennison 2007; Li and Ma 2010; Cheung and Slavin 2011). This chapter provides some suggestions and cautions on managing implementation to capitalize on the use of technology.

1. The excerpt from the Massachusetts Curriculum Frameworks is included by permission of the Massachusetts Department of Elementary and Secondary Education. The complete and current version of each Massachusetts curriculum framework is available at http://www.doe.mass.edu/frameworks/current.html (accessed September 2, 2015).

Educational Technology and the Common Core

The use of technology is directly integrated into the California Common Core State Standards for Mathematics (CA CCSSM). The mathematics content standards encourage the use of multiple representations and modeling to help students understand the mathematical concepts behind a problem. This is an area where the use of technology can be helpful. The standards specifically refer to using technology tools in a number of cases, especially in the middle grades and high school. For example, Geometry standard **7.G.2** states the following:

> Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. (California Department of Education [CDE] 2013a, 50)

Similarly, the higher mathematics standards for algebra, functions, geometry, and statistics and probability include references to using technology to develop mathematical models, test assumptions, and conduct appropriate computations.

Technology is also an integral part of the Standards for Mathematical Practice (MP standards) that are emphasized throughout the CA CCSSM, starting in kindergarten and continuing through grade twelve. It is expected that students will be able to integrate technology tools into their mathematical work. For example, the descriptive text for standard **MP.5** (Use appropriate tools strategically) states the following:

> Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations . . . They are able to use technological tools to explore and deepen their understanding of concepts. (National Governors Association Center for Best Practices, Council of Chief State School Officers [NGA/CCSSO] 2010q)

Students who gain proficiency in the CA CCSSM are expected to know not only how to use technology tools, but also when to use them.

Technology and the Common Core: Illustrative Examples

Grade Level or Course	Content Standards	Practice Standards	Instructional Strategy Using Technology
			Elementary Grades
Kindergarten	K.CC.4	MP.2 MP.6 MP.7	Using a free application, such as "Concentration" or "Okta's Rescue" from the National Council of Teachers of Mathematics (NCTM) Illuminations[2] resources, students work in pairs to match number names with the corresponding numeral.
Grade One	1.OA.6	MP.2 MP.7 MP.8	Using a free application, such as "Deep Sea Duel" from NCTM Illuminations, students work in pairs to find various number combinations that sum to a particular number.
Grade Two	2.NBT.7	MP.1 MP.6 MP.7	Using a free application, such as "Grouping and Grazing" from NCTM Illuminations, students work on addition.
Grade Three	3.OA.7	MP.1 MP.6 MP.7	Using a free application, such as "Pick-a-Path" from NCTM Illuminations, the teacher assigns a group of students to solve problems on tablet computers while other students work directly with the teacher.
			Middle Grades
Grade Six	6.SP.3 6.SP.4	MP.3 MP.5	Using a computer, students find a data set online. They use a spreadsheet formula to calculate measures of center and variability, create a graphical representation, and write a description of the data based on the numerical and graphical evidence.
Grade Eight	8.SP.1 8.SP.2 8.SP.3	MP.4 MP.5	Students work in pairs, using two graphing calculators and one ultrasonic ranging device to collect data. The first student walks *toward* his or her partner, who uses the ranging device to record the distance between them. The two students attempt to produce a graph that is a straight line, repeating the measurements until both partners are happy with the result. The pair now reverses roles, but with the second student walking away from his or her recording partner. When the data are collected, the pair answer the following questions by manipulating the Time List and Distance List data stored in their two calculators: • How far away was your partner when he or she started? • How far away was your partner at the end of the experiment? • How long did the experiment last? • By computing $\frac{Dist(last) - Dist(first)}{Time(last) - Time(first)}$, calculate your velocity and your partner's velocity. How are these alike? How are these different? Explain your observations. • Compute your partner's velocity over the first half, second half, first quarter, second quarter, third quarter, and fourth quarter of the experiment to determine if your velocity was constant. How constant was the velocity? How do you know? • Manually or otherwise (e.g., using Median-Median or Least Squares), fit a line to your partner's data and obtain an equation for the line. What are the slope and y-intercept of your line? What do the slope and y-intercept represent in terms of the experiment? How do these compare to your earlier calculations of velocity?

Continued on next page

2 . The NCTM Illuminations resources are available at https://illuminations.nctm.org/Default.aspx (accessed September 2, 2015).

Technology and the Common Core: Illustrative Examples (*continued*)

Grade Level or Course	Content Standards	Practice Standards	Instructional Strategy Using Technology
Higher Mathematics			
Geometry Mathematics I Mathematics II	G-CO.9 G-CO.10 G-CO.11 G-CO.12 G-CO.13	MP.5 MP.7	Using dynamic geometry software and an interactive whiteboard, students investigate and create conjectures of geometric theorems and constructions.
Mathematics I Algebra I	S-ID.6 S-ID.7 S-ID.8 S-ID.9	MP.4 MP.5	Students use a computer to locate a bivariate data set. Then they use statistical software to create a scatter plot and calculate the least squares regression line. Students explore the properties of this line and use it to predict and interpret relevant results.
Mathematics I Algebra I	F-LE.3 S-ID.6a	MP.2 MP.4 MP.5	For a whole-class activity, the teacher needs a graphing calculator, one ultrasonic ranging device, a wooden plank ranging from 6 to 9 feet in length, and a large (family or industrial size) can of a non-liquid, such as refried beans or ravioli. The plank is raised to a small incline by propping up one end with one or two textbooks. The experiment consists of collecting data on the distance between the ranging device placed at the top of the ramp and the can placed at the bottom of the ramp. The can's position on the ramp and its velocity are recorded by the ultrasonic ranging device as the can is rolled up and allowed to roll back down the ramp. Preparing for the experiment, an assistant practices rolling the can up and down. From these practice rolls, the class decides on the length of the experiment (the number of trials), and students are asked to describe what they see. [The can's speed slows on the way up, there is an apparent pause at the top, and the can speeds up as it descends the ramp.] Having decided on the length of the experiment and possibly the rate of sampling, students then collect data. The rolling process is repeated until a clean run, one in which the can does not roll off the ramp, is obtained. Note that it is common for the can to roll off the ramp. The resulting graph is discussed. How close did the can get to the ranging device? The descending part of the graph corresponds to the ascent of the can. When did the can change direction (begin rolling *down* the ramp instead of *up*)? Students perform a quadratic regression and plot the resulting equation. Then they compute and examine the residuals, the number negative, the number positive, and the Mean Absolute Deviation to discuss the goodness of fit.

Technology is also an integral part of the assessment system used by the multi-state Smarter Balanced Assessment Consortium (Smarter Balanced), of which California is a governing member. Smarter Balanced has implemented computer-adaptive assessments that respond to a student's initial performance to more rapidly and accurately identify which skills the student has mastered. These assessments also allow for a quick turnaround of test results so that the results can be used to inform instruction. The Smarter Balanced test protocols allow the use of calculators on certain test items for middle and high school assessments, including integrating calculators directly into the assessment software. (For additional information, see the Assessment chapter.)

Educational Technology in the Classroom

Two basic types of educational technology are commonly used in mathematics lessons: handheld devices, which include calculators and wireless devices; and computer applications, which include software and online resources.

Handheld Devices

For decades, one of the biggest questions and controversies about educational technology was the use of calculators in classroom instruction and assessment. Previous studies (Hembree and Dessart 1986; Loveless 2004) raised cautions about the use of calculators in elementary grades, especially in terms of undermining students' skills at basic computation. Ellington (2003) found that calculators had the greatest benefit when used for both instruction and assessment. She noted, however, that "[s]tudents received the most benefit when calculators had a pedagogical role in the classroom and were not just available for drill and practice or checking work" (Ellington 2003, 456). Instruction needs to be structured to use technology in non-routine ways (i.e., not in drill and computational practice)—in other words, where students are using it to make decisions and solve problems (Guerrero, Walker, and Dugdale 2004). Graphing calculators can be used in conjunction with the technology emphasis in the CA CCSSM to allow students to actively participate in the process of developing mathematical ideas and solving problems. These devices can help students better understand spatial concepts, connect functions with their graphs, and visualize written problems to develop solutions (Ellington 2003).

In its 2011 research brief titled *Using Calculators for Teaching and Learning Mathematics*, the NCTM stated the following conclusion after a synthesis of nearly 200 studies conducted from 1976 through 2009:

> In general, we found that the body of research consistently shows that the use of calculators in the teaching and learning of mathematics does not contribute to any negative outcomes for skill development or procedural proficiency, but instead enhances the understanding of mathematics concepts and student orientation toward mathematics. (NCTM 2011b)

Although these findings do not prove that calculators enhance rather than supplant students' computational and procedural skills, they do provide reassurance that calculators can be integrated into instruction and assessment without harming student progress toward mathematical proficiency. It is important to remember that curriculum and instruction involving the use of calculators should be designed to emphasize the problem-solving and conceptual skills of students.

The next generation of handheld devices with networking capabilities also offers opportunities for using technology effectively in a classroom environment. Clark-Wilson (2010) conducted a study of seven teachers using handheld graphing computers connected via wireless network to the teacher's computer. These devices allowed teachers to monitor student work and provide live feedback and enabled students to lead the classroom discussion via a projector connected to the network. The advantages of using these devices included promoting a "collaborative classroom" where students were able to learn from each other. Clark-Wilson also noted the benefits of added student engagement, a finding that was duplicated in many other studies on the use of classroom technology.

Smartphones and tablet computers are other forms of handheld technology that are becoming increasingly common in schools. Likewise, educational applications (frequently referred to as “apps”) that are designed to work with these devices are proliferating. Smartphones and tablet computers offer the advantages of built-in networking capability and access to the Internet, which enables immediate access to content and feedback from the teacher (as noted above). Tablet computers, with advantages in terms of weight and convenience, are being used by some school districts to provide delivery of instructional materials.

However, there are challenges associated with the use of handheld devices (including smartphones) to assist or provide instruction. Recent studies suggest that schools need to educate students as well as parents about the need for policies regarding student access to and use of technology at school, but comprehensive bans on cell phones may not be the most effective means of addressing these problems. Smartphones and tablet computers are easily lost or stolen and can be expensive to replace. Furthermore, having networked technology that is “always on” can also be detrimental. In a 2009 national survey of middle and high school students, 35 percent of the students admitted to using a cell phone to cheat, and 52 percent admitted to cheating by using the Internet. The survey indicated the cultural gaps involved with new technology; for example, many students did not consider texting a warning about a pop quiz to fellow students or copying text available online to turn in as their own work to be “cheating” (Common Sense Media 2009). This same survey also found that although 92 percent of parents indicated the belief that cheating through the use of cell phones happens at their child’s school, only 3 percent of parents said that their child had engaged in such behavior (Challenge Success 2012). To discourage this type of cheating, teachers might include student assessments that support content mastery and require students to demonstrate their knowledge in multiple ways, explain how they solved a problem, or describe their reasoning behind a method.

Computers and Software

Computers have become a ubiquitous presence in schools over the past fifteen years. The number of computers has increased from approximately one computer for every 11 students in the late 1990s to one computer per 5.8 students in 2010–11 (Education Data Partnership 2011).

Research on the use of computer-mediated learning tools has demonstrated the potential for increased student achievement and proficiency in mathematical concepts. However, these benefits depend on the teaching approaches, types of programs, and the learners themselves (Li and Ma 2010). Kahveci and Imamoglu (2007) note that mathematics instruction is most effective when students frequently interact with the content and that the most successful instructional systems are those that adapt to students, allow them to work collaboratively, and give immediate feedback. Similarly, Li and Ma (2010) found that computer technology was most effective when used with constructivist teaching methods where students gain conceptual understanding through an inquiry-based, collaborative learning model.

As a more concrete example, Ruthven, Deaney, and Hennessy (2009) studied the use of graphing software in secondary schools for investigating algebraic equations. They found that the use of the software to enable students to graph linear and quadratic equations in the classroom had positive results in terms of efficiency of instruction, student engagement, and understanding. Students could use the software to explore the topic and share their results—for example, by immediately seeing the effects on a graph of altering the coefficient in a formula-defining function. However, the authors of this study noted that despite the fact that the software had been "designed to do things easily," the teacher's role was still vital in structuring the activity and designing tasks that would help students master the mathematical content at the core of the lesson.

Online Learning

Online delivery of instruction is becoming increasingly popular. More than one million students in kindergarten through grade twelve enrolled in at least one online course in 2007–08 (United States Department of Education 2010). Online courses offer distinct advantages to school districts in terms of cost and convenience, especially for districts where students are distributed across a wide geographic area and challenges may exist in delivering instruction in particular content areas.

While more research is being conducted on the efficacy of online instruction, preliminary findings provide reason for optimism. In a 2009–10 study of online learning, the United States Department of Education found only five studies on K–12 education in its survey of research from 1996 through 2008. Of those five, only one dealt with mathematics, but in general, the study's authors found that the outcomes for online learning were not significantly different from those involving face-to-face instruction, and programs that combined online and face-to-face learning (a "blended" or "hybrid" model) could actually produce higher outcomes in terms of student performance. The study noted that newer online applications are able to combine delayed communication using asynchronous tools (e.g., e-mail, newsgroups, and discussion boards) with real-time communication using synchronous tools such as Web casting, chat, and video conferencing sessions. These combinations allow students to approach the subject with more interaction between the content, their peers, and their teacher, which is more conducive to the "deep learning" that is the goal of mathematics instruction. This interactive approach is consistent with a sociocultural perspective on learning, which holds that learning takes place in social environments where social activity provides support and assistance for learning (Vygotsky 1978; Cobb 1994). However, the relative newness of online learning and the limited number of studies available suggest that districts should approach online instruction with caution, especially when the material is intended to replace face-to-face instruction, rather than enhance it.

Professional Development and Teacher Support

The various research studies cited in this chapter share a consensus that educational technology cannot improve student outcomes without the classroom teacher playing a central role. The teacher must ensure that technological tools are used to support student understanding of mathematical concepts and practices; technology should not be used simply to entertain students or shield them from developing mathematical practices.

Moreover, teacher attitudes toward technology can affect its implementation and effectiveness. Guerrero, Walker, and Dugdale (2004) have noted that many teachers are cautious about technology and believe that its use may potentially hinder students' understanding and learning of mathematics. Some teachers fear, for example, that the inappropriate use of technology may interfere with students' ability to learn number facts or basic computational skills. Others are wary of the changes to instruction that are necessary to make use of new technology in the classroom. In some cases, teachers are willing to use technology but face opposition in the form of reluctant administrators or an organizational culture that is resistant to change (Goos and Bennison 2007). It is also important for administrators to ensure that teachers have the technical support necessary to keep technology functioning and available.

Merely providing teachers with greater access to technology will not lead to its successful use (Goos and Bennison 2007; Walden University 2010). Using technology effectively requires changes in pedagogical approaches. The technological tools referenced in this chapter may involve changes to the working environment, to the format and timing of lessons and activities, and to the curriculum. Therefore, any innovation in technology must be accompanied by adaptations to the teacher's craft knowledge (Ruthven, Deaney, and Hennessy 2009).

A study by Walden University (2010) examined several myths about the relationship between educators, technology, and twenty-first-century skills. The study found that it is not necessarily true that newer teachers use technology more frequently than more experienced teachers do. The study also suggested that teachers and administrators often have very different ideas about classroom technology, with administrators more likely to assume that technology is used more often and is more effective than is actually the case. Teachers surveyed indicated that they did not feel particularly well prepared by their pre-service training programs for implementing technology and twenty-first-century skills. However, the study also reiterated the importance of the teacher's role in successful implementation of classroom technology.

These findings emphasize the critical importance of providing professional learning for teachers in the effective use of educational technology. Specifically, mathematics teachers need professional learning on how to use technology to enhance mathematics learning, not just how the tools work. This professional learning should be ongoing—not just a one-time event. Using technology to teach the same mathematical topics in fundamentally the same way does not take advantage of the capabilities of technology, and it may even be harmful in that it can show that technology is not worth the cost or effort of implementation (Garofalo et al. 2000).

The Digital Divide and the Achievement Gap

The term *digital divide* was coined in the 1990s to reference the gap in access to computers and to the Internet that separated different demographic and socioeconomic groups in the United States. The concept was popularized by a series of reports (titled *Falling Through the Net*) issued by the National Telecommunications and Information Administration (NTIA) [NTIA 1995, 1998, 1999, 2000]. These reports found that rural Americans, the socioeconomically disadvantaged, and ethnic minorities tended to have less access to modern information and communication technology and the benefits provided by those connections.

Although the gap in access has closed somewhat over the past two decades, especially in terms of access to broadband connections, it remains significant (Smith 2010). In 2009, 79.2 percent of white households had Internet access; the percentages for African American and Hispanic households were

60.0 and 57.4, respectively (United States Census Bureau 2009). Furthermore, there are concerns that minorities are less likely to be involved with social media and "Web 2.0" applications that include rich content and technologies for networking and collaboration online (Payton 2003; Trotter 2007). Given the overlap between the groups involved in the digital divide and the achievement gap in student performance, it is important that districts, schools, and teachers remain alert to the issue of equitable access to technology. Federal grants and other funding have helped to make the technology available to schools that have disproportionate populations of students from disadvantaged groups. However, despite these attempts to ensure that all students have equitable access to technology at school, many students still lack such access outside of their school environments. Studies have shown that gaps in access to reading material affect outcomes in reading achievement, and gaps in access to technology will have a similar impact on student success in twenty-first-century learning environments. The following solutions may help address these gaps (Davis et al. 2007):

- Giving students access to computer resources outside of school hours
- Issuing technology devices to students to take home
- Training teachers to be aware of access issues and providing them with strategies to address these issues as part of their professional development

Accessibility

Educational technology can help ensure that all children have access to the standards-based academic curriculum. Issues of universal access are discussed in more detail in the Universal Access chapter of this framework, but the specific ability of technology to support students with special needs should be addressed. One advantage of educational technology—the ability to differentiate instruction to meet varied learning needs—makes it a potentially effective tool to support the learning goals of these students.

Assistive technology can be used to help students with disabilities gain access to the core curriculum and perform functions that might otherwise be difficult or impossible. This technology may be a hardware device that helps a student overcome a physical disability or adaptive software that modifies content so that a student can access the curriculum. One example is a digital talking book that reads content for a student who has a visual handicap or a learning disability that affects his or her reading comprehension. Other examples include an enlarged, simplified computer keyboard; a talking computer with a joystick; headgear; or eye selection devices that could be used by students with motor difficulties. Li and Ma (2010) found that students in special education programs were a subgroup that tended to show higher gains than other students when computer technology was used to support instruction. Software that differentiates instruction can also be used to meet the needs of students who are below grade level in mathematics. The CDE's Clearinghouse for Specialized Media and Translations (http://www.cde.ca.gov/re/pn/sm/ [accessed September 2, 2015]) produces accessible versions of textbooks, workbooks, assessments, and ancillary student instructional materials. Accessible formats include braille, large print, audio, and digital files that may consist of Rich Text Format (RTF), HyperText Markup Language (HTML), the Digital Accessible Information System (DAISY), or Portable Document Format (PDF).

Educational technology may also be used to support English learners. Software that uses visual cues to assist in the teaching of mathematics concepts can help someone with limited English proficiency gain understanding. A 2010 study of one district's Digital Learning Classroom project found that interactive whiteboard technology used in grades three and five increased English learners' achievement and helped to close the achievement gap between English learners and students who are proficient in English (Lopez 2010).

Finally, educational technology can help to provide a challenging and interesting educational environment for advanced learners. Computer programs that include self-paced options and allow students to explore advanced concepts can keep these students engaged in the learning process. Technology that facilitates a collaborative learning environment can also help advanced students become involved in their peers' study of mathematics, which is a more useful outcome than simply giving advanced learners a longer list of problems to solve or sending them off to study independently. Adaptive-learning software provides individualized instruction that focuses on the needs of all students and challenges them to improve in mathematics achievement.