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Attachment 2

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# Overview of the 2023 California *Mathematics Framework* and Revision Process

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Citations included in this document can be found in appendix B of the *Mathematics Framework*.

## **What Is the Mathematics Framework?**

The purpose of the 2023 *Mathematics Framework for California Public Schools (Mathematics Framework)* is to support implementation of the California Common Core State Standards for Mathematics (CA CCSSM). Adopted in 2010 and updated in 2013, the standards put forth the state’s vision for mathematics, kindergarten through grade twelve (K–12), mapping out what students need to know and be able to do in math by the end of each grade level. The framework offers guidance for enacting the standards through curriculum and instructional approaches grounded in research and reflecting best practices across the globe. The goal is to ensure equity and excellence in math learning so that all California students become powerful users of mathematics to better understand and positively impact the world—in their careers, in college, and in civic life.

## **Why a New Framework?**

Given the status of mathematics access and achievement, California and the United States (US) need to develop new approaches for deepening math learning, addressing achievement gaps, and modernizing instruction to meet the demands of a fast-changing world. The US has long rated below the international average on the Programme for International Student Assessment (PISA). Currently, US 15-year-olds score lower than 30 other education systems, and their scores have declined since PISA began in 2000. While California saw less pandemic-related decline than other states on the most recent National Assessment of Educational Progress (NAEP), California still ranks below the national average on NAEP’s fourth- and eighth-grade math assessments. On California’s most recently reported Smarter Balanced assessment, only 33 percent of students met or exceeded mathematics proficiency standards in 2022, and substantial achievement gaps exist, especially for Black, American Indian or Alaska Native, and Latino students.

To address the urgency to improve mathematics learning, the framework provides new guidance for curriculum and instruction that builds on a body of research on effective mathematics teaching and learning and the successful approaches used in nations that produce high and equitable achievement in math. Helping students develop the thinking skills and content knowledge necessary to master the standards is critical to improving academic performance and creating conditions for later success. The 2023 *Mathematics Framework* includes strategies to support educators to integrate California’s Standards for Mathematical Practice in ways that challenge, engage, and support all students in deep and relevant math content learning. Drawing on the experiences of educators who have worked for years to develop successful strategies for teaching California’s rigorous standards, the framework’s strategies and practices aim to ensure that all students develop deep skills and a love of mathematics and that many more choose to pursue a science, technology, engineering, or math (STEM) major in college or pursue other careers that benefit from quantitative knowledge and reasoning. Strong mathematical knowledge will better enable all of California’s students to thrive as citizens in a complex world.

## **What’s in the Mathematics Framework?**

The 2023 *Mathematics Framework* includes guidance that will help educators to structure the teaching of the state’s rigorous standards around “Big Ideas” that integrate rather than isolate TK–12 math concepts—a best practice in high-performing countries. This approach encourages teachers to think about how the Big Ideas in mathematics connect both within and across grade levels in developmental progressions. For example, students develop informal number sense even before they begin their formal schooling, and mathematics instruction should continue to deepen students’ understandings of number and operations as they progress through their elementary years, providing a strong foundation for more complex mathematical ideas such as algebra. It emphasizes development of the habits of mind and habits of interaction described in the Standards for Mathematical Practice—for example, persevering in problem solving, explaining one’s thinking, constructing arguments—that students need in order to become powerful users of mathematics and better interpret and understand their world. The framework’s instructional approach connects learning to the “real world” through authentic examples and use of data, an approach that not only addresses “why do I need to learn this?” but sparks student curiosity and encourages inquiry, problem solving, and deep learning. The framework also emphasizes students’ need to have opportunities to explicitly connect their conceptual understanding with facts and procedures in ways that make sense to them so that they can use known facts, including those drawn from memory, to determine unknown facts and develop fluency.

The framework dispels myths around who can and who can’t learn math and encourages development in students of a growth (rather than fixed) mindset, which bolsters perseverance, confidence, and achievement. It also includes guidance to help educators make math instruction culturally relevant and empowering, thus allowing students to “see themselves” in curriculum and in math-related careers. The pedagogical strategies it highlights have been found to promote higher and more equitable engagement and achievement for English learners and for other students who have been historically underrepresented in STEM careers. The framework supports instruction for linguistically and culturally diverse English learners who are developing mathematical proficiency informed by the California English Language Development Standards (CA ELD Standards) and the California Department of Education’s advice for integrating the CA ELD Standards into mathematics teaching (CDE, 2021a). It incorporates explicit guidance for planning assessment and instructional materials adoptions that are responsive to English learner needs.

The framework clarifies guidance around meeting the needs of high-achieving learners as well as those who need greater assistance. The aim is to ensure attention to a range of instructional options, including personalized learning, differentiated instruction, strategic grouping, additional course-taking opportunities, and acceleration. Relatedly, the framework discusses the conditions research has found to be associated with successful acceleration, and it provides guidance about multiple strategies, in addition to acceleration, for ensuring that students have the opportunity to reach advanced mathematics courses, such as calculus and statistics, during high school.

In response to the explosion of data use in our daily lives and across fields, the framework highlights the opportunities data present for bringing mathematics alive for students. It discusses ways to integrate data use into math instruction throughout the grades and to encourage students to become proficient at understanding and using data—including helping students use data to make decisions in their roles as global citizens and identify misleading data, so that their decisions are sound.

To align with State Board of Education (SBE) guidelines and the CA CCSSM, the framework affirms two high school pathways—Traditional and Integrated—and clarifies the autonomy of local schools and districts in designing courses and pathways that best meet the needs of local families. The guidance helps educators to integrate and align math concepts taught at the middle and high school levels, ensure that all high school math pathways are open to all students, and expand high school math course options to encourage more students to go beyond minimum course-taking requirements.

### The framework’s content by chapter is as follows:

Chapter 1—*Mathematics for All: Purpose, Understanding, and Connection*—introduces and sets up the *Mathematics Framework*, describing how it lays out the curricular and instructional approaches that evidence suggests will afford all students the opportunities they need to learn meaningful and rigorous mathematics, meet state mathematics standards, access pathways to high level math courses, and achieve success.

Chapter 2—*Teaching for Equity and Engagement*—focuses on how teachers can create equitable and engaging mathematics classroom environments, TK–12, that support all students and improve mathematics access and outcomes.

Chapter 3—*Number Sense*—provides an in-depth discussion of the progression of the Big Ideas of number sense and the growth of number fluency—the ability to use mathematics flexibly, efficiently, and accurately—across the grade levels.

Chapter 4—*Exploring, Discovering, and Reasoning With and About Mathematics*—goes deeply into California’s Standards for Mathematical Practice, providing important background for discussing how elementary, middle, and high school teachers teach the Big Ideas of mathematics. (See also chapters 6, 7, and 8).

Chapter 5—*Mathematical Foundations for Data Science*—helps educators identify how the CA CCSSM can support PK–12 students to develop foundational knowledge and skills for the use of data as a tool for mathematical problem solving.

Chapter 6—*Mathematics: Investigating and Connecting, Transitional Kindergarten through Grade Five*—discusses how the framework’s approach to mathematics teaching unfolds throughout elementary school where mathematics is a vibrant, interactive, student-centered endeavor of investigating and connecting the Big Ideas of mathematics. Students develop an understanding of concepts that include place value, arithmetic operations, fractions, geometric shapes and properties, and measurement. These concepts are studied with a goal to develop both understanding and fluency, which requires knowing, efficiently retrieving, and appropriately using facts, procedures, and strategies, including from memory.

Chapter 7—*Mathematics: Investigating and Connecting, Grades Six through Eight* —discusses how the framework’s approach to mathematics teaching unfolds throughout middle school, where major topics include proportional reasoning, rational numbers, measurement in geometrical and data science scenarios, all developed through activities that are situated in intriguing, authentic contexts and require students to build connections among ideas.

Chapter 8—*Mathematics: Investigating and Connecting,* *High School*—discusses how the framework’s approach to mathematics teaching unfolds throughout high school, where the mathematics content standards are organized into conceptual categories (number and quantity; algebra; functions; modeling; geometry; and statistics and probability). It describes the traditional and integrated pathways schools may offer and the course options that students and their families may choose depending on their interests and aspirations. It also clarifies the expectations for coursework leading to different higher education options.

Chapter 9—*Structuring School Experiences for Equity and Engagement—*describes methods of teaching and coursework designs that can enable all students to be appropriately challenged and many more to reach advanced mathematics, without requiring that all students work on the same mathematics or be placed in inflexible course sequences that make it difficult for them to move into or between STEM or non-STEM pathways if they so choose.

Chapter 10—*Supporting Educators in Offering Equitable and Engaging Mathematics Instruction—*discusses how to plan and design structured, ongoing, high-quality programs of professional learning that support teachers throughout their careers as they enact the framework’s approach to teaching mathematics.

Chapter 11—*Technology and Distance Learning in the Teaching of Mathematics—*provides guidance to help educators enrich students’ learning of the mathematics content and practice standards through the use of technology.

Chapter 12—*Mathematics Assessment in the 21st Century—*encourages educators, administrators, and policymakers to focus on assessment that engages students in continuous improvement efforts by using mastery-based approaches—notably, by assessing with rubrics and using self, peer, and teacher feedback.

Chapter 13—*Instructional Materials to Support Equitable and Engaging Learning of the California Common Core State Standards for Mathematics*—is directed to the developers and publishers of instructional materials and conveys information needed to participate in the SBE’s instructional materials adoption process.

## **Revision Process**

California has followed a careful process for developing the framework that provides transparency, follows the California *Education Code* and complies with regulations aimed at maximizing public input. Development of the *Mathematics Framework* started in 2019 and has included input from teacher and student focus groups, a committee of educators, public meetings of the state’s Instructional Quality Commission (IQC), and two 60-day public comment periods during which a wide range of interest holders contributed feedback. At a May 2021 public hearing on the draft, the IQC noted concerns raised by the public in the first 60-day field review and recommended hundreds of edits and other changes. A revised draft was then posted on March 14, 2022, for a second 60-day field review. Over those two months, CDE received more than 900 comments, many quite detailed, requesting further changes.

As is often the case with projects that require additional capacity, the CDE and SBE worked in partnership with the Region 15 Comprehensive Center (R15CC), a federally -funded technical assistance center led by WestEd, to carefully review and analyze feedback from the second field review. The R15CC tapped its network of in-house mathematics education experts to assist in the process. The revised draft is responsive to public concerns, responsive to direction from the IQC, and aligned with SBE guidelines.

## **Notable Improvements Responsive to Public Comment**

Guidance around meeting the needs of high-achieving learners, as well as those who need greater assistance, has been clarified and strengthened to ensure attention to a range of instructional options, including personalized learning, differentiated instruction, strategic grouping, additional course-taking opportunities, and acceleration. The recommended edits to the draft incorporate the conditions research has found to be associated with successful acceleration, as well as strategies for reaching advanced coursework without acceleration. It also describes means for ensuring students can gain access to advanced level mathematics courses at various junctures.

To bring the guidance into alignment with SBE guidelines and the CA CCSSM, the recommended edits to the draft affirm two high school pathways––Traditional and Integrated. In response to the explosion of data use across fields and the opportunities to bring mathematics alive for students, the recommended edits to the draft incorporate ways to integrate data use into math instruction throughout the grade spans in both high school pathways. The draft also clarifies the autonomy of local schools and districts in designing courses and pathways that best meet the needs of their students and families. The high school pathway discussion and graphic were revamped to more clearly articulate course sequencing and course options, including pathways to pre-calculus and calculus, as well as options for financial literacy, data science, statistics, and discrete math. Citations to research and other sources have been reviewed and updated to ensure accuracy.

The revised draft reflects hundreds of individual line edits throughout all of the chapters responsive to hundreds of specific public suggestions fielded during the second of two 60-day public comment periods. It also includes structural and content changes responsive to more general public comments, direction from the IQC and the SBE-adopted framework guidelines. The public comments generally underscored the importance of clear phrasing and reader-friendly structuring and formatting of information throughout the framework, as well as reducing the volume of text. Recognizing this, the editing of each chapter attended to providing clear and concise wording as well as to removing redundancies, shortening where possible, and improving graphics. In some places text was rearranged, subheads or paragraph heads were added, and/or transitions were created to improve reader friendliness. To improve the flow and readability of the text, longer vignettes were moved to an appendix, with hyperlinks. To support framework-wide cohesion, discussion of the instructional design approach, which recurs across chapters, was clarified and standardized.

### Chapters 1, 2, 5, 8, 9, and 13 reflect more significant content changes. A summary of revisions to those chapters is as follows:

*Chapter 1:* Addressing public comments in the second field review included updating and revising the neuroscientific discussions to avoid overgeneralizations and ensure that citations are correct and research findings are clearly and accurately captured. To clarify the core ideas introduced in this first chapter of the framework, some sections were moved and section headers reworded to more effectively communicate the central ideas of the framework: an understanding of the Big Ideas that organize the standards, the use of multidimensional teaching approaches and open tasks, and a focus on equity. Material from other chapters was incorporated to more clearly explain the Drivers of Investigation, Standards for Mathematical Practice, and Content Connections, and how they interact.

*Chapter 2:* In responding to public comment, this chapter revision began with data on mathematics achievement and inequities in student outcomes that warrant instructional change. It then presents the five instructional strategies described in the chapter, incorporating additional discussion and research evidence to help readers understand what the strategy is and how its use may support equitable student outcomes. The chapter was shortened by rearranging text and removing several tables not previously discussed in the narrative as well as some information specific to implementation of the instructional strategies.

*Chapter 5:* In responding to public comments in the second field review, the revisions of this chapter seek to help readers understand how the foundations of data science currently reside within existing content standards across grade levels, the chapter was reorganized around three thematic topics derived directly from the CA CCSSM: understanding and describing variability in data and data distributions; data collection, sampling, and random processes; and comparing distributions and identifying associations between variables. The revised chapter also seeks to clarify how data literacy and data science are part of a continuum. While data literacy—the ability to understand and use data to answer questions—is part of data science, the field of data science also includes advanced mathematics and computational skills that build upon—and go far beyond—the content contained in the kindergarten through grade twelve CA CCSSM. The chapter also contains new content that highlights how the use of computational tools and bigger data sets can support students’ understanding of statistics in middle school and high school.

*Chapter 8:* Chapter 8 was revised to address public comments in the second field review, particularly to remove references to the Mathematics: Investigating and Connecting (MIC) pathway, which had encouraged more intensive use of data science. Instead the framework encourages the greater use of data in all course pathways. The revision also pulled portions of appendix A into the chapter to better explain the course content within each of the two pathways. With the removal of the MIC pathway, the draft affirms two high school pathways—Traditional and Integrated. The pathways graphic (figure 8.4) was also updated to clarify that students should be able to choose from more course offerings during their third- and fourth-year math classes, as well as to clarify which courses are needed to be ready to pursue a STEM pathway in college. Redundancies within the chapter were removed and research citations were added to support the discussion of topics such as integration, algebra enrollment, and acceleration. Information about University of California mathematics (area C) course criteria was checked and clarified.

*Chapter 9:* Chapter 9 was revised to address public comments in the second field review, specifically to clarify and ensure accuracy in discussions of the research on tracking, provide additional explanation and research evidence related to personalization, one-on-one tutoring, and technology integration, and eliminate redundancies within the chapter.

*Chapter 13*: To align with public comments throughout the framework in the second field review, the revisions of this chapter included adding and adapting language from the 2013 evaluation criteria and piloting guidelines into a new introduction with context around state and local adoptions with added emphasis on alternatives to traditional textbooks as technology evolves. The State Adoption of Instructional Materials section has added emphasis of the standards organization around Big Ideas along the learning progressions, and addresses local educational agencies’ responsibility and authority to evaluate instructional materials and adopt those that best meet the needs of their students. Changes to this section further clarify the categories of criteria for the evaluation of instructional resources.

California Department of Education, June 2023