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# **Item 11 Addendum**: Mathematics Framework–– Staff-Recommended Edits Based on Public Comment

The California Department of Education and State Board of Education (SBE) staff are recommending the amendments listed below, based on public comment received, to the SBE action on the *Mathematics Framework.* Differential markup is required for document accessibility purposes for all web postings and is included as follows: proposed additions are marked as <begin add>underlined<end add>, while proposed deletions are marked as <begin delete>~~strikethrough~~ <end delete>.

Chapter 1, page 8, lines 205–206 (make corresponding edit in appendix B):

<begin delete>~~In many other countries, the standards guiding content in each grade are fewer, higher, and deeper, with greater coherence and integration~~.<end delete> <begin add>Studies of high achieving countries, find that their standards (or national course of study) guiding content are fewer and higher, with greater coherence (Schmidt, Houang, & Cogan, 2002).<end add>

Chapter 1, pages 8–9, lines 206–209:

Topics are studied more deeply, with applications to real world problems. <begin delete>~~Mathematical~~<end delete> <begin add>Instructional<end add> practices include collaborative problem-solving strategies, heterogeneously grouped classrooms, and an integrated approach to mathematics from grade school through high school.

Chapter 3, pages 3–4, lines 60–62:

Similarly, chapter 4 describes how the Standards for Mathematical Practice can be instilled across grade levels, and chapter 5 describes progressions for <begin add>the development of data science for all the grades, K–12<end add>.

Chapter 3, page 19, lines 471–476:

Class discussions that allow students to express and critique their own and others’ reasoning are instrumental in supporting flexible thinking about numbers and the development of generalizable methods for addition and subtraction (SMP.2, 3, 4, 6, 7). <begin delete> ~~Note that while students in first grade do begin to add two-digit numbers, they do so using strategies as distinguished from formal algorithms. The CA CCSSM intentionally place the standard algorithms for addition and subtraction in fourth grade (4.NBT.4).~~<end delete> <begin add> Note that students in first grade are not expected to add two-digit numbers using a formal algorithm; the CA CCSSM places the standard for students’ fluent addition and subtraction of multi-digit whole numbers using the standard algorithm in grade four.<end add>

Chapter 3, page 20, line 503:

Change the labels on the graphic so that the upper box reads “3 + 4” and the lower one reads “4 + 3”.

Chapter 3, page 32, lines 793–795:

For example, if two columns plus an additional five small squares are shaded on the grid, the student can visualize that value as <begin delete>~~1.25 or 1-1/4~~<end delete> <begin add>0.25 or 1/4<end add> of the whole.

Chapter 6, page 48, line 979 (third row, third column of table):

**MD.8, MD.5, NBT.1, NBT.2, NBT.5, NBT.6, NBT.7<begin add>, NBT.9<end add>:** Understand the unit values of money and compute different values when combining dollars and cents. <begin add> Connect these money values to place values and to 2-digit and 3-digit methods of adding and subtracting and explain such methods using drawings as needed.<end add>

Chapter 6, page 48, line 979 (fifth row, third column of table):

**NBT.1, NBT.3, NBT.7, <begin add>NBT.9, <end add>OA.4, G.2:** Use skip counting, counting bundles of 10, and expanded notation to understand the composition and place value of numbers up to 1,000. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. <begin add>Use these place values to develop understanding with 3-digit adding and subtracting.<end add>

Chapter 6, page 51, line 1065:

(fourth row, second column of table):

<begin delete>~~Addition and subtraction patterns~~<end delete> <begin add> Patterns in Four Operations<end add>

(fifth row, second column):

Number flexibility to 100 <begin add>for all four operations<end add>

Chapter 6, page 76, lines 1563–1567:

<begin delete>*~~Everyday Mathematics~~* ~~offers guidance for families, explaining how premature instruction in standard algorithms can often lead to erroneous and even harmful ideas. Students may come to believe that mathematics is mostly about memorizing, that mathematics problems should be solved in a few minutes, and that there is just one right way to solve a problem.~~<end delete>

Chapter 6, page 76, lines 1568–1574:

<begin delete> ~~Note that the CA CCSSM do not include standard algorithms in transitional kindergarten through grade three, although there are standards addressing fluencies needed for proficiency in standard algorithms in later grades. Instead, as shown in figure 6.31, the progression related to standard algorithms begins with the standard algorithm for addition and subtraction in grade four; the algorithm for multiplication is addressed in grade five; and the introduction of the standard algorithm for whole number division comes in grade six. (Chapter seven addresses grade six.)~~<end delete> <begin add> The CA CCSSM do not call for fluency with standard algorithms in grades TK–3, so there is time to develop meanings for accessible standard algorithms with drawings in these grades. The CA CCSSM do say that first-grade “Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10.” (CDE 2013, 14). And second-grade students “solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations.” (CDE 2013, 18). The CA CCSSM place fluent use of standard algorithms at the grades indicated below in the table.<end add>

Chapter 6, page 81, lines 1643–1644:

Acquiring fluency with multiplication <begin add> and division of whole numbers <end add> <begin delete> ~~facts~~ <end delete> begins in third grade and development continues in grades four and five.

Chapter 8, page 9, line 130 (add the following text after the last bullet):

<begin add> One fundamental use of mathematics is to model real-world situations, for example the costs of different cellphone plans, the changing values of used cars, or the different sizes and comparative costs of different pizzas. What all these situations have in common is that the objects and relationships between them can be expressed in mathematical terms, often with one such object being expressible as a function of another. Once a situation has been described in that way, it can be analyzed mathematically to discover relationships, find patterns, and make predictions.

As students progress through the secondary curriculum, they encounter increasingly complex mathematical functions and relationships, and increasingly sophisticated ways to represent and analyze data. They begin by working with linear functions, sets of linear functions, and some polynomial families of functions (for example quadratics). Later on they encounter logarithmic, exponential, and trigonometric functions. The mathematical objects or analytic methods they encounter may be new – but the processes of mathematizing and sensemaking are the same. The goal, whether for applications or the study of mathematical objects and relations in their own right, is to develop robust understandings and habits of sensemaking, with an increasingly large toolkit of concepts and functions. <end add>

Chapter 6, page 118, line 2519:

(fourth row, second column of table)

<begin delete>~~Addition and Subtraction Patterns~~<end delete>

(fifth row, second column of table)

Number Flexibility to 100 <begin add>For All Four Operations<end add>

Make corresponding changes to figure 6.52.

Chapter 8, page 29, lines 700–706:

**Structuring High School Pathway<begin delete>~~s~~<end delete>**

High schools are free to organize their mathematics pathways in different ways. <begin add> By completing Algebra I and Geometry or Mathematics I and II,1 students will satisfy the requirements of California Assembly Bill 220 of the 2015 legislative session that requires students to complete two mathematics courses in order to receive a diploma of graduation from high school, with at least one course meeting the rigor of Algebra I. Depending upon their post-secondary goals, students may choose different third- and fourth-year courses, and all college-intending students should complete four years of mathematics in high school to meet California State University and University of California recommendations. <end add>

Figure 8.4 below indicates possible pathways for high school coursework, reflecting a common ninth- and tenth-grade experience, and a broader array of options in eleventh and twelfth grade <begin add> relevant to students’ interests <end add>. High schools will typically offer either the Integrated or Traditional pathway during students’ first two or three years as well as an array of more advanced courses<begin add>, some of which qualify for Area C credit in the UC/CSU admissions process (see discussion below) <end add>.

Insert footnote: <begin add>1 Note that the second course (beyond Algebra I or Mathematics I) can be any mathematics course of the student’s choosing. <end add>

Chapter 8, page 30, line 721–724:

For the figure, add asterisks to “Algebra I” and “Mathematics I,” and make the asterisk on “Calculus” a double asterisk. Change the text on lines 723–724 as follows:

<begin add>\* Students may take Algebra 1 or Mathematics 1 in middle school.<end add>

\*<begin add>\*<end add> Calculus, which can be taken during or after high school, is an important course to support student selection of a STEM career.

Chapter 8, page 32, lines 766–773:

Delete paragraph and corresponding footnote.

Chapter 8, page 34, lines 836–838:

The update includes the allowance of courses in <begin add> advanced mathematics <end add> <begin delete> ~~data science~~ <end delete> to serve as the required third (or recommended fourth) year of mathematics coursework. <begin delete> ~~For additional information on data science, see chapter 5~~ <end delete>.

Chapter 8, page 35, lines 842–851:

Courses that substantially align with Common Core (+) standards (see chapters on *Higher Mathematics Courses: Advanced Mathematics* and *Higher Mathematics Standards by Conceptual Category* and the Standards for Mathematical Practice [SMPs] in the *California Common Core State Standards: Mathematics* [2013]), and are intended for eleventh- and/or twelfth-grade levels are eligible for area C approval and may satisfy the required third year or recommended fourth year of the mathematics subject requirement if approved as an advanced mathematics course. <begin delete> ~~Examples of such courses include, but are not limited to, applied mathematics, computer science, data science, precalculus, probability, statistics, and trigonometry~~ <end delete>.

Chapter 8, pages 36–37, lines 887–899:

This statement <begin delete> ~~and UC’s 2020 policy shift (Johnson, 2020)~~ <end delete> encouraging <begin add> students to choose an individually appropriate course of study <end add> <begin delete> ~~more flexibility in high school courses show the commitment of the University of California to~~ <end delete> <begin add> reinforces the <end add> value <begin add> of <end add> a range of mathematics courses as pathways to college <begin add> and careers <end add>. For some students—particularly those intending to major in mathematics, engineering and other STEM fields, a strong pathway to calculus in high school or the first year of college is valuable. Many other students with different future intentions, such as social science <begin add> or business <end add> degrees, may <begin add> undertake a pathway <end add> <begin delete> ~~be better served with courses~~ <end delete> that lead<begin add>s<end add> to <begin delete> ~~data science and~~ <end delete> statistics <begin add> or financial algebra <end add>. Such courses should be designed so that they can also lead to a possible future in STEM. They are inherently mathematical and can be designed to include the topics enumerated at the beginning of this chapter and the competencies described as desired for entering college students <begin delete> ~~by the UC, CSU, and community college system (Intersegmental Committee of the Academic Senates of the California Community Colleges, the California State University, and the University of California, 2010, 2013)~~ <end delete>:

Chapter 8, page 38, lines 944–945:

The main purpose of Algebra I is to develop students’ <begin add> understanding of and <end add> fluency with linear, quadratic, and exponential functions<begin add>, and their use to model real-world phenomena <end add>.

Chapter 8, page 45, lines 1008–1010:

The major mathematical ideas in the geometry course include geometric transformations, proving geometric theorems, congruence and similarity, analytic geometry, right-triangle trigonometry, and probability. <begin add> Producing a proof should not be seen as a way to meet abstract requirements regarding the ways that mathematical claims should be presented, but rather as the end product of reasoning and sensemaking, organized and presented in ways that make it easier to convey the resulting understandings.<end add>

Chapter 8, page 74, lines 1333–1338:

Diagram indicating two pathways of courses indicating a variety of course offerings for Years 3 and 4 in high school. The preparatory courses are Algebra I and Mathematics I, followed by Geometry and Mathematics II. The later course options include <begin delete>~~(in no particular order): Computer Science,~~ <end delete> Algebra II, Mathematics III, <begin add>Computer Science,<end add> Statistics, Data Science I, II, Precalculus, Calculus, Discrete Math, Financial Algebra, and Other Math. All of these options lead to STEM and Non-STEM Majors and Careers.

Chapter 9, page 15, lines 372–378:

In one successful teacher-developed approach, students engage in blended, self-paced, mastery-based learning with teacher-made videos supplementing in-class problem-solving individually and in collaborative groups, with continual assessment and revision of work moving students toward confidence and competence (Modern Classroom, 2021). <begin add> A similar model developed by a middle school teacher and now taught in many schools, uses diagnostic assessments to create a tailored set of assignments for each student that the teacher can use in technology-infused mix of direct instruction, collaborative work with peers, and individualized learning. <end add> A study of this model found that participating students improved at a faster rate, on average, on mathematics assessments than did a nationally representative comparison group (Margolis, 2019).

Appendix A, page 1, lines 5–8:

Appendix A: <begin delete> ~~Mathematical Progressions within the High School Pathways and~~ <end delete> Key Mathematical Ideas to Promote Student Success in Introductory University Courses in Quantitative Fields

Appendix A, pages 2–8, lines 23–207:

Delete these lines and make corresponding changes to the table of contents.