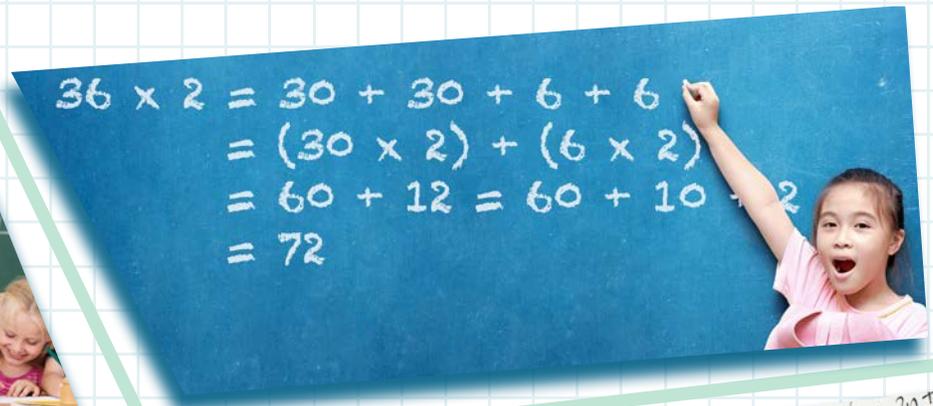


Mathematics Framework

for California Public Schools

Kindergarten Through Grade Twelve



Adopted by the California State Board of Education, November 2013

Published by the California Department of Education
Sacramento, 2015

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Developed by the Instructional Quality Commission

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Foreword

With the *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve* (framework), we take the next steps on a path toward our goal of college and career readiness for all of California’s students. This journey began with the adoption of the California Common Core State Standards for Mathematics (CA CCSSM) in August 2010, which sparked exciting and important shifts in mathematics instruction and learning. The journey will continue as schools and districts bring the CA CCSSM to life in classrooms throughout the state. This framework is part of that effort.

Focus, coherence, and rigor—the three major instructional shifts under the CA CCSSM—are the principles underlying a new direction for student learning. The promise of the CA CCSSM is instruction and learning that promotes problem solving, communication skills, and critical thinking. To meet this promise, students must experience a balanced approach to instruction and learning that supports conceptual understanding, procedural skill and fluency, and application of mathematics to real-world problems.

Mathematics is essential to living in and understanding the world. We apply mathematics when we check our pockets to be sure we have enough money for a purchase, measure ingredients for making dinner, or evaluate the evidence in a debate on local government spending. Mathematicians and scientists apply mathematics to determine how much thrust is needed to launch a spacecraft, to plot its course into space, and ensure that it lands safely. Understanding and being able to apply mathematics opens the doors to college and to careers in fields as varied as automobile mechanics, architecture, construction, medicine, engineering, economics, and the arts. In addition, exploring mathematical concepts, working collaboratively on engaging tasks, and presenting and critiquing arguments—all of which are required by the CA CCSSM—help prepare students for life after high school.

It will take a concerted effort for all students to meet the goal of college and career readiness, and the path will not always be easy. Teachers, administrators, other educators, parents and family members, community members, education stakeholders, policymakers, institutes of higher education, early learning programs, and students all have vital roles to play. Working together, embracing the challenge and the promise of providing all students with high-quality, standards-based mathematics instruction, we can help our children reach their personal college and career goals.

We invite you to join us on the journey toward college and career readiness for California’s students and offer this mathematics framework as a guide.



TOM TORLAKSON
State Superintendent of Public Instruction



MICHAEL W. KIRST
President, California State Board of Education

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This edition of the *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve* was adopted by the California State Board of Education (SBE) on November 6, 2013. When this edition was approved, the following persons were serving on the SBE:

Michael W. Kirst, President
Ilene Straus, Vice President
Sue Burr
Carl A. Cohn
Bruce Holaday
Aida Molina
Patricia Ann Rucker
Nicolasa Sandoval
Trish Boyd Williams
Jesse Zhang, Student Member

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Introduction

The highest form of pure thought is mathematics.

—Plato (427–347 BCE)

Focus, coherence, and rigor, the underlying principles of the California Common Core State Standards for Mathematics (CA CCSSM), hold the promise of preparing all California students for college, careers, and civic life—and developing mathematically competent individuals who can use mathematics as a tool for making wise decisions in their personal lives, a foundation for rewarding work, and a means for comprehending and influencing the world in which they will live. This framework supports these ambitious goals by emphasizing mathematical instruction and learning that focus on key topics, build mathematical understanding and fluency in a coherent manner, and develop students’ ability to apply mathematics creatively to analyze and solve complex problems.

Why Is Mathematics Important?

Mathematics impacts everyday life, future careers, and good citizenship. A solid foundation in mathematics prepares students for future occupations in fields such as business, medicine, science, engineering, and technology. Students’ understanding of probability and the ability to quantify and analyze information enable them to interpret economic data, participate in political discussions, and make wise personal financial decisions. Mathematical modeling is a tool for solving everyday problems, making informed decisions, improving life skills (e.g., logical thinking, reasoning, and problem solving), planning, designing, predicting, and developing financial literacy.

Success in mathematics education provides students with college and career options and increases prospects for future income. Knowledge and understanding of high school mathematics correlates to access to college, graduation from college, and earnings in the top quartile of income from employment. The value of such preparation promises to be even greater in the future. The National Science Board indicates that the growth of jobs in the mathematics-intensive science and engineering workforce is outpacing overall job growth by a 3-to-1 ratio (National Mathematics Advisory Panel 2008).

Mathematics Achievement

With regard to achievement in mathematics, students in the United States have not kept pace with their international peers. Achievement gaps still exist throughout the country, college remediation rates are too high, and some students are unprepared to perform and thrive in the workforce. California’s student achievement data reflect similar challenges for some students. The 2011 National Assessment of Educational Progress (NAEP) results indicate that California’s fourth- and eighth-grade students continue to make incremental gains in their mathematics scores; however, too many students also continue to place at the “Basic” achievement level, which denotes partial mastery of fundamental skills (California Department of Education [CDE] 2011).

Standards Implementation

The CA CCSSM resemble the standards of the highest-achieving nations and reflect the importance of focus, coherence, and rigor. California’s implementation of the CA CCSSM demonstrates a commitment to providing a world-class education for all students, narrowing the achievement gap, supporting lifelong learning, and helping students develop the skills and knowledge necessary to fully participate in the global economy of the twenty-first century. The CA CCSSM build on California’s standards-based educational system in which standards, curriculum, instruction, assessment, and accountability are aligned to support student attainment of the standards. Teachers and local school officials, in collaboration with families and community partners, use standards to help students achieve academic success (CDE 2012c).

California Common Core State Standards for Mathematics

For more than a decade, research conducted on mathematics education in high-performing countries has pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent to improve mathematics achievement in this country. The national Common Core State Standards for Mathematics, as well as the CA CCSSM, were established to address the problem of having a curriculum that is “a mile wide and an inch deep” (National Governors Association Center for Best Practices, Council of Chief State School Officers [NGA/CCSSO] 2010c).

These standards were informed by international benchmarking and began with research-based learning progressions detailing what is known about how students’ mathematical knowledge, skills, and understanding develop over time. The progression from kindergarten standards to standards for higher mathematics exemplifies the three principles of focus, coherence, and rigor that underlie the CA CCSSM. The standards stress conceptual understanding, procedural skill and fluency, and application to ensure that students will learn and absorb the critical information necessary to succeed at higher levels of mathematics and can apply their learning in increasingly complex situations.

The CA CCSSM include two types of standards: Standards for Mathematical Practice, which are the same at each grade level; and Standards for Mathematical Content, which are different at each grade level. These two types of standards address both “habits of mind” that students should develop to foster mathematical understanding and expertise, and skills and knowledge—what students need to know and be able to do. The standards also call for mathematical practices and mathematical content to be connected as students engage in mathematics. The Standards for Mathematical Practice are defined in the Overview of the Standards Chapters. In addition, the Standards for Mathematical Content and the Standards for Mathematical Practice are listed at the end of each grade level (K–8) and higher mathematics course.

Guiding Principles for Mathematics Programs in California

Five guiding principles¹ underlie the Standards for Mathematical Practice, Standards for Mathematical Content, and other resources in this framework; see table IN-1. These philosophical statements should guide the construction and evaluation of mathematics programs in schools and the broader community. The Standards for Mathematical Practice are interwoven throughout the guiding principles.

Table IN-1. Guiding Principles for Mathematics Programs in California

Guiding Principle 1: Learning

Mathematical ideas should be explored in ways that stimulate curiosity, create enjoyment of mathematics, and develop depth of understanding.

Guiding Principle 2: Teaching

An effective mathematics program is based on a carefully designed set of content standards that are clear and specific, focused, and articulated over time as a coherent sequence.

Guiding Principle 3: Technology

Technology is an essential tool that should be used strategically in mathematics education.

Guiding Principle 4: Equity

All students should have a high-quality mathematics program that prepares them for college and careers.

Guiding Principle 5: Assessment

Assessment of student learning in mathematics should take many forms to inform instruction and learning.

Guiding Principle 1: Learning

Mathematical ideas should be explored in ways that stimulate curiosity, create enjoyment of mathematics, and develop depth of understanding.

Students need to understand mathematics deeply and use it effectively. The Standards for Mathematical Practice describe ways in which students increasingly engage with the subject matter as they grow in mathematical maturity and expertise through the elementary, middle, and high school years.

For students to achieve mathematical understanding, instruction and learning must balance mathematical procedures and conceptual understanding. Students should be actively engaged in doing meaningful mathematics, discussing mathematical ideas, and applying mathematics in interesting, thought-provoking situations. Student understanding is further developed through ongoing reflection about cognitively demanding and worthwhile tasks.

1. The guiding principles were adapted from the Massachusetts Curriculum Frameworks and are included by permission of the Massachusetts Department of Elementary and Secondary Education. The complete and current version of each Massachusetts curriculum framework is available at <http://www.doe.mass.edu/frameworks/current.html> (accessed May 9, 2014).

Tasks should be designed to challenge students in multiple ways. Short- and long-term investigations that connect procedures and skills with conceptual understanding are integral components of an effective mathematics program. Activities should build upon students' curiosity and prior knowledge and enable them to solve progressively deeper, broader, and more sophisticated problems; see **MP.1** (Make sense of problems and persevere in solving them) in table OV-2 of the Overview of the Standards Chapters. Mathematical tasks reflecting sound and significant mathematics should generate active classroom discourse, promote the development of conjectures, and lead to an understanding of the necessity for mathematical reasoning; see **MP.2** (Reason abstractly and quantitatively) in table OV-2 of the Overview of the Standards Chapters.

Guiding Principle 2: Teaching

An effective mathematics program is based on a carefully designed set of content standards that are clear and specific, focused, and articulated over time as a coherent sequence.

The sequence of topics and instruction should be based on what is known about how students' mathematical knowledge, skill, and understanding develop over time. What and how students are taught should reflect not only the topics within mathematics but also the key ideas that determine how knowledge is organized and generated within mathematics; see **MP.7** (Look for and make use of structure) in table OV-2 of the Overview of the Standards Chapters. Students should be asked to apply their learning and to show their mathematical thinking and understanding. This high-quality instruction requires teachers to have a deep knowledge of mathematics.

Mathematical problem solving is the hallmark of an effective mathematics program. Skill in mathematical problem solving requires practice with a variety of mathematical problems as well as a firm grasp of mathematical techniques and their underlying principles. Armed with this deeper knowledge, students can use mathematics in flexible ways to attack various problems and devise different ways to solve any particular problem; see **MP.8** (Look for and express regularity in repeated reasoning) in table OV-2 of the Overview of the Standards Chapters. Mathematical problem solving calls for reflective thinking, persistence, learning from the ideas of others, and reviewing one's own work with a critical eye. Students should be able to construct viable arguments and critique the reasoning of others. They should analyze situations and justify their conclusions, communicate their conclusions to others, and respond to the arguments of others; see **MP.3** (Construct viable arguments and critique the reasoning of others) in table OV-2 of the Overview of the Standards Chapters. Students at all grades should be able to listen to or read the arguments of others, decide whether they make sense, and ask questions to clarify or improve the arguments.

Mathematical problem solving provides students with experiences to develop other mathematical practices. Success in solving mathematical problems helps to create an abiding interest in mathematics. Students learn to model with mathematics and to apply the mathematics that they know to solve problems arising in everyday life, society, and the workplace; see **MP.4** (Model with mathematics) in table OV-2 of the Overview of the Standards Chapters.

For a mathematics program to be effective, it must be taught by knowledgeable teachers. According to Liping Ma, “The real mathematical thinking going on in a classroom, in fact, depends heavily on the teacher’s understanding of mathematics” (Ma 2010). Research on the relationship between teachers’ mathematical knowledge and student achievement confirms the importance of teachers’ content knowledge (National Mathematics Advisory Panel 2008). The message from the research is clear: having knowledgeable teachers really does matter, and teacher expertise in a subject drives student achievement. As Liping Ma states, “Improving teachers’ content subject matter knowledge and improving students’ mathematics education are thus interwoven and interdependent processes that must occur simultaneously” (Ma 2010). See the Instructional Strategies chapter and the Supporting High-Quality Common Core Mathematics Instruction chapter for more information.

Guiding Principle 3: Technology

Technology is an essential tool that should be used strategically in mathematics education.

Technology enhances the mathematics curriculum in many ways. Tools such as measuring instruments, manipulatives (such as base-ten blocks and fraction pieces), scientific and graphing calculators, and computers with appropriate software, if properly used, contribute to a rich learning environment for investigating, exploring, developing, and applying mathematical concepts. Appropriate use of calculators is essential; calculators should not be used as a replacement for basic understanding and skills. Elementary students should learn how to perform the basic arithmetic operations independent of the use of a calculator (National Center for Education Statistics 1995). The use of a graphing calculator can help middle school and secondary students visualize properties of functions and their graphs. Graphing calculators should be used to enhance—not replace—student understanding and skills.

When presenting or solving mathematical problems, teachers and students should consider the tools available to them. Students should be familiar with tools appropriate for their grade level so that they can make sound decisions about which tools will be helpful; see **MP.5** (Use appropriate tools strategically) in table OV-2 of the Overview of the Standards Chapters.

Technology enables students to communicate ideas in the classroom or to search information sources such as the Internet, which is an important addition to a school’s internal library resources. Technology can also be especially helpful in assisting students with special needs in the classroom, at home, and in the community.

Technology changes the mathematics to be learned, as well as when and how it is learned. For example, currently available technology provides a dynamic and exploration-driven approach to mathematical concepts such as functions, rates of change, geometry, and averages that was not possible in the past. Some mathematics becomes more important because technology requires it, some becomes less important because technology replaces it, and some becomes possible because technology allows it. See the Technology in the Teaching of Mathematics chapter for additional information.

Guiding Principle 4: Equity

All students should have a high-quality mathematics program that prepares them for college and careers.

All California students should have a high-quality mathematics program that meets the goals and expectations of the CA CCSSM and addresses students' individual interests and talents. The standards provide clear signposts along the way to the goal of college and career readiness for all students; they also accommodate a broad range of students, from those requiring a significant amount of extra support in mathematics to others needing minimal support or enrichment opportunities. To promote achievement of these standards, teachers should plan for, instruct, model, and support classroom discourse, reflection, use of multiple problem-solving strategies, and a positive disposition toward mathematics. They should have high expectations for all students. At every level of the education system, teachers should act on the belief that every child can and should learn challenging mathematics. Teachers and guidance personnel should advise students and parents about why it is important to take advanced courses in mathematics and how this will prepare students for success in college and the workplace.

All students should have the benefit of quality instructional materials, good libraries, and adequate technology—and all students must have the opportunity to learn and meet the same high standards. In order to meet the needs of the greatest range of students, mathematics programs should provide the necessary intervention and support for those students who are below or above grade-level expectations. Practice and enrichment should extend beyond the classroom. Tutorial sessions, mathematics clubs, competitions, robotics, and apprenticeships are examples of mathematics activities that promote learning.

Because mathematics is the cornerstone of many disciplines, a comprehensive curriculum should include applications to everyday life and modeling activities that demonstrate the connections among disciplines. Schools should also provide opportunities for communicating with experts in applied fields to enhance students' knowledge of these connections; see **MP.4** (Model with mathematics) in table OV-2 of the Overview of the Standards Chapters.

An important part of preparing students for college and careers is to ensure that they have the mathematics and problem-solving skills necessary to make sound financial decisions in everyday life—for example, to set up a bank account, learn about saving money and earning interest, understand student loans, read credit and debit statements, select the best bargains when shopping, and choose the most cost-effective cell phone plan based on monthly usage. See the Universal Access chapter and appendixes A and B for additional information.

Guiding Principle 5: Assessment

Assessment of student learning in mathematics should take many forms to inform instruction and learning.

A comprehensive assessment program is an integral component of an instructional program. It provides students with frequent feedback on their performance, teachers with diagnostic tools for gauging

students' depth of understanding of mathematical concepts and skills, parents with information about their children's performance in the context of program goals, and administrators with a means for measuring student achievement.

Assessments take a variety of forms, require different amounts of time, and address various aspects of student learning. Gaps in knowledge and errors in reasoning can be identified when students “think aloud” or talk through their reasoning. By observing and questioning students as they work, teachers can gain insight into students' abilities to apply appropriate mathematical concepts and skills, make conjectures, and draw conclusions. Homework, mathematics journals, portfolios, oral presentations, and group projects offer additional means for capturing students' thinking, knowledge of mathematics, facility with the language of mathematics, and ability to communicate what they know to others. Tests and quizzes assess knowledge of mathematical concepts, operations, and skills and their efficient application to problem solving; they can also pinpoint areas that require more practice or teaching. Taken together, the results of these different forms of assessment provide rich profiles of students' achievements in mathematics and serve as the basis for identifying curricula and instructional approaches to best develop students' talents.

Assessment should also be a major component of the learning process. As students help identify goals for lessons or investigations, they gain greater awareness of what they need to learn and how they will demonstrate that learning. Engaging students in this kind of goal setting can help them reflect on their work, understand the standards to which they are held accountable, and take ownership of their learning. See the Assessment chapter for additional information.

Supporting Twenty-First-Century Learning

California is part of a growing national movement to teach students the problem-solving skills and critical thinking they need for college, careers, and civic life. The Partnership for 21st Century Skills (P21) developed a framework for twenty-first-century learning comprising student outcomes and support systems. The student outcomes consist of the following elements:

1. Core subjects and twenty-first-century interdisciplinary themes, which include global awareness; financial, economic, business, and entrepreneurial literacy; civic literacy; health literacy; and environmental literacy
2. Life and career skills, which include flexibility and adaptability, initiative and self-direction, social and cross-cultural skills, productivity and accountability, and leadership and responsibility
3. Learning and innovation skills, often referred to as the “4 Cs”: creativity and innovation, critical thinking and problem solving, communication, and collaboration
4. Information, media, and technology skills, which include information literacy, media literacy, and ICT (information, communications, and technology) literacy

Support systems provided by P21 include standards and assessments, curriculum and instruction, professional development, and learning environments.

California educators need to intentionally include the 4 Cs in mathematics instruction. A fundamental goal is to promote higher-order mathematical thinking skills and interdisciplinary approaches that integrate the use of supportive technologies, inquiry, and problem-based learning to provide contexts for pupils to apply learning in relevant, real-world scenarios and that prepare all pupils for college, careers, and citizenship in the twenty-first century. Mathematics instruction and learning are instrumental to mastering P21 interdisciplinary themes, particularly financial, economic, business, and entrepreneurial literacy. Resources connecting the Partnership for 21st Century Skills with the Common Core State Standards are available at <http://www.p21.org/> (accessed May 15, 2014).

Purpose of the Framework

The *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve* is meant to guide teachers in curriculum development and instruction as they work to ensure that all students meet or exceed the CA CCSSM. The framework also provides educators and developers of instructional materials with a context for implementing the standards. Building on the standards, the framework addresses how all students in California public schools can best meet those standards. California’s mathematics framework is available online and, as such, will remain a “living” document that will be updated regularly.

Implementation of the CA CCSSM will take time and effort, but it also provides a new opportunity to ensure that California’s students are held to the same high expectations in mathematics as their national and global peers. Educators and administrators, as well as parents, guardians, and community members, are challenged to become familiar with the standards and to support raising the bar for student achievement through rigorous curriculum and instruction that develops students’ conceptual understanding, procedural skill and fluency, and application of mathematics to solve problems.