

# Supporting High-Quality Common Core Mathematics Instruction Chapter

of the

## ***Mathematics Framework***

*for California Public Schools:  
Kindergarten Through Grade Twelve*

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# Supporting High-Quality Common Core Mathematics Instruction

**B**road support is required to plan and implement effective and efficient mathematics instruction that meets the needs of every student. This is an important obligation shared by administrators, teacher leaders, college and university personnel, community members, parents, and other groups. The stakeholders at each school or school district form a support system that assists in the design, implementation, and evaluation of effective mathematics instructional programs. These stakeholders also serve an important function as advocates for a sustained focus on mastery of the California Common Core State Standards for Mathematics (CA CCSSM) by every student. This chapter addresses the roles and responsibilities of stakeholders in developing, implementing, and maintaining high-quality, standards-based mathematics instructional programs.

A comprehensive report titled *Greatness by Design: Supporting Outstanding Teaching to Sustain the Golden State* (Educator Excellence Task Force [EETF] 2012) calls for teachers, administrators, and other supervisors of mathematics instruction to take certain actions in response to the need for continual improvement of mathematics instruction in California. Recommendations from this report that are relevant to supporting high-quality mathematics instructional programs are summarized below (EETF 2012, 5–6):

- *Teacher education is uneven in duration and quality.* Some educators are given excellent preparation, while others receive minimal training. Education for and development of teacher leaders is more uneven in quality. Steps must be taken to ensure that every teacher participates in a high-quality preparation program and that mechanisms for developing leadership exist and are supported.
- *Mentoring for beginner teachers is decreasing.* Due to several factors, not the least of which is a decrease in funding, fewer and fewer teachers in California are receiving the benefits of high-quality mentoring. New teachers need to be supported through the difficult transition they experience in their first few years of teaching.
- *Professional learning time and opportunities are sorely underfunded.* California teachers have little time for collaboration or learning—usually only about three to five hours per week of individual planning time. Opportunities for professional learning and teacher collaboration must be seen as an integral part of the teaching profession.
- *Evaluation of teachers is inadequate.* Teacher evaluation is frequently spotty and rarely designed to give instructors and administrators the feedback and support that would help them improve or provide a fair and focused way to make personnel decisions. Evaluation efforts should be focused on helping teachers grow and improve, as opposed to being used to reprimand.
- *In most school districts, leadership pathways are poorly defined and inadequately supported.* There are few opportunities for expert teachers to share practices with their peers or to take

on leadership roles. Most teachers are still isolated from each other, teaching in self-contained classrooms and performing the same functions that they did when they first entered the profession. The spread of teaching expertise is uncommon, and this needs to change if high-quality mathematics instruction is to be available to every student in California. Instructional quality can improve when professionals work together.

It is evident that substantial professional learning for teachers will be needed to successfully implement the CA CCSSM. Although no single district or school has the absolute power or resources to address all of the concerns discussed in the *Greatness by Design* report, both the mathematics teaching community and stakeholders in mathematics instruction should consider these issues as major roadblocks to true progress. It is time for school and university educators throughout the state to combine their efforts and unite behind a common goal of improving mathematics instruction for all California students.

## Administrative Role and Support

The role of school board members, district administrators, and school administrators is crucial to the success of any mathematics instructional program. Establishing and clearly articulating high expectations for all teachers and for every student are the foundation of a successful program. It is essential for administrators to express a positive attitude toward mathematics and an appreciation for the importance of mathematics in the future of every student. One of the most important jobs of principals and administrators is to help create a system of collaboration among teachers for developing CA CCSSM instructional practices—and to recognize that creating such a system will take time and support.

In order to effectively support programs of instruction, district and school administrators, as well as school board members, need to understand that high-quality mathematics instruction involves these elements:

- Knowledge of the Standards for Mathematical Practice (MP standards) contained in the CA CCSSM, as well as the Standards for Mathematical Content
- An understanding of the role of the MP standards and how they contribute to establishing effective mathematics learning environments
- An understanding that the MP standards are equal in importance to the content standards in the CA CCSSM—in particular, if students are not engaging in the MP standards, then the CA CCSSM are not being fully implemented

The following resources may help administrators understand the implications of the CA CCSSM for teaching:

1. Inside Mathematics—video excerpts of mathematics lessons highlighting the MP standards: <http://www.insidemathematics.org/index.php/mathematical-practice-standards> (accessed July 7, 2015)
2. The Mathematics Common Core Toolbox (PARCC prototyping project): <http://www.ccsstoolbox.com> (accessed July 7, 2015)

3. Implementing the Mathematical Practice Standards (“Browse Illustrations” tab): <http://mathpractices.edc.org/browse-by-mps> (accessed July 7, 2015)
4. Mathematics Assessment Project (“Standards” tab): <http://map.mathshell.org/materials/stds.php> (accessed July 7, 2015)
5. Kansas Association of Teachers of Mathematics (KATM) flipbooks: <http://www.katm.org/baker/pages/common-core-resources.php> (accessed August 28, 2015)

Together with their teaching staff, administrators may need to seek opportunities for learning more about the CA CCSSM through professional workshops, conferences, or professional learning. Administrators must become informed instructional leaders for mathematics education. They should also rely on teacher leaders at their school sites or within their districts to offer support and knowledge of such practices. Additionally, administrators must be aware of the assessment strategies that can be utilized in the mathematics classroom and have a balanced approach in assessing the effectiveness of mathematics instruction. They understand that the results of multiple assessment strategies—rather than a student’s score on a single test—reflect an accurate understanding of student learning. In the same vein, a short walk-through of a classroom once a year is typically insufficient to accurately judge the effectiveness of instruction. To this end, district and school administrators should participate in ongoing professional learning on the topic of mathematics education and assessment of learning.

Administrators convey high expectations for mathematics instruction by supporting teachers with resources, including time for planning lessons, professional learning, and collaboration. Administrators also provide constructive, informative feedback while the teachers implement their plans. Frequent mathematics-lesson observations allow the school administrator to provide those teachers with relevant feedback regarding their instructional practices. Administrators engage with students and teachers to glean a full picture of the instructional practices used by the teacher and whether those practices are effective.

The MP standards play a crucial role in any CA CCSSM classroom. Administrators may be unfamiliar with these standards, and many would benefit from their own professional learning experiences that are centered on the CA CCSSM. The MP standards describe ways in which students engage in mathematics to develop deep conceptual understanding and procedural fluency. As students grow in mathematical maturity, the MP standards become evident in their classrooms. Students should be actively engaged in doing meaningful mathematics, discussing mathematical ideas and reasoning, applying mathematics in interesting situations, and discovering new mathematical ideas through modeling the world around them. The MP standards appear in different forms (depending on the grade level of the classroom), but in any classroom, they represent the ways in which students engage in *doing* mathematics and play a core role in instruction (adapted from Massachusetts Department of Elementary and Secondary Education [MDESE] 2011, 9). The MP standards are also described in the Overview of the Standards Chapters in this framework.

Table HQ-1 lists the MP standards and provides a few examples of what implementation of each practice may look like in the classroom.

**Table HQ-1. Implementation of the Standards for Mathematical Practice**

Standards for Mathematical Practice	Students	Teachers
<p><b>MP.1</b> Make sense of problems and persevere in solving them.</p>	<ul style="list-style-type: none"> <li>Analyze information and explain the meaning of the problem.</li> <li>Actively engage in problem solving (develop, carry out, and refine a plan).</li> <li>Show patience and positive attitudes.</li> <li>Ask themselves if their answers make sense.</li> <li>Check their answers with a different method.</li> </ul>	<ul style="list-style-type: none"> <li>Pose rich problems and ask open-ended questions.</li> <li>Provide wait-time for processing or finding solutions.</li> <li>Circulate to pose probing questions and monitor student progress.</li> <li>Provide opportunities and time for cooperative problem solving and reciprocal teaching.</li> </ul>
<p><b>MP.2</b> Reason abstractly and quantitatively.</p>	<ul style="list-style-type: none"> <li>Represent a problem symbolically.</li> <li>Explain their thinking.</li> <li>Use numbers and quantities flexibly by applying properties of operations and place value.</li> <li>Examine the reasonableness of answers and calculations.</li> </ul>	<ul style="list-style-type: none"> <li>Ask students to explain their thinking regardless of accuracy.</li> <li>Highlight flexible use of numbers.</li> <li>Facilitate discussion through guided questions and representations.</li> <li>Accept varied solutions or representations.</li> </ul>
<p><b>MP.3</b> Construct viable arguments and critique the reasoning of others. <b>Students build proofs by induction and proofs by contradiction.</b> CA 3.1 (for higher mathematics only).</p>	<ul style="list-style-type: none"> <li>Make conjectures to explore their ideas.</li> <li>Justify solutions and approaches.</li> <li>Listen to the reasoning of others, compare arguments, and decide whether the arguments make sense.</li> <li>Ask clarifying and probing questions.</li> </ul>	<ul style="list-style-type: none"> <li>Provide opportunities for students to listen to or read the conclusions and arguments of others.</li> <li>Establish a safe environment for discussion.</li> <li>Ask clarifying and probing questions.</li> <li>Avoid giving too much assistance (e.g., providing answers or procedures).</li> </ul>
<p><b>MP.4</b> Model with mathematics.</p>	<ul style="list-style-type: none"> <li>Apply prior knowledge to new problems and reflect.</li> <li>Use representations to solve real-life problems.</li> <li>Apply formulas and equations where appropriate.</li> <li>Ask questions about the world around them and attempt to attach meaningful mathematics to the world.</li> </ul>	<ul style="list-style-type: none"> <li>Pose problems connected to previous concepts.</li> <li>Provide a variety of real-world contexts.</li> <li>Use intentional representations.</li> <li>Provide students the space to ask questions and pose problems about the world around them.</li> </ul>

**Table HQ-1** (continued)

<b>MP.5</b> Use appropriate tools strategically.	<ul style="list-style-type: none"> <li>• Select and use tools strategically (and flexibly) to visualize, explore, and compare information.</li> <li>• Use technological tools and resources to solve problems and deepen understanding.</li> </ul>	<ul style="list-style-type: none"> <li>• Make appropriate tools available for learning (e.g., calculators, concrete models, digital resources, pencils and paper, compasses, protractors, and the like).</li> <li>• Embed tools within instruction.</li> </ul>
<b>MP.6</b> Attend to precision.	<ul style="list-style-type: none"> <li>• Calculate accurately and efficiently.</li> <li>• Explain thinking using mathematics vocabulary.</li> <li>• Use appropriate symbols and specify units of measure.</li> </ul>	<ul style="list-style-type: none"> <li>• Recognize and model efficient strategies for computation.</li> <li>• Use mathematics vocabulary precisely and consistently, and challenge students to do the same.</li> </ul>
<b>MP.7</b> Look for and make use of structure.	<ul style="list-style-type: none"> <li>• Look for, develop, and generalize relationships and patterns.</li> <li>• Apply conjectures about patterns and properties to new situations.</li> </ul>	<ul style="list-style-type: none"> <li>• Provide time for applying and discussing properties.</li> <li>• Ask questions about the application of patterns.</li> <li>• Highlight different approaches for solving problems.</li> </ul>
<b>MP.8</b> Look for and make use of regularity in repeated reasoning.	<ul style="list-style-type: none"> <li>• Look for methods and shortcuts through patterns in repeated calculations.</li> <li>• Evaluate the reasonableness of intermediate results and solutions.</li> </ul>	<ul style="list-style-type: none"> <li>• Provide tasks and problems with patterns.</li> <li>• Ask about possible answers before computations are made and inquire about reasonableness of answers after computations are made.</li> </ul>

Adapted from Howard County Public School System 2011.

Administrators play an important role in supporting teachers during the transition to a CA CCSSM classroom and beyond. The MP standards represent a different vision of what students should be doing in classrooms. Students may investigate mathematical concepts with manipulatives for an entire class period or work on the same mathematics problem for a substantial amount of time. Parents may not understand this style of instruction or these new expectations for California students. Administrators will need to provide opportunities and support for teachers to introduce and explain the CA CCSSM during interactions with parents.

## Mathematics Professional Learning for Teachers

For California mathematics teachers to provide highly effective mathematics instruction, there must be professional learning opportunities that deepen mathematics teachers' content knowledge and knowledge of effective instructional strategies. The content of such programs must be aligned with the goals and standards for teaching mathematics in California. As the *Greatness by Design* report notes, California must rebuild its professional learning system to make it “sustained, content-embedded, collegial and connected to practice; focused on student learning; and aligned with school improvement efforts” (EETF 2012, 16). Some of the important features of professional learning programs for teachers of mathematics are discussed in the following section.

## Content of Professional Learning Programs

For a mathematics program to be effective, it must be taught by knowledgeable teachers. According to Liping Ma, “The real mathematical thinking going on in a classroom, in fact, depends heavily on the teacher’s understanding of mathematics” (Ma 2010). A landmark study in 1996 found that students with initially comparable academic achievement levels had vastly different academic outcomes when teachers’ knowledge of the subject matter differed (Milken 1999). The message from the research is clear: having knowledgeable teachers really does matter, and teacher expertise in a subject drives student achievement. “Improving teachers’ content subject matter knowledge and improving students’ mathematics education are thus interwoven and interdependent processes that must occur simultaneously” (Ma 2010, 125).

Professional learning for mathematics teachers must address the teachers’ content knowledge of the topics taught at their grade level(s), as well as mathematics relevant to prior and later grade levels where appropriate. Research over the past decade has shown a positive correlation between teacher content knowledge and student learning (Hill and Lubienski 2007; Hill, Rowan, and Ball 2005). The content knowledge required of teachers at each grade level has changed significantly with the adoption of the CA CCSSM. These changes in content must be considered when professional learning programs for teachers are designed. Specific guidelines for the mathematics content knowledge at various grade spans that might appear in such programs are provided later in this chapter.

The MP standards represent a shift toward students “doing mathematics” in the classroom. As noted in *The Mathematical Education of Teachers II*, teachers “must not only understand the practices of the discipline, but how these practices can occur in school mathematics and be acquired by students” (The Conference Board of the Mathematical Sciences 2012, 8). To develop an understanding of the MP standards and the implications for mathematics instruction, teachers should engage in solving problems through the mathematical practices. Intensive, content-focused professional learning workshops—such as Saturday meetings or multi-day summer workshops—provide a forum where teachers can do this. For example, professional learning should accomplish the following results:

- Engage teachers in the posing and solving of problems, requiring teachers to make sense out of problems and learn to persevere in solving them (MP.1).
- Encourage teachers to explain their reasoning, make conjectures, and critique each other’s reasoning in a safe environment (MP.3).
- Allow teachers to learn which tools are appropriate for the mathematics at hand and gather experience with the use of those tools in the classroom (MP.5).

Professional learning programs that incorporate teacher collaboration across schools or districts can draw on successful experiences of other teachers in teaching the MP standards.

In addition to a teacher’s grade-level mathematics knowledge, contemporary mathematics education research points to the importance of teacher acquisition of a specific body of content knowledge for teaching mathematics, often referred to as *pedagogical content knowledge* (see Hill et al. [2007] for a comprehensive discussion of this idea). This body of knowledge includes understanding problem-solving strategies that arise through student thinking, knowledge of multiple representations of mathematical

concepts (e.g., multiple representations of fractions), comprehension of the relationships embedded in content areas, an understanding of common student thinking and misconceptions, knowledge of specific teaching strategies for different topics, and ways to differentiate instruction, among others.

Of note are strategies that involve students in classroom discourse as a means of implementing the MP standards. Also, formative assessment strategies can help inform teachers about the efficacy of lessons, units, or modules and the extent of student understanding. Finally, paying attention to the needs of certain populations, including students with disabilities and English learners, is crucial to providing high-quality mathematics instruction for all students in California. To the extent possible, mathematics professional learning for teachers should be attentive to these areas and rely on the most current materials and research.

Suggested mathematics content for teachers' professional learning is presented below, according to the domains and conceptual categories in the CA CCSSM. These suggestions are based on recommendations in two documents: *Gearing Up for the Common Core State Standards in Mathematics* (Institute for Mathematics and Education 2011) and *The Mathematical Education of Teachers II* (The Conference Board of the Mathematical Sciences 2012).

- Grades K–2: Counting and Cardinality; Number and Operations in Base Ten; Operations and Algebraic Thinking
- Grades 3–5: Number and Operations—Fractions; Number and Operations in Base Ten; Operations and Algebraic Thinking
- Grades K–5: Measurement and Data; Geometry
- Grades 6–8: Ratios and Proportional Relationships; The Number System; Geometry; Statistics and Probability
- Grades 9–12: Functions; Modeling; Transformational Geometry

The University of Arizona (UA) Progressions Documents for the Common Core Math Standards (<http://ime.math.arizona.edu/progressions/> [UA 2011–13]) are useful tools for teachers exploring these topics. The documents can be used as starting points for content-based professional learning programs.

School administrators and teachers should strive to develop an understanding of students and youth culture to enhance mathematics instruction. Teachers have the potential to act as institutional agents with the capacity and commitment to provide institutional resources and opportunities to students (Stanton-Salazar 1997). Teacher–student relationships are potential *social capital*—that is, forms of support that help students become effective participants in the school system (Bourdieu 1977, 1986; Stanton-Salazar 1997). In the context of schools, teacher–student relationships include student learning and achievement (Katz 1999). Katz (1999) also states that two signs of productive teacher–student relationships are high expectations and caring for students. Many students value care and respect. When relationships between teachers and students become supportive, these relationships have the potential to alter students' lives in positive ways (Stanton-Salazar 2001). This notion of *teacher–student relationship* is derived from the social capital framework, which was cultivated by Bourdieu to examine the role of relationships between institutional agents and their students (Stanton-Salazar 2001).



Finally, supervisors who provide mathematics professional learning opportunities for teachers should be well versed in mathematics knowledge, knowledge of students and instructional strategies, and classroom issues that teachers face. Strong partnerships are encouraged between (1) schools, districts, and county offices of education; and (2) mathematics education faculty and mathematics faculty from nearby institutions of higher education. All have a stake in the mathematics instruction of California students, and all have something to offer to professional learning programs for teachers.

## Forms of Mathematics Professional Learning Programs

The types of mathematics professional learning programs for teachers vary, but there are some common characteristics of effective professional learning programs that should be attended to when designing such programs. Professional learning programs for teachers should include mathematics content instruction for teachers, as well as effective and appropriate pedagogical strategies for the classroom. Programs for teachers should be sustained, with a focus on long-term goals. A one-shot, single-day workshop is unlikely to have a lasting effect on classroom instruction without consistent and long-term follow-up and support. Both research and the collective experience of thousands of teachers, administrators, and teacher educators in California confirm this (Darling-Hammond et al. 2009; Blank and de las Alas 2010).

Below are common models of lasting, supportive professional learning programs:

- *Summer intensive workshops or university courses for teachers.* One- or two-week summer professional learning institutes allow teachers to focus solely on the development of their knowledge of content and instructional strategies. Multimedia resources allow teachers to examine mathematics teaching in a collaborative environment and develop plans for implementation during the school year. Summer workshops, however, are most effective when paired with follow-up programs.
- *Teacher collaboration (coaching, math circles, professional learning communities).* Site-based professional learning engages teachers in real-time study of their practice. A lone teacher has a difficult road ahead if he or she wishes to implement new strategies in the classroom without the support and understanding of colleagues. Teacher collaboration has been a feature of successful professional learning programs that serve to help teachers make larger-scale changes in mathematics instruction at their schools. Such efforts are needed to implement the CA CCSSM.
- *Lesson study.* The challenges to collaboration include a tradition of autonomy in classrooms, time and scheduling constraints, lack of supportive leadership, and pressure for individual accountability. One innovative way that provides a structure for teacher collaboration is the Lesson Study Model. Lesson study, adapted from Japan, is a form of long-term professional development in which teams of teachers collaboratively plan, observe, analyze, and refine actual classroom lessons. Each lesson-study cycle consists of three phases: planning a lesson, observing student reactions to the lesson, and then analyzing those reactions. Because the focus is on the effectiveness of a lesson itself and what students learn rather than on an individual teacher's performance, the method helps reduce teachers' anxiety and resistance to being observed. To watch a full lesson-study cycle, visit <https://www.collaborativeclassroom.org/lesson-study> (Center for the Collaborative Classroom 2015).

- *Fostering of teacher leadership.* Teachers may be encouraged to use their expertise in formal or informal leadership roles. Teachers who attend workshops or conferences should be given the opportunity to share what they have learned with peer teachers. A teacher who shows commitment to professional learning can become a mathematics coach or start a lesson-study group at his or her school. Teachers may participate on a textbook committee, take a role in designing benchmark assessments, or be part of the school or district academic planning team. Many teachers are unaware of the leadership roles they can play in their school or district unless they are encouraged to take on such roles.

A final feature of effective mathematics professional learning is schoolwide administrative support. Teachers face many pressures in the classroom that may make them less willing to take risks when implementing new instructional techniques or using new materials. If principals and other administrators support teacher efforts to improve their instructional practices, then such changes are more likely to be integrated into classroom practice.

## Induction and Support for New Teachers

Induction and support for new teachers should be given special attention in California schools. As of the writing of this document, the research of Ingersoll and Perda (2010) indicates that the recruitment and retention of mathematics teachers is of crucial importance nationwide. Data show that large numbers of teachers report dissatisfaction with their jobs because of feelings of isolation, a lack of schoolwide support and collaboration, and a lack of effective professional learning. Research indicates that this dissatisfaction can be alleviated to a large degree by the implementation of effective support programs tailored to new teachers (Ingersoll and Perda 2010). Features of such programs are similar to those described above, but also include these elements:

- Mentoring by knowledgeable, effective, reflective, and experienced teachers in the same grade level and content area as the novice teachers
- Content knowledge development to draw connections between the university mathematics courses that novice teachers just completed and the mathematics they are now required to teach
- Classroom strategies that address classroom management issues and difficulties with engaging students in the MP standards

## Evaluation of Instruction

As described in the *Greatness by Design* report (EETF 2012), successful evaluation systems for teachers should provide useful feedback over time while also identifying those teachers who are struggling (and need intensive assistance) and removing those who do not improve. *Greatness by Design* (EETF 2012, 17) recommends that evaluation systems:

- *be based on the California Standards for the Teaching Profession* and assess an educator's practices—from pre-service preparation to induction—and throughout the remainder of the career;
- *tie evaluation to useful feedback and to professional learning opportunities* that are relevant to an educator's goals and needs;

- *assess the extent to which instruction aligns with the CA CCSSM*, including focus on both mathematics content and the MP standards;
- *combine data from a variety of sources*, including valid measures of educator practice, student learning, and professional contributions, which are examined in relation to one another;
- *include both formative and summative assessments*, providing information to both improve practice and support personnel decisions;
- *differentiate support* based on the educator’s level of experience and individual needs;
- *build on successful Peer Assistance and Review models* for educators who need assistance in order to ensure intensive, expert support and well-grounded personnel decisions;
- *value and promote collaboration*, which supports improvement of the whole school;
- *be a priority in the district*, providing time, training, and support for evaluators and those who mentor educators needing assistance.

## Expanded Learning Time

In 2012, the California Department of Education unveiled its *Common Core State Standards System Implementation Plan for California* (CDE 2012c). The plan recommends that districts “integrate the CCSS into programs and activities beyond the K–12 school setting” and suggests providing “professional development to district administrators, school principals, and after school program directors on how to collaborate to incorporate, into after school/extended day programs, activities that enrich the CCSS-related learning initiated during the regular day.”<sup>1</sup>

### Definition

“Expanded learning time” is an approach to enhance and integrate active learning experiences beyond the traditional school day—after school, before school, during summer, and with extended days, weeks, or school years—to reduce the achievement gap and improve student success.<sup>2</sup> These strategies utilize time outside the classroom as a unique opportunity to address the academic, social, emotional, and physical needs and interests of students through individualized and engaging learning that results in improved student achievement. Programs should be high-quality, include community partners, be results-driven, and flexible to student and community needs.

According to a report by the Forum for Youth Investment (Devaney and Yohalem 2012), traditional education and expanded learning partners may collaborate effectively on Common Core implementation by:

- increasing alignment and communication between the school staff and after school staff about learning supports and opportunities;
- increasing alignment of skills and knowledge emphasized in the Common Core standards;
- increasing awareness and sharing knowledge between school staff and after school staff;

1. The terms *extended learning programs* and *expanded learning programs* are used interchangeably and broadly refer to the learning times and experiences outside of the regular school day and year—for example, before school, after school, intersessions, and summer. The CDE has chosen to use the term *expanded learning time*.

2. This is the working definition of *expanded learning time* as of December 2012 and adopted by the California Department of Education’s After School Division. This definition was developed in collaboration with the Partnership for Children and Youth.

- increasing shared professional development and planning time;
- supporting strategies on how to communicate the role and implementation of the California Common Core State Standards for Mathematics to parents and community partners.

As outlined in the definition of expanded learning time referenced above, the underlying principles explicitly reinforce and complement key aspects of the MP standards, such as making learning relevant, project-based, and engaging.

School staff and administrators may invite expanded learning providers to school and community meetings and trainings and then develop plans for more intentional alignment. This first step of collaboration may also include sharing resources and materials on Common Core implementation and the CA CCSSM. Additional resources and partners for schools and school staff include a range of providers of technical assistance in expanded learning, such as county offices of education and contracted entities that can facilitate local partnerships and share best practices for Common Core implementation. Because of the demands of Common Core implementation, the differences in programming and skill development between the traditional school day and expanded learning environments should be clearly understood by students, parents, teachers, and providers.

## **College, University, and Professional Support**

The support of college and university personnel for high-quality mathematics instruction is also crucial. Personnel from institutions of higher education support K–12 mathematics education by joining in partnership with their local schools. By becoming more involved with other institutions of learning, college and university personnel become more aware of the research that needs to be done in the school settings. Armed with research conducted in their profession, college and university personnel can be strong advocates of high-quality mathematics instruction.

Teachers who are well prepared to teach mathematics are vital to the success of mathematics education in California. The adoption of new mathematics content standards and the forthcoming changes in assessment require many teachers to gain new knowledge and alter classroom practices. Even experienced teachers need support in learning and instituting new curriculum and instructional strategies, and new teachers and teacher candidates need even greater support in learning to teach mathematics as they acquire the fundamentals of teaching. Colleges and university personnel can provide support for those teachers through school visits and through the learning opportunities offered by higher education.

Additionally, the introduction of new mathematics standards means that the curriculum of college teacher-preparation courses that address mathematics must change to reflect new content and the MP standards. Developers of teacher-preparation programs must take the initiative to create programs that ensure knowledge of the CA CCSSM through appropriate course work and pedagogical preparation to teach higher-order thinking and performance skills for students, in addition to culturally and linguistically responsive pedagogy (EETF 2012, 29). Teacher credentialing programs should include a focus on implementing the CA CCSSM.

Local county offices of education are linked with the California Department of Education and can provide resources for the implementation of the CA CCSSM and professional learning for instruction.

County offices of education have access to the latest CA CCSSM resources and can provide support for administrators to understand the CA CCSSM and opportunities for collaboration among schools.

Finally, local, state, and national professional organizations can play a role in supporting schools, administrators, and teachers as they transition to the CA CCSSM. Examples of such organizations include the California Mathematics Project (<http://csmp.ucop.edu/cmp> [accessed July 9, 2015]), which has numerous regional sites; the California Mathematics Council (CMC); local affiliates of the CMC; the National Council of Teachers of Mathematics (NCTM); and the National Council of Supervisors of Mathematics (NCSM).

## Community and Parent Support

Although schools are the primary learning environments for formal mathematics, students' homes and communities also play significant roles. When schools collaborate with students' family members and the community, students become fully prepared for a lifelong appreciation of mathematics. Mathematics can have a place outside the classroom: in mathematics clubs, through local and national mathematics competition teams, and through school mathematics activities that promote parent and family involvement (e.g., "Family Math Night").

Schools and districts can create formal and informal partnerships with a variety of public and private organizations, agencies, and businesses to seek support and participation in the mathematics education of children in California. Many private companies and organizations have education departments that seek opportunities to work with youngsters. Schools are encouraged to (1) use community resources to provide the additional adult support and instructional materials that students need to meet mathematics education requirements; and (2) start to develop students' ideas about the workforce, careers, and their relationship to mathematics and how to benefit their communities with that relationship.

Parental involvement in the mathematics education of children can take many different forms. Some parents may show support by voicing to their children consistent respect for the value of education in general and mathematics in particular. Parents help their children with homework or projects and take an active approach in their children's learning when they can. They mirror the appreciation for reasoning and the learning of mathematics that they hope to see in their children. Parents may volunteer in the classroom or serve in an advisory capacity on an appropriate committee. They may attend mathematics nights and workshops that are sponsored by the school or district. Regardless of how parents or family members support education, they are always made to feel welcome at their children's schools and know that their contributions are valued and appreciated.

Parents and families need to be advised of school district goals and plans for mathematics education programs. They need to be informed about the CA CCSSM and the grade- or course-level expectations for their children and how to support their children's mastery of the standards, including the habits of mind inculcated in the MP standards. In particular, parents must be aware that the CA CCSSM represent a change in mathematics instruction—a shift toward active student participation in the reasoning and discovery involved in learning mathematics. Community efforts are needed to propel California schools forward as they move to full implementation of the CA CCSSM.