Universal Access Chapter

of the

Mathematics Framework

for California Public Schools: Kindergarten Through Grade Twelve

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Universal Access

The California Common Core State Standards for Mathematics (CA CCSSM) articulate rigorous grade-level expectations. These standards provide a historic opportunity to improve access to rigorous academic content for all students, including students with special needs. All students should be held to the same high expectations outlined in the mathematical practices and the content standards (both of which compose the CA CCSSM), although some students may require additional time, language support, and appropriate instructional support as they acquire knowledge of mathematics. Effective education of all students includes closely monitoring student progress, identifying student learning needs, and adjusting instruction accordingly. Regular and active participation in the classroom—not only solving problems and listening, but also discussing, explaining, reading, writing, representing, and presenting—is critical to each student's success in mathematics.

This chapter uses an overarching approach to address the instructional needs of students in California. Although suggestions and strategies for mathematics instruction are provided, they are not intended to—nor could they be expected to—offer teachers and other educators a road map for effectively meeting the instructional needs of every student. The instructional needs of each student are unique and change over time. Therefore, high-quality curriculum, purposeful planning, uninterrupted and protected instructional time, scaffolding, flexible grouping strategies, differentiation, and progress monitoring are essential components of ensuring universal access to mathematics learning.

The first sections in this chapter discuss planning for universal access, differentiation, Universal Design for Learning, the new language demands of the CA CCSSM, assessment for learning, and California's Multi-Tiered System of Supports (MTSS). Later sections focus on students with targeted instructional needs: students with disabilities, English learners, at-risk learners, and advanced learners.

Planning for Universal Access

The ultimate goal of mathematics programs in California is to ensure universal access to high-quality curriculum and instruction so that all students are prepared for college and careers. By carefully planning to modify curriculum, instruction, grouping, and assessment techniques, teachers can be well prepared to adapt to the diversity in their classrooms. *Universal access* in education is a concept that encompasses planning for the widest variety of learners from the beginning of the lesson design process; it should not be "added on" as an afterthought. Likewise, universal access is not a set of curriculum materials or specific time set aside for additional assistance; rather, it is a schema. For students to benefit from universal access, some teachers may need assistance in planning instruction, differentiating curriculum, utilizing flexible grouping strategies, and using the California English Language Development Standards (CA ELD standards) in tandem with the CA CCSSM. Teachers need to employ many different strategies to help all students meet the increased demands of the CA CCSSM.

For all students, it is important that teachers use a variety of instructional strategies—but this is *essential* for students with special needs. Below are some of the strategies that are important to consider when planning for universal access:

- Assess each student's mathematical skills and understandings at the start of instruction to uncover strengths and weaknesses.
- Assess or be aware of the English language development level of English learners.
- Differentiate instruction, focusing on the mathematical practice standards, the concepts within the content standards, and the needs of the students.
- Utilize formative assessments on an ongoing basis to modify instruction and reevaluate student placement or grouping.
- Create a safe environment and encourage students to ask questions.
- Draw upon students' literacy skills and content knowledge in their primary language.
- Engage in careful planning and organization with the various needs of all learners in mind and in collaboration with specialists (e.g., instructional coaches, teachers of special education, and so forth).
- Engage in backward and cognitive planning[1] to fill in gaps involving skills and knowledge and to address common misunderstandings.
- Use the principles of Universal Design for Learning (UDL) when modifying curriculum and planning lessons.
- Utilize the University of Arizona (UA) Progressions Documents for the Common Core Math Standards (UA 2011–13) to understand how mathematical concepts are developed at each grade level and to identify strategies to address individual student needs. The Progressions documents are available at http://ime.math.arizona.edu/progressions/ (accessed July 16, 2015).
- When necessary, organize lessons in a manner that includes sufficient modeling and guided practice before moving to independent practice. This is also known as *gradual release of responsibility*.
- Pre-teach routines to address changing seating arrangements (e.g., groups) and other classroom procedures.
- Use multiple representations (e.g., math drawings, manipulatives, and other forms of technology) to explain concepts and procedures.
- Allow students to demonstrate their understanding and skills in a variety of ways.
- Employ flexible grouping strategies.
- Provide frequent opportunities for students to collaborate and engage in mathematical discourse.

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1. *Backward planning* identifies key areas such as prior knowledge needed, common misunderstandings, organizing information, key vocabulary, and student engagement. Backward planning is what will be included in a lesson or unit to support intended student learning. *Cognitive planning* focuses on how instruction will be delivered, anticipates potential student responses and misunderstandings, and provides opportunities to check for understanding and re-teaching during the delivery of the lesson. Backward planning determines what elements will be included; cognitive planning determines how those elements will be delivered.

- Include activities that allow students to discuss concepts and their thought processes.
- Emphasize and pre-teach (when necessary) academic and discipline-specific vocabulary.
- When students are learning to engage in mathematical discourse, provide them with language models and structures (such as sentence frames).
- Explore technology and consider using it along with other instructional devices.
- For advanced learners, deepen the complexity of lessons or accelerate the pace of student learning.

Additional suggestions to support students who have learning difficulties are provided in appendix E (Possible Adaptations for Students with Learning Difficulties in Mathematics). This list of possible adaptations addresses a range of students, some of whom may have identified instructional needs and others who are struggling unproductively for unidentified reasons. If a student has an individualized education program (IEP) or 504 Plan, the strategies, accommodations, or modifications in the plan guide the teacher on how to differentiate instruction. Additional adaptions should be used only when they are consistent with the IEP or 504 Plan.

Differentiation

Differentiated (or modified) instruction helps students with diverse academic needs master the same challenging grade-level academic content as students without special needs (California Department of Education [CDE] 2015b). In differentiated instruction, the method of delivery changes—not the topic of the instruction. Instructional decisions are based on the results of appropriate and meaningful student assessments. Differentiated instruction helps to provide a variety of ways for individual students to take in new information, assimilate it, and demonstrate what they have learned (CDE 2015b).

Differentiation is the foundation for universal access. As Carol Ann Tomlinson has written, "In a differentiated classroom, the teacher proactively plans and carries out varied approaches to content, process, and product in anticipation of and response to student differences in readiness, interest, and learning needs" (Tomlinson 2001, 7). For example, a teacher could differentiate content (*what* the student learns) based on readiness, interest, or learning profile. The same holds true for differentiating process (*how* the student learns) and product (the way the student communicates what he has learned) based on readiness, interest, or learning profile. These pieces of differentiation are all closely intertwined and often cannot be separated into individual practices.

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>
> —Carol Ann Tomlinson (Tomlinson 2001, 7)

Research indicates that a student is most likely to learn content when the lesson presents tasks that may be "moderately challenging." When a student can complete an assignment independently, with little effort, new learning does not occur. On the other hand, when the material is presented in a manner that is too difficult, then "frustration, not learning, is the result" (Cooper 2006, 154). This idea is also at the heart of Vygotsky's "Zone of Proximal Development" (Vygotsky 1978). Advanced learners and students with learning difficulties in mathematics often require systematically planned differentiation strategies to ensure that they experience appropriately challenging curriculum and instruction. This section looks at four modes of differentiation: depth, pacing, complexity, and novelty. Many of the strategies presented can benefit all students, not just those with special needs.

Depth

Depth of understanding refers to how concepts are represented and connected by learners. The greater the number and strength of the connections, the deeper the understanding is. In order to help students develop depth of understanding, teachers need to provide opportunities to build on students' current understanding and assist them in making connections between previously learned content and new content (Grotzer 1999).

Differentiation is achieved by increasing the depth to which a student explores a curricular topic. The CA CCSSM raise the level of cognitive demand through the Standards for Mathematical Practice (MP) as well as grade-level and course-level Standards for Mathematical Content. Targeted instruction is beneficial when it is coupled with adjustments to the level of cognitive demand (LCD). The LCD is the degree of thinking and ownership required in the learning situation. The more complex the thinking and the more ownership (invested interest) students have for learning, the higher the LCD. Likewise, a lower LCD requires straightforward, more simplistic thinking and less ownership by the students. Having high expectations for all students is critically important; however, posing a consistently high LCD can actually set up some students for failure. Similarly, posing a consistently low LCD for students is not pedagogically appropriate and is unlikely to result in new learning. To meet the instructional needs of the students, the LCD must be adjusted at the time of instruction (Taylor-Cox 2008). One strategy that teachers can use is tiered assignments with varied levels of cognitive demand to ensure that students explore the same essential ideas at a level that builds on their prior knowledge; this is appropriately challenging and prompts continual growth.

Pacing

Slowing down or speeding up instruction is referred to as *pacing*. This is perhaps the most common strategy that teachers employ for differentiation; it can be simple and inexpensive to implement, yet it can prove effective for many students with special needs (Benbow and Stanley 1996; Geary 1994). An example of pacing for advanced learners is to collapse a year's course into one semester by moving quickly through the material the students already know (curriculum compacting) without sacrificing either depth of understanding or application of mathematics to novel situations. Alternatively, students may move on to the content standards for the next grade level (accelerating). Caution is warranted to ensure that students are not placed in mathematics courses for which they are not adequately prepared—in particular, placing unprepared students in Mathematics I or Algebra I at middle school (see appendix D, Course Placements and Sequences, for additional information and guidance). Two recent studies on middle school mathematics report that grade-eight students are often placed in Mathematics I or Algebra I courses for which they are not ready, a practice that sets up many students for failure (Finkelstein et al. 2012; Williams et al. 2011).

For students whose achievement is below grade level in mathematics, an increase in instructional time may be appropriate. The amount of additional instructional time, in terms of both duration and frequency, depends on the unique needs of each student. Frequent use of formal and informal formative assessments of conceptual understanding, procedural skill and fluency, and application informs both the teacher and the student about progress toward instructional goals, and instructional pacing should be modified based on the student's progress (Newman-Gonchar, Clarke, and Gersten 2009).

Complexity

Understanding within and across disciplines is referred to as *complexity*. Modifying instruction by complexity requires teacher professional learning and collaboration and instructional materials that lend themselves to such variations. Complexity involves uncovering relationships between and among ideas, connecting other concepts, and using an interdisciplinary approach to the content. When students engage in a performance task or real-world problem, they must apply their mathematical knowledge and skills and knowledge of other subjects (Kaplan, Gould, and Siegel 1995).

For all students, but especially students who experience difficulty in mathematics, teachers should focus on the foundational skills, procedures, and concepts in the standards. Several studies have found that the use of visual representations and manipulatives can improve students' proficiency. Number lines, math drawings, pictorial representations, and other types of visual representations are effective scaffolds. However, if visual representations are not sufficient, concrete manipulatives should be incorporated into instruction (Gersten et al. 2009).

Teachers can differentiate the complexity of a task to maximize student learning outcomes. For students with special needs, differentiation is sometimes questioned by those who say that struggling students never progress to more interesting or complex assignments. It is important to focus on essential concepts embedded in the standards and on frequent assessment to ensure that students are prepared with the understanding and skills they will need to succeed in subsequent grades. Struggling students are expected to learn the concepts well so that they develop a foundation on which further mathematical understanding can be built; this can be accomplished through well-chosen and interesting tasks and problems. See the section on California's MTSS and Response to Instruction and Intervention (RtI^2) for additional information. Advanced students benefit from a combination of self-paced instruction and enrichment (National Mathematics Advisory Panel 2008).

Novelty

Keeping students engaged in learning is an ongoing instructional challenge that can be complicated by the varied instructional needs of students. Novelty is one differentiation strategy that is primarily student-initiated and can increase student engagement. Teachers can introduce novelty by encouraging students to re-examine or reinterpret their understanding of previously learned information. Students can look for ways to connect knowledge and skills across disciplines or between topics in the same discipline. Teachers can work with students to help them learn in more personalized, individualistic, and non-traditional ways. This approach may involve a performance task or real-world problem on a subject that interests the student and requires the student to use mathematics understandings and skills in new or more in-depth ways (Kaplan, Gould, and Siegel 1995).

Universal Design for Learning

As noted by Diamond (2004, 1), "*Universal access* refers to the teacher's scaffolding of instruction so all students have the tools they need to be able to access information. *Universal design* typically refers to those design principles and elements that make materials more accessible to more children—larger fonts, headings, and graphic organizers, for example." Diamond also comments that "[j]ust as designing entrance ramps into buildings makes access to individuals in wheelchairs easier, curriculum may also be designed to be easier to use. When principles of universal design are applied to curriculum materials, universal access is more likely" (Diamond 2004, 1).

Universal Design for Learning (UDL) is a framework for implementing the concepts of universal access by providing equal opportunities to learn for *all* students. Based on the premise that one-size-fits-all curricula create barriers to learning for many students, UDL helps teachers design curricula to meet the varied instructional needs of all of their students.

The purpose of UDL curricula is to help students become "expert learners" who are (a) strategic, skillful, and goal directed; (b) knowledgeable; and (c) purposeful and motivated to learn more (Center for Applied Special Technology [CAST] 2011, 7).

Goals of UDL

- Improve access, participation, and achievement for students.
- Eliminate or reduce physical and academic barriers.
- Value diversity through proactive design.

Source: CAST 2011.

The UDL guidelines developed by CAST are strategies to help teachers make curricula more accessible to all students. The guidelines are based on three primary principles of UDL and are organized under each of the principles as follows.[2]

Principle I: Provide Multiple Means of Representation (the "what" of learning)

Guideline 1: Provide options for perception.

Guideline 2: Provide options for language, mathematical expressions, and symbols.

Guideline 3: Provide options for comprehension.

The first principle allows flexibility so that mathematical concepts can be taught in a variety of ways to address the background knowledge and learning needs of students. For example, presentation of content for a geometry lesson could utilize multiple media that include written, graphic, audio, and interactive technology. Similarly, the presentation of content will include a variety of lesson formats, instructional strategies, and student grouping arrangements (Miller 2009, 493).

Principle II: Provide Multiple Means of Action and Expression (the "how" of learning)

Guideline 4: Provide options for physical action.

Guideline 5: Provide options for expression and communication.

Guideline 6: Provide options for executive functions.

2. For more information on UDL, including explanations of the principles and guidelines and the detailed checkpoints for each guideline, visit the National Center on Universal Design for Learning Web page at http://www.udlcenter.org/aboutudl/ udlguidelines (CAST 2011).

The second principle allows for flexibility in how students demonstrate understanding of mathematical content. For example, when explaining the subtraction algorithm, students in grade four may use concrete materials, draw diagrams, create a graphic organizer, or deliver an oral report or a multimedia presentation (Miller 2009, 493).

Principle III: Provide Multiple Means of Engagement (the "why" of learning)

Guideline 7: Provide options for recruiting interest.

Guideline 8: Provide options for sustaining effort and persistence.

Guideline 9: Provide options for self-regulation.

The third principle aims to ensure that all students maintain their motivation to participate in mathematical learning. Alternatives are provided that are based upon student needs and interests, as well as "(a) the amount of support and challenge provided, (b) novelty and familiarity of activities, and (c) developmental and cultural interests" (Miller 2009, 493). Assignments provide multiple entry points with adjustable challenge levels. For example, students in grade six may gather, organize, summarize, and present data to describe the results of a survey of their own design. In order to develop self-regulation, students reflect upon their mathematical learning through a choice of journals, check sheets, learning logs, or portfolios and are provided with encouraging and constructive teacher feedback through a variety of formative assessment measures that demonstrate student strengths and areas where growth is still necessary.

Although it takes considerable time and effort to develop curriculum and plan instruction based on UDL principles, all students can benefit from an accessible and inclusive environment that reflects a universal design approach—and this type of environment is essential for learners with special needs. Teachers and other educators should be provided with opportunities for professional learning on UDL, time for curriculum development and instructional planning, and necessary resources (e.g., equipment, software, instructional materials) to effectively implement UDL. For example, interactive whiteboards can be a useful tool for providing universally designed instruction and engaging students in learning. Teachers and students can use these whiteboards to explain concepts or illustrate procedures. The large images projected onto whiteboards can be seen by most students, including those who have visual disabilities (DO-IT 2012).

New Language Demands of the CA CCSSM

Students who learn mathematics based on the CA CCSSM face increased language demands during mathematics instruction. Students are asked to engage in discussions about mathematics topics, explain their reasoning, demonstrate their understanding, and listen to and critique the reasoning of others. These increased language demands may pose challenges for all students and even greater challenges for both English learners and students who are reading or writing below grade level. These language expectations are made explicit in several of the standards for mathematical practice. Standard **MP.3**, "Construct viable arguments and critique the reasoning of others," states an expectation that students will justify their conclusions, communicate their conclusions to others, and respond to the arguments of others. It also states that students at all grade levels can listen to or read the arguments of others, decide whether those arguments make sense, and ask useful questions to clarify or improve arguments. Standard **MP.6**, "Attend to precision," asks students to communicate precisely with each other, use clear definitions in discussions with others and in their own reasoning, and that beginning in the elementary grades, students offer carefully formulated explanations to each other. Standard **MP.1**, "Make sense of problems and persevere in solving them," states that students can explain correspondences between equations, verbal descriptions, tables, and graphs.

Standards that call for students to *describe*, *explain*, *demonstrate*, and *understand* provide opportunities for students to engage in speaking and writing about mathematics. These standards appear at all grade levels. For example, in grade two, standard **2.OA.9** asks students to explain why addition and subtraction strategies work. Another example occurs in the Algebra conceptual category of higher mathematics: standard **A-REI.1** requires students to explain each step in solving a simple equation and to construct a viable argument to justify a solution method.

To support students' ability to express their understanding of mathematics, teachers need to explicitly teach not only the language of mathematics, but also academic language for argumentation (*proof, theory, evidence, in conclusion, therefore*), sequencing (*furthermore, additionally*), and relationships (*compare, contrast, inverse, opposite*). Pre-teaching vocabulary and key concepts allows students to be actively engaged in learning during lessons. To help students organize their thinking, teachers may need to scaffold with graphic organizers and sentence frames (also called *communication guides*).

The CA CCSSM call for students to read and write in mathematics to support their learning. According to Bosse and Faulconer (2008), "Students learn mathematics more effectively and more deeply when reading and writing is directed at learning mathematics" (Bosse and Faulconer 2008, 8). Mathematics text is informational text that requires different skills to read than those used when reading narrative texts. The pages in a mathematics textbook or journal article can include text, diagrams, tables, and symbols that are not necessarily read from left to right. Students may need specific instruction on how to read and comprehend mathematics text.

Writing in mathematics also requires different skills than writing in other subjects. Students will need instruction in writing informational or explanatory text that requires facility with the symbols of mathematics and graphic representations, as well as understanding of mathematical content and concepts. Instructional time and effort focused on reading and writing in mathematics benefits students by "requiring them to investigate and consider mathematical concepts and connections" (Bosse and Faulconer 2008, 10), which supports the mathematical practices standards. Writing in mathematics needs to be explicitly taught, because skills do not automatically transfer from English language arts or English language development. Therefore, students benefit from modeled writing, interactive writing, and guided writing in mathematics.

As teachers and curriculum leaders design instruction to support students' reading, writing, speaking, and listening in mathematics, the California Common Core State Standards for English Language Arts and Literacy in History/Social Studies, Science, and Technical Subjects (CA CCSS for ELA/Literacy) and the California English Language Development Standards (http://www.cde.ca.gov/sp/el/er/eldstandards.asp [CDE 2013b]) are essential resources. The standards for reading informational text in the CA CCSS for ELA/Literacy specify the skills students must master in order to comprehend and apply what they read. Writing Standard 2 of the CA CCSS for ELA/Literacy provides explicit guidance on writing informational or explanatory texts by clearly stating the expectations for students' writing according to grade level. Engaging in mathematical discourse can be challenging for students who have not had many opportunities to explain their reasoning, formulate questions, or critique the reasoning of others. Standard 1 in the Speaking and Listening strand of the CA CCSS for ELA/Literacy, as well as Part I of the CA ELD standards, calls for students to engage in collaborative discussions and set expectations for a progression in the sophistication of student discourse from kindergarten through grade twelve and from the emerging level to the bridging level for English learners. Teachers and curriculum leaders should utilize the CA CCSS for ELA/Literacy and the CA ELD standards in tandem with the CA CCSSM when planning instruction. In grades six through twelve, there are standards for literacy in science and technical subjects that include reading and writing focused on domain-specific content and that can provide guidance, as students are required to read and write more complex mathematics text.

It is a common misconception that mathematics is limited to numbers and symbols. Mathematics instruction is often delivered verbally or through text that is written in academic language, not everyday language. Francis et al. (2006a) note, "The skills and ideas of mathematics are conveyed to students primarily through oral and written language—language that is very precise and unambiguous" (Francis et al. 2006a, 35). Words that have one meaning in everyday language have a different meaning in the context of mathematics. Also, many individual words, such as *root*, *point*, and *table*, have technical meanings in mathematics that are different from what a student might use in other contexts. Reading a mathematics text can be difficult because of the special use of symbols and spatial aspects of notations (e.g., exponents and stacked fractions, diagrams, and charts), as well as the structural differences between informational and narrative text, with which students are often more familiar. For example, a student might misread 5^2 (five squared) as 52 (fifty-two). Language difficulties may also occur when students are translating a word problem into an algebraic or numeric expression or equation. As early as grade one, students will encounter phrases such as "seven less than 10"; and in grade eight, students are asked to translate "7 fewer than twice Ann's age is 16" into an equation. In higher mathematics, it is essential to understand the concept that the language is conveying.

As students explore mathematical concepts, engage in discussions about mathematics topics, explain their reasoning, and justify their procedures and conclusions, the mathematics classroom will be vibrant with conversation.

Mathematics has specialized language that requires different interpretation than everyday language. Attention must be paid to particular terms that may be problematic. Table UA-1 provides examples of mathematical language that may cause difficulties for English learners, depending on context or usage.

Table UA-1. Mathematical Language That May Cause Difficulties for English Learners

Words whose meanings are found only in mathematics (used only in academic English)	Hypotenuse, parallelogram, coefficient, quadratic, circumference, polygon, polynomial
Symbolic language (used almost universally)	$+, -, \times, \div, \pi, \frac{1}{2}$
Words with multiple meanings in everyday English	The floor is <u>even</u>. The picture is <u>even</u> with the window. Breathing develops an <u>even</u> rhythm during sleep. The dog has an <u>even</u> temperament. I looked sick and felt <u>even</u> worse. <u>Even</u> a three-year-old child knows the answer.
Words with multiple meanings in academic English	Number: <u>Even</u> numbers (e.g., 2, 4, 6, and so on) Number: <u>Even</u> amounts (e.g., even amounts of sugar and flour) Measurement: An <u>even</u> pound (i.e., an exact amount) Function: An <u>even</u> function (e.g., $f(x)=f(-x)$ or cosine function)
Phonologically similar words.	***tens*** versus ***tenths*** ***sixty*** versus ***sixteen*** ***sum*** versus ***some*** ***whole*** versus ***hole*** ***off*** versus ***of*** How many **halves** do you **have**? ***then*** versus ***than***

Adapted from Asturias 2010.

Helping all students meet mathematical language demands requires careful planning; attention to the language demands of each lesson, unit, and module; and ongoing monitoring of students' understanding and their ability to communicate what they know and can do. As students explore mathematical concepts, engage in discussions about mathematics topics, explain their reasoning, and justify their procedures and conclusions, the mathematics classroom will be vibrant with conversation.

Assessment for Learning

There are many types of assessment in education. This section focuses on assessment *for* learning: formative and diagnostic assessment. Teachers should determine their students' current achievement levels in mathematics so that each student or group of students can be offered mathematics instruction leading to mastery of all grade-level or course-level mathematics standards. Given the vertical alignment of the CA CCSSM, the concept that what students have already learned in mathematics should form the basis for further learning is particularly true. Assessments may help identify those students who are ready to move on or are ready for greater challenges. Assessments may also identify students' misconceptions, overgeneralizations, and overspecializations so that these types of errors can be corrected. (Refer to the Assessment chapter for additional information.)

Formative Assessment

Formative assessment is key to ensuring that all students are provided with mathematics instruction designed to help them progress at an appropriate pace from what they already know to higher levels of learning. Formative assessment is assessment *for* learning. Formative assessment allows the teacher to gather information about student learning as it is happening. Armed with this knowledge, teachers can alter their lesson or instructional strategies and offer academic support and enrichment to students who need it. The Glossary of Education Reform (Great Schools Partnership 2014) describes formative assessment in this way:

> Many educators and experts believe that formative assessment is an integral part of effective teaching. In contrast with most summative assessments, which are deliberately set apart from instruction, formative assessments are integrated into the teaching and learning process. For example, a formative-assessment technique could be as simple as a teacher asking students to raise their hands if they feel they have understood a newly introduced concept, or it could be as sophisticated as having students complete a self-assessment of their own writing (typically using a rubric outlining the criteria) that the teacher then reviews and comments on. While formative assessments help teachers identify learning needs and difficulties, in many cases the assessments also help students develop a stronger understanding of their own academic strengths and weaknesses. When students know what they do well and what they need to work harder on, it can help them take greater responsibility over their own learning and academic progress. (Great Schools Partnership 2014)

Diagnostic Assessment

Diagnostic assessment of students often reveals both strengths and weaknesses (sometimes referred to as *gaps*) in students' learning. Diagnostic assessment also may reveal learning difficulties and the extent to which limited English language proficiency is interfering with mathematics learning. When gaps are discovered, instruction can be designed to remediate specific weaknesses while taking into consideration identified strengths. With effective support, students' weaknesses can be addressed without slowing down the students' mathematics learning progression. For example, the development of fluency with division using the standard algorithm in grade six is an opportunity to identify and address learning gaps in place-value understanding. This approach, in which place-value instruction and learning support students' fluency with division, is more productive than postponing grade-level work to focus on earlier standards that address place value (National Governors Association Center for Best Practices, Council of Chief State School Officers [NGA/CCSSO] 2012, 12). Additionally, assessments may indicate that a student already possesses mathematical skills and conceptual understanding beyond that of his or her peers and requires a modified curriculum to remain engaged. For example, a more advanced student could be challenged to complete an investigation such as a "Problem of the Month" from the Inside Mathe-matics Web site (http://www.insidemathematics.org/ [Inside Mathematics 2015]).

If a student is struggling unproductively to complete grade-level tasks, the teacher needs to determine the cause of the student's lack of achievement. Contributing factors might include:

- a lack of content-area knowledge;
- limited English proficiency;
- inappropriate instructional pacing;
- learning difficulties;
- frequent absences from school;
- homelessness;
- family issues;
- reading difficulties.

Teachers need to know their students in order to address each student's instructional needs. Sometimes a student may have a persistent misunderstanding of a concept or skill in mathematics, or the student may have consistently repeated an error until it has become routine. These problems may affect the student's ability to understand and solve problems. Intervention may be necessary to help students with these types of difficulties.

Diagnostic testing may also uncover students who appear to be struggling, when in fact they have already mastered the content and need more of a challenge to remain engaged. These students also need creative intervention, such as investigations and challenging problems.

California's Multi-Tiered System of Supports and Response to Instruction and Intervention

The California Multi-Tiered System of Supports (MTSS) provides a basis for understanding how California educators can work together to ensure equitable access and opportunity for all students to master the CA CCSSM. California's MTSS includes Response to Instruction and Intervention (RtI^2) as well as additional philosophies and concepts.

In California, the MTSS is an integrated, comprehensive framework that focuses on the CA CCSS and other state-adopted content standards, core instruction, differentiated learning, student-centered learning, individualized student needs, and the alignment of systems necessary for all students' academic, behavioral, and social success. The MTSS offers the potential to create systematic change through intentional design, as well as redesign of services and supports that quickly identify and match the needs of all students.

Comparing the MTSS to RtI²

The CDE's RtI² processes focus on students who are struggling and provide a vehicle for teamwork and data-based decision making to strengthen student performance before and after educational and behavioral problems increase in intensity. For additional information, please visit the CDE's RtI² Resources Web page (http://www.cde.ca.gov/ci/cr/ri/rtiresources.asp [CDE 2015c]).

MTSS Differences with RtI²

The MTSS has a broader scope than does RtI². The MTSS also includes these elements:

- Focusing on aligning the entire system of initiatives, supports, and resources
- Promoting district, site, and grade-level participation in identifying and supporting systems for alignment of resources
- Systematically addressing support for all students, including gifted and high achievers
- Enabling a paradigm shift for providing support and setting higher expectations for all students through intentional design and redesign of integrated services and supports, rather than selection of a few components of RtI and intensive interventions
- Endorsing UDL instructional strategies so all students have opportunities for learning through differentiated content, processes, and product
- Integrating instructional and intervention support so that systemic changes are sustainable and based on CA CCSS–aligned classroom instruction
- Challenging all school staff members to change the ways in which they work across all school settings

The MTSS is not designed solely for consideration in special education placement; it focuses on all students.

MTSS Similarities to RtI²

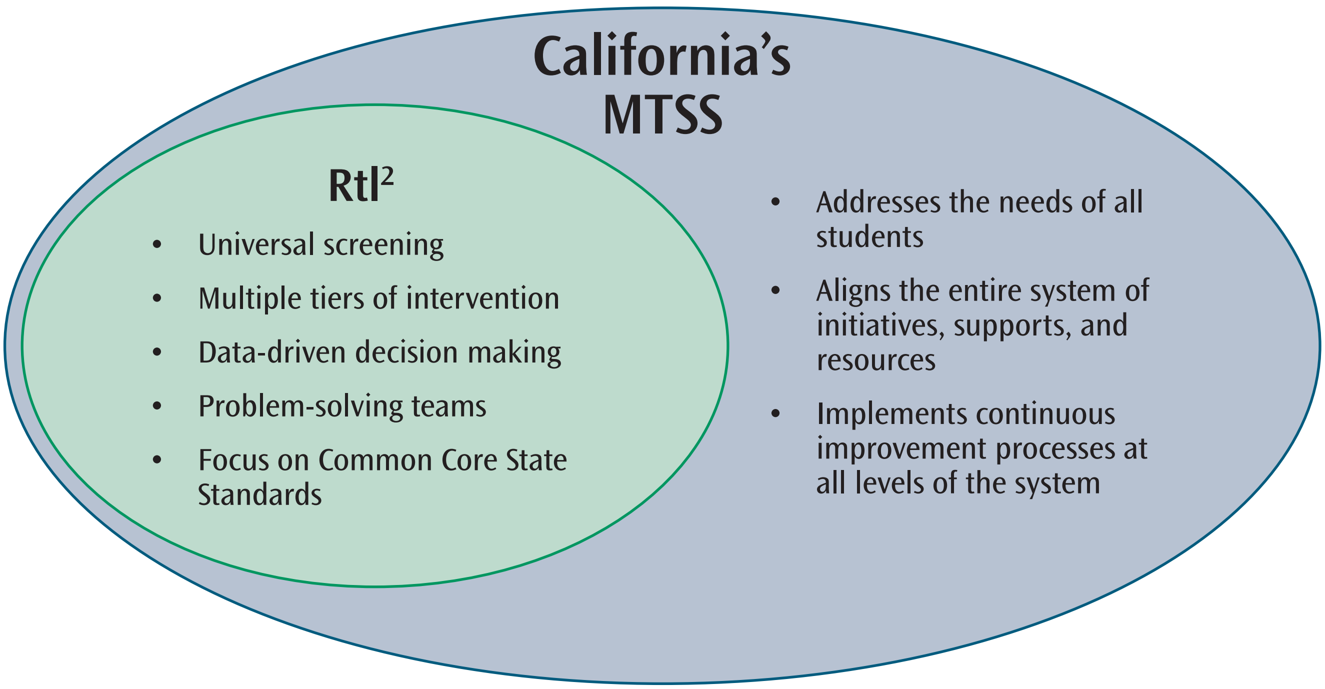
The MTSS incorporates many of the same components of RtI², such as these:

- Supporting high-quality standards and research-based, culturally and linguistically relevant instruction with the belief that every student can learn—including students who live in poverty, students with disabilities, English learners, and students from all ethnicities present in the school and district cultures
- Integrating a data collection and assessment system (including universal screening, diagnostics, and progress monitoring) to inform decisions appropriate for each tier of service delivery
- Relying on a problem-solving systems process and method to identify problems, develop interventions, and evaluate the effectiveness of interventions in a multi-tiered system of service delivery
- Seeking and implementing appropriate research-based interventions for improving student learning

- Using positive, research-based behavioral supports schoolwide and in classrooms to achieve important social and learning outcomes
- Implementing a collaborative approach to analyzing student data and working within the intervention process

Figure UA-1 provides a Venn diagram showing the similarities and differences between California's MTSS and RtI[2] processes. Both rely on RtI[2]'s data gathering through universal screening, data-driven decision making, and problem-solving teams, and both focus on the CA CCSS. However, the MTSS addresses the needs of all students by aligning the entire system of initiatives, supports, and resources and by implementing continuous improvement processes at all levels of the system.

Figure UA-1. Venn Diagram of the Similarities and Differences Between the MTSS and RtI[2]



Tier 1, Tier 2, and Tier 3 Mathematics Interventions

With the caveat that there has been little research on effective RtI[2] interventions for mathematics, Gersten et al. (2009) provide eight recommendations (see table UA-2) to identify and support the needs of students who are struggling in mathematics.[3] The authors note that systematic and explicit instruction is a "recurrent theme in the body of scientific research." They cite evidence for the effectiveness of combinations of systematic and explicit instruction that include teacher demonstrations and think-alouds early in the lesson, unit, or module; student verbalization of how a problem was solved; scaffolded practice; and immediate corrective feedback (Gersten et al. 2009).

3. For additional information on the eight recommendations and detailed suggestions on implementing them in the classroom, see Gersten et al. 2009.

Table UA-2. Recommendations for Identifying and Supporting Students Who Are Struggling in Mathematics

Tier 1
Recommendation 1. All students should be screened to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.
Tiers 2 and 3
Recommendation 2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade five and on rational numbers in grades four through eight. These materials should be selected by committee.
Recommendation 3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.
Recommendation 4. Interventions should include instruction on solving word problems that is based on common underlying structures.
Recommendation 5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas, and interventionists should be proficient in the use of visual representations of mathematical ideas.
Recommendation 6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.
Recommendation 7. Progress of students who receive supplemental instruction and other students who are at risk should be monitored.
Recommendation 8. Motivational strategies in Tier 2 and Tier 3 interventions should be included.

Adapted from Gersten et al. 2009.

With systematic instruction, concepts are introduced in a logical, coherent order, and students have many opportunities to apply each concept. As an example, students develop their understanding of place value in a variety of contexts before learning procedures for addition and subtraction of two-digit numbers. To help students learn to communicate their reasoning and the strategies they used to solve a problem, teachers model thinking aloud and ask students to explain their solutions. These recommendations fit within the overall framework of the MTSS described previously.

Planning Instruction for Students with Disabilities

Some students who receive their mathematics instruction in the general education classroom (Tier 1) or receive Tier 2 or Tier 3 interventions may also have disabilities that require accommodations or placements in programs other than general education. Students with disabilities who have difficulty remembering and retrieving basic mathematics facts may not be able to retain the information necessary to solve mathematics problems.

Students with disabilities are provided access to all the mathematics standards through a rich and supported program that uses instructional materials and strategies that best meet the students' needs.

A student's 504 accommodation plan or IEP often includes suggestions for a variety of teaching and learning techniques. This is to ensure that the student has full access to a program that will allow him or her to master the CA CCSSM, including the MP standards. Teachers must familiarize themselves with each student's 504 accommodation plan or IEP to help the student achieve mastery of the grade-level CA CCSSM.

Section 504 Plan

A Section 504 accommodation plan is typically produced by school districts in compliance with the requirements of Section 504 of the federal Rehabilitation Act of 1973. The plan specifies agreed-on services and accommodations for a student who, as a result of an evaluation, is determined to have a physical or mental impairment that substantially limits one or more major life activities. Section 504 allows a wide range of information to be contained in a plan: (1) the nature of the disability; (2) the basis for determining the disability; (3) the educational impact of the disability; (4) the necessary accommodations; and (5) the least restrictive environment in which the student may be placed.

Individualized Education Program (IEP)

An IEP is a comprehensive written statement of the educational needs of a child with a disability and the specially designed instruction and related services to be employed to meet those needs. An IEP is developed (and periodically reviewed and revised) by a team of individuals knowledgeable about the child's disability, including the parent(s) or guardian(s). The IEP complies with the requirements of the Individuals with Disabilities Education Act (IDEA) and covers items such as (1) the child's present level of performance in relation to the curriculum; (2) measurable annual goals related to the child's involvement and progress in the curriculum; (3) specialized programs (or program modifications) and services to be provided; (4) participation in general education classes and activities; and (5) accommodation and modification in assessments.

In recent years, five different meta-analyses of effective mathematics instruction for students with disabilities have been conducted. The studies included students who have learning disabilities, but also students with mild intellectual disabilities, attention deficit hyperactive disorder (ADHD), behavioral disorders, and students with significant cognitive disabilities (Adams and Carnine 2003; Baker, Gersten, and Lee 2002; Browder et al. 2008; Kroesbergen and Van Luit 2003; Xin and Jitendra 1999). These meta-analyses, along with the National Mathematics Advisory Panel (2008) report titled *Foundations for Success*, suggest that the following four methods of instruction show promise for improving mathematics achievement in students with disabilities:

1. **Systematic and explicit instruction.** Teachers guide students through a defined instructional sequence with explicit (direct) instructional practice. Teachers model a strategy for solving a particular type of problem so that students can see when and how to use the strategy and what they can gain by doing so. This type of instruction helps students learn to regularly apply strategies that effective learners use as a fundamental part of mastering concepts.

2. **Self-instruction.** Students manage their own learning through a variety of self-regulation strategies with specific prompting or solution-oriented questions.

3. **Peer tutoring.** This refers to many different types of tutoring arrangements, but most often involves pairing students together to learn or practice an academic task. Peer tutoring works best when students of different ability levels work together.

4. **Visual representation.** This type of instruction involves the use of manipulatives, pictures, number lines, and graphs of functions and relationships to teach mathematical concepts. The Concrete–Representational–Abstract (CRA) sequence of instruction is an evidence-based instructional practice involving manipulatives to promote conceptual understanding (Witzel, Riccomini, and Schneider 2008). It is the most common example of visual representation and shows promise for improving understanding of mathematical concepts for students with disabilities. The CRA instructional sequence consists of three tiers of learning: (1) concrete learning through hands-on instruction using actual manipulative objects; (2) representational learning through pictorial representations of the previously used manipulative objects during concrete instruction; and (3) learning through abstract notations such as operational symbols. Each tier is interconnected and builds upon the previous one, promoting conceptual understanding, procedural accuracy, and fluency and leading toward mathematical proficiency for students. The CRA sequence is built upon the premise of UDL, which calls for multi-modal forms of learning (e.g., seeing, hearing, moving muscles, and touching). This sequence allows learners to interact in multiple ways, which may increase student engagement and the desire to attend to the task at hand. Using manipulatives in concrete and representational ways helps learners to gain meaning from abstract mathematics by breaking down the steps into understandable concepts. To that end, the CRA instructional sequence provides a more meaningful and contextually relevant solution to rote memorization of algorithms and rules taught in isolation.

In order to improve mathematics performance in students with learning difficulties, Vaughn, Bos, and Schumm (2010) also suggest that when new mathematical concepts are introduced or when students have difficulty learning a concept, teachers need to “begin with the concrete and then move to the abstract” (Vaughn, Bos, and Schumm 2010, 385). Furthermore, these authors suggest that student improvement will occur when teachers provide:

- explicit instruction that is highly sequenced and indicates to students why the learning is important;
- assurance that students understand the teacher’s directions as well as the demands of the task by closely monitoring student work;
- systematic use of learning principles such as positive reinforcement, varied practice, and student motivation;
- real-world examples that are understandable to students (Vaughn, Bos, and Schumm 2010, 385).

For students with significant cognitive disabilities, systematic instruction—which includes teacher modeling, repeated practice, and consistent prompting and feedback—was found to be an effective instructional strategy. Studies focused on skills such as counting money and basic operations. Students also learned from instruction in real-world settings, such as a store or restaurant (Browder et al. 2008).

Although direct instruction has been shown to be an effective strategy for teaching basic mathematical skills, the CA CCSSM emphasize a *balance* of conceptual understanding, fluency with skills and procedures, and application of mathematics concepts to real-world contexts. This balance can be achieved by connecting mathematical practices to mathematical content. Helping students to develop mathematical practices, including analyzing problems and persevering in solving them, constructing arguments and critiquing others, and reasoning abstractly and quantitatively, requires a different approach. Based on their work with students who have disabilities and those working below grade level, Stephan and Smith (2012) offer suggestions for creating a standards-based learning environment. Three key components of this type of learning environment are the selection of appropriate problems, the role of the teacher(s), and the role of the students. The problems students are asked to solve must be engaging to students, open-ended, and rich enough to support mathematical discourse.

Stephan and Smith recommend that problems be "grounded in real-world contexts" (Stephan and Smith 2012, 174) and accessible to all students, and they should require little direct instruction to introduce. The teacher introduces the problem to be solved, reminds students of what they have already learned that may help them with the problem, and answers clarifying questions. The teacher does not provide direct instruction, but quickly sets the context for the students' work. To foster student discussion, the teacher takes the role of information gatherer and asks questions of the students that help them reason through a problem. If students are working in small groups, the teacher moves from group to group to ensure all students are explaining their reasoning and asking their peers for information and explanations. Students take on the role of active learners who must figure out how to solve the problem instead of being given the steps for solving it. They work with their peers to solve problems, analyze their own solutions, and apply previous learning to new situations. Depending on the problem posed, students find more than one possible answer and more than one way to solve the problem. When teachers utilize diverse pairings for group work (e.g., students working at or above grade level collaborate with students who are not), students can accomplish content- or language-task goals as well as mathematics goals. Collaborative work between the partners facilitates inclusion through the learning of mathematical content. Vaughn, Bos, and Schumm (2010) note that collaborative learning has proven to be an effective method of instruction for students with developmental disabilities in the general education classroom.

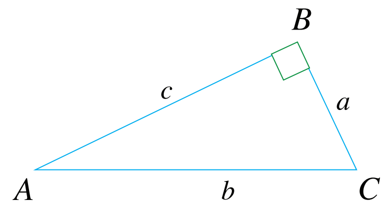
Patterns of Error in Computation

Vaughn, Bos, and Schumm (2010) indicate that many of the computation errors made by students fall into certain patterns. Ashlock (1998) theorizes that errors are generated when students "overgeneralize" during the learning process. On the other hand, other errors occur when students "overspecialize" during the learning process by restricting procedures in solving the problem (Ashlock 1998, 15). To diagnose the computational errors of students who are experiencing difficulty, assessment tools must alert the teacher to both overgeneralization and overspecialization. Teachers need to probe deeply as they examine written work—looking for misconceptions and erroneous procedures that form patterns across examples—and try to find out why specific procedures were learned. These discoveries will help teachers plan for and provide instruction to meet the needs of their students.

Errors also occur when students have not learned basic facts, perform an incorrect operation, do not complete the algorithm in the correct sequence, lack understanding of place value within the algorithm, or provide a random response. Figure UA-2 presents some examples of student errors.

Figure UA-2. Examples of Student Error Patterns

Overgeneralization

Because $23 = 20 + 3$, the student thinks that $2y = 20 + y$ (so that if $y = 5$, $2y = 25$)	 $c^2 = a^2 + b^2$ The student cites the Pythagorean Theorem (ignoring the actual location of the hypotenuse).	$4 + 2 = 6$ $6 - 2 = 8$ The student does not pay attention to the addition and subtraction signs and thinks both answers are sums because they appear to the right side of the equal sign.

Overspecialization

The student writes $100.36 + 12.57$ as $100.36 + 125.70$ because the two addends must have the same number of digits on either side of the decimal point.	The altitude of a triangle has to be contained within the triangle. 

Improper composing and decomposing

$\begin{array}{r} 47 \\ +34 \\ \hline 71 \end{array}$	$\begin{array}{r} 65 \\ +38 \\ \hline 93 \end{array}$	$\begin{array}{r} 37 \\ +25 \\ \hline 52 \end{array}$	The composed ten is not added. The student may be composing the ten in her head and forgetting to add it, or she may be adding left to right and does not know what to do when the addition results in a two-digit answer, so she records only the ones digit. An interview with the student would provide further diagnostic information (Miller 2009, 230).
$\begin{array}{r} 45 \\ -37 \\ \hline 12 \end{array}$	$\begin{array}{r} 46 \\ -28 \\ \hline 22 \end{array}$	$\begin{array}{r} 36 \\ -17 \\ \hline 21 \end{array}$	The student does not decompose the tens when needed. Instead, he subtracts the smaller "ones" number from the larger "ones" number (Miller 2009, 230).
$\begin{array}{r} 25 \\ \times 32 \\ \hline 50 \\ 75 \\ \hline 125 \end{array}$	$\begin{array}{r} 44 \\ \times 27 \\ \hline 308 \\ 88 \\ \hline 396 \end{array}$	$\begin{array}{r} 53 \\ \times 37 \\ \hline 371 \\ 159 \\ \hline 530 \end{array}$	The student misaligns the second partial product (Miller 2009, 230).

As mentioned in figure UA-2, interviewing students to find out how they solved a problem can provide teachers with insights on students' misunderstandings or learning difficulties. Teachers can employ these and other remediation strategies:

- Returning to simpler problems
- Analyzing student errors and bringing to light students' misconceptions
- Estimating
- Demonstrating or providing students with concrete models to develop conceptual understanding (moving to a representational model and then abstract thinking as students progress)
- Using grid paper so students can align numbers by place value
- Designing graphic organizers and flowcharts
- Providing students with meaningful opportunities and sequential practice to learn basic facts for fluency

Students with disabilities can successfully study higher mathematics. They may require accommodations, such as access to a calculator or learning strategies that provide alternatives to memorizing computation facts, but no child should be denied the opportunity to study higher mathematics based on his or her disabilities.

Accommodations for Students with Disabilities

Accommodations support equitable instruction and assessment for students by lessening the effects of a student's disability. Without accommodations, students with disabilities may have difficulty accessing grade-level instruction and participating fully in assessments. When possible, accommodations should be the same or similar across classroom instruction, classroom tests, and state and district assessments. However, some accommodations may be appropriate only for instructional use and may not be appropriate for use on a standardized assessment. It is crucial for educators to be familiar with state policies regarding accommodations used for statewide assessment.

A small number of students with significant disabilities will struggle to achieve at or near grade level. These students, who will participate in alternative assessments, account for approximately 1 percent of the total student population. Substantial supports and accommodations are often necessary for these students to have meaningful access to academic content standards and to standards-aligned assessments that are appropriate for the students' academic and functional needs.

All students with disabilities can work toward grade-level academic content standards, and most of these students will be able to accomplish this goal when the following three conditions are met (Thompson et al. 2005):

1. Standards are implemented within the foundational principles of UDL.
2. A variety of evidence-based instructional strategies are considered to align materials, curriculum, and production to reflect the interests, preferences, and readiness of diverse learners maximizing students' potential to accelerate learning.
3. Appropriate accommodations are provided to help students access grade-level content.

Accommodations play an important role in helping students with disabilities to access the core curriculum and demonstrate what they know and can do. A student's IEP or 504 Plan team determines the appropriate accommodations for both instruction and state and district assessments. Decisions about accommodations must be made on an individual student basis, not on the basis of category of disability or administrative convenience. For example, rather than selecting accommodations from a generic checklist, IEP and 504 Plan team members (including families and the student) need to carefully consider and evaluate the effectiveness of accommodations for each student.

> **"The [Common Core State Standards] should . . . be read as allowing for the widest possible range of students to participate fully from the outset and as permitting appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities *reading* should allow for the use of Braille, screen-reader technology, or other assistive devices, while *writing* should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, *speaking* and *listening* should be interpreted broadly to include sign language."**
>
> —NGA/CCSSO 2010b

Accommodations are typically made in presentation, response, setting, and timing and scheduling so that learners are provided with equitable access during instruction and assessment.

- **Presentation.** Accommodations in presentation allow students to access information in ways that do not require them to visually read standard print. These alternative modes of access are auditory, multi-sensory, tactile, and manual. For example, a student with a visual impairment may require that a test be presented in a different manner, such as in a digital format accompanied with a text-to-speech software application or with a braille test booklet.

- **Response.** Accommodations in response allow students to complete activities, assignments, and assessments in different ways or to solve or organize problems using some type of assistive device or organizer. For example, a student may require an alternative method of completing multi-step computational problems due to weak fine motor skills or physical impairments; such methods may include computer access with a specialized keyboard, a speech-to-text application, or other specialized software.

- **Setting.** Accommodations in setting allow for a change in the location where a test or assignment is given or in the conditions of an assessment setting. For example, a student may require that an assessment be administered in a setting appropriate to the student's individual needs (such as testing an individual student separately from the group to provide visual or auditory supports).

- **Timing and Scheduling.** Accommodations in timing and scheduling allow for an increase in the typical length of time to complete an assessment or assignment and perhaps change the way the time allotted is organized. For example, a student may take as long as reasonably needed to complete an assessment, including taking portions over several days to avoid fatigue caused by a chronic health condition.

The Council of Chief State School Officers provides guidance in its *Accommodations Manual: How to Select, Administer, and Evaluate Use of Accommodations for Instruction and Assessment of Students with Disabilities* (http://www.ccsso.org/Documents/2005/Accommodations_Manual_How_2005.pdf [Thompson et al. 2005]).

The selection and evaluation of accommodations for students with disabilities who are also English learners must include collaboration among educational specialists, the classroom teacher, teachers providing instruction in English language development, families, and the student. It is important to note that English learners are disproportionately represented (in high numbers) in the population of students who are identified as having disabilities. This suggests that some of these students may not have disabilities and that the identification process is inappropriate for English learners.

Accommodations are available to all students—those who have disabilities and those who do not. Accommodations do not reduce learning expectations; rather, they provide access. Accommodations can reduce or even eliminate the effects of a student's disability. It is important to note that although some accommodations may be appropriate for instructional use, they may not be appropriate for use on a standardized assessment.

Assistive Technology

A fundamental goal of the CA CCSSM is to promote a culture in which all students are challenged to meet high expectations. To ensure that all students have access to general education instruction, standards, and curriculum, students with disabilities may be provided additional supports and services, as appropriate. These supports are often provided through the use of assistive technology. Assistive technology is used by individuals to gain access and perform functions that might otherwise be difficult or impossible. Assistive technology is defined in federal law (the Individuals with Disabilities Education Improvement Act of 2004) as "any item, piece of equipment, or product system, whether acquired commercially off the shelf, modified, or customized, that is used to increase, maintain, or improve functional capabilities of a child with a disability" (Pub. L. No. 108–466, 118 Stat. 2652 [2004]). Assistive technology can include a wide variety of learning enhancements, including mobility devices, writing implements, communication boards, and grid paper, as well as hardware, software, and peripherals that assist in accessing lessons. For more information about assistive technology, visit https://www.washington.edu/accessit/ (National Center on Accessible Information Technology in Education 2015).

Teachers implement accommodations and modifications in mathematics instruction in numerous ways, including through the use of assistive technology. Students with physical, sensory, or cognitive disabilities may face additional learning challenges or may learn differently. For example, students with fine motor disabilities may not be able to hold a pencil to write answers on a test or use a standard calculator to solve mathematics problems. Students who have difficulty decoding text and symbols may struggle to comprehend text. When assistive technology is appropriately integrated into the classroom, students are provided with a variety of ways to access the information and to complete their work.

Disabilities vary widely, and accommodations must be tailored to each student's unique needs. Assistive technology helps teachers provide accommodations and modifications for students with disabilities. Accommodations change how a student learns material, and modifications change what a student is taught or expected to learn.

- *Assistive technology accommodations.* Assistive technology provides access to the course curriculum. Students can receive assistance from a computer that scans and reads text or digital content to incorporate images, sound, video clips, and additional information. Students with visual impairments can gain access to instructional materials through digital large print with a contrasting background, the ability to change the font as it appears on the screen, or text-to-speech devices. Software that converts text to braille characters, using a refreshable display, provides students with access to printed information. Students can use mobile devices to create or record notes so that they can later print out assignments or use the notes to study for a test. A student with motor difficulties might use an enlarged or simplified computer keyboard, a talking computer with a joystick, or other modified input device such as a switch, headgear, or eye selection devices. The American Speech–Language–Hearing Association (ASHA) presents information on augmentative and alternative communication systems or applications[4] that help students with severe speech or language disabilities express thoughts, needs, or ideas. These and other types of assistance can provide access, but they do not change content and are therefore considered accommodations.
- *Assistive technology modifications.* Assistive technology provides additional help to students who otherwise would not be able learn a concept or show what they have learned. Examples of modifications provided by assistive technology include the use of speech-to-text devices, calculators, or other devices that provide information not otherwise available to students. Of course, there are many other types of modifications that do not involve the use of assistive technology.

Although assistive technology helps to level the playing field for students with special needs, many types of assistive technology (both software and hardware) are beneficial for all students. The flexibility of assistive technology allows a teacher to use tools and materials that support a student's individual strengths and also address his or her disability in the least restrictive environment.

The CDE provides information that clarifies basic requirements for consideration and provision of assistive technology and services to individuals with disabilities. Information is also available for local educational agencies, particularly members of IEP teams, to effectively address these requirements. For other examples of assistive technology, please visit the CDE Assistive Technology Checklist Web page at http://www.cde.ca.gov/sp/se/sr/atexmpl.asp (CDE 2015a).

Planning Instruction for California's English Learners

Students in California demonstrate a wide variety of skills, abilities, and interests as well as different levels of proficiency in English and other languages. California's students come from diverse cultural, linguistic, ethnic, and religious backgrounds, have different experiences, and live with different familial and socioeconomic circumstances. The greater the variation of the student population, the richer the learning experiences for all, and the more assets upon which teachers may draw. At the same time,

4. For more information about augmentative and alternative communication, visit http://www.asha.org/public/speech/disorders/AAC/ (ASHA 2015).

the teacher's role in providing high-quality curriculum and instruction that is sensitive to the needs of individuals becomes more complex. In diverse settings, the notion of *shared responsibility* is particularly crucial. Teachers need the support of one another, administrators, specialists, and the community in order to best serve all students.

Approximately 25 percent of California's public school students are learning English as an additional language. These students come to California schools from all over the world, but the majority were born in California. Schools and districts are responsible for ensuring that all English learners have full access to an intellectually rich and comprehensive curriculum, via appropriately designed instruction, and that they make steady—and even accelerated—progress in their English language development.

English learners come to school with a range of cultural and linguistic backgrounds; experiences with formal schooling; proficiency with mathematics, their native language, and English; migrant and socio-economic statuses; and interactions in the home, school, and community. All of these factors inform how educators support English learners to achieve school success through the implementation of the CA ELD standards in tandem with the CA CCSSM. Educators should not confuse students' language ability with their mathematical understanding.

Ethnically and racially diverse students make up approximately 74 percent of California's student population, making it the most diverse student population in the nation. In 2012–13, more than 1.3 million students—or roughly 25 percent of the California public school population—were identified as English learners. Of those English learners, 84.6 percent identified Spanish as their home language. The next largest group of English learners, 2.3 percent, identified Vietnamese as their home language (CDE 2013c). Given the large number of English learners in California's schools, it is essential to provide these students with effective mathematics instruction.

English learners face a significant challenge in learning subject-area content while simultaneously developing proficiency in English. Planning mathematical instruction for English learners is most effective when the instruction takes into consideration the students' mathematics skills and understandings as well as their assessed levels of proficiency in English and their primary language. Because of variations in academic background and age, some students may advance more quickly in mathematics or English language development than other students who require more support to make academic progress. Many districts use assessment tools such as the statewide assessment,[5] which measures the progress of English learners in acquiring the skills of listening, speaking, reading, and writing in English. The statewide assessment is designed to identify a student's proficiency level in English and to monitor the student's progress in English language development. Other tools for measuring progress in English language development are academic progress, teacher and parent evaluation, and tests of basic skills (such as district benchmarks).

The role of English language proficiency must be a consideration for English learners who experience difficulties in learning mathematics. Even students who have good conversational English skills may lack the academic language necessary to fully access mathematics curriculum (Francis et al. 2006a).

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5. This statewide assessment was formerly known as the California English Language Development Test (CELDT) and will be replaced by the English Language Proficiency Assessments for California (ELPAC) in 2016.

Academic language, as described by Saunders and Goldenberg, "entails all aspects of language from grammatical elements to vocabulary and discourse structures and conventions" (Saunders and Goldenberg 2010, 106).

> **"[E]very teacher must incorporate into his or her curriculum instructional support for oral and written language as it relates to the mathematics standards and content. It is not possible to separate the content of mathematics from the language in which it is discussed and taught."**
>
> —Francis et al. 2006a, 38

Moschkovich (2012b) cautions that communicating in mathematics is more than a matter of learning vocabulary; students must also be able to participate in discussions about mathematical ideas, make generalizations, and support their claims. She states, "While vocabulary is necessary, it is not sufficient. Learning to communicate mathematically is not merely or primarily a matter of learning vocabulary" (Moschkovich 2012b, 18). Providing instruction that focuses on teaching for understanding, helping students use multiple representations to comprehend mathematical concepts and explain their reasoning, and supporting students' communication about mathematics is challenging (Moschkovich 2012a, 1). Moschkovich's recommendations for connecting mathematical content to language are provided in table UA-3.

Table UA-3. Recommendations for Connecting Mathematical Content to Language

1. Focus on students' mathematical reasoning, not accuracy in using language.
2. Shift to a focus on mathematical discourse practices; move away from simplified views of language.
3. Recognize and support students to engage with the complexity of language in math classrooms.
4. Treat everyday language and experiences as resources, not as obstacles.
5. Uncover the mathematics in what students say and do.
Source: Moschkovich 2012a, 5–8.

Teachers can take the following steps to support English learners in the acquisition of mathematical skills and knowledge as well as academic language:

- Explicitly teach academic vocabulary for mathematics, and structure activities in which students regularly employ key mathematical terms. Be aware of words that have multiple meanings (such as *root*, *plane*, *table*, and so forth).
- Provide communication guides, sometimes called *sentence frames*, as a temporary scaffold to help students express themselves not just in complete sentences but articulately within the MP standards.
- Use graphic organizers and visuals to help students understand mathematical processes and vocabulary.

For English learners who are of elementary-school age, progress in mathematics may be supported through intentional lesson planning for content, mathematical practice, and language objectives. Language objectives "articulate for learners the academic language functions and skills that they need to master to fully participate in the lesson and meet the grade-level content standards" (Echevarria, Vogt, and Short 2008). In mathematics, students' use of the MP standards requires students to translate between various representations of mathematics and to develop a command of receptive (listening, reading) and productive (speaking, writing) language. Language is crucial for schema-building; learners construct new understandings and knowledge through language, whether unpacking new learning for themselves or justifying their reasoning to a peer.

The following are examples of possible language objectives for a student in grade two:

- Read word problems fluently.
- Explain in writing the strategies used to solve addition and subtraction problems within 100.
- Describe orally the relationship between addition and subtraction.

Francis et al. (2006a) examined research on instruction and intervention in mathematics for English learners. The consensus among the researchers was that a lack of development of academic language is a primary cause of English learners' academic difficulties and that more attention needs to be paid to the development of academic language. Like Moschkovich, Francis et al. (2006a) make clear that understanding and using academic language involve many skills beyond merely learning new vocabulary words; these skills include using increasingly complex words, comprehending and using sentence structures and syntax, understanding the organization of text, and producing writing appropriate to the content and to the students' grade level.

One approach to improve students' academic language is to "amplify, rather than simplify" new vocabulary and mathematical terms (Wilson 2010). When new or challenging language is continually simplified for English learners, they cannot gain the academic language necessary to learn mathematics. New vocabulary, complex text, and the meanings of mathematical symbols need to be taught in context with appropriate scaffolding or amplified. Amplification helps increase students' vocabulary and makes mathematics more accessible to students with limited vocabulary. In the progression of rational-number learning throughout the grades, particularly relevant to upper elementary and middle school, students encounter increasingly complex uses of mathematical language (words, symbols) that may contradict student sense-making and associations of a term or phrase from earlier grades. For example, *half* is interpreted as either a call to divide a certain quantity by two, or to double that quantity, depending upon the context:

Half of 6 is ______?

Six (6) divided by one-half is ______?

The standards distinguish between *number* and *quantity*, where quantity is a numerical value of a specific unit of measure. By middle school, students are expected to articulate that a "unit rate for Sandy's bike ride is one-half mile per hour," based upon reading the slope of a distance-versus-time line graph of a bike ride traveled at this constant rate. Here, "one-half" represents the distance traveled for each hour, rather than the equivalent ratio of one mile traveled for every two hours. The same symbols that students encountered in early elementary grade levels to represent parts of a whole—for example, partitioning in grade two, formalized unit fractions in grade three—are now attached to new language and concepts in upper elementary grade levels and middle school.

Mathematical Discourse

According to the New Zealand Council for Educational Research (2014), "Mathematical classroom discourse is about whole-class discussions in which students talk about mathematics in such a way that they reveal their understanding of concepts. Students also learn to engage in mathematical reasoning and debate." Teachers ask "strategic questions that elicit from students both how a problem was solved and why a particular method was chosen" (New Zealand Council for Educational Research 2014). Students learn to critique ideas (their own and those of other students), and they look for efficient mathematical solutions.

Researchers caution that focusing on academic language alone may promote teaching vocabulary without a context or lead to the misconception that students are lacking because of their inability to use academic language (Edelsky 2006; MacSwan and Rolstad 2003). It is essential for instruction to include teaching vocabulary in context so that the mathematical meaning can be emphasized. Classroom discourse is one instructional strategy that promotes the use of academic and mathematical language within a meaningful context. *Mathematics discourse* is defined as communication that centers on making meaning of mathematical concepts; it is more than just knowing vocabulary. It involves negotiating meanings by listening and responding, describing understanding, making conjectures, presenting solutions, challenging the thinking of others, and connecting mathematical notations and representations (Celedón-Pattichis and Ramirez 2012, 20).

Lesson plans that include objectives for language, mathematical content standards, and mathematical practice standards need to identify where these three objectives intersect and what specific scaffolds are necessary for English learners' mathematical discourse. As one example, a high school teacher of long-term English learners has planned a lesson that requires students to identify whether four points on a coordinate graph belong to a quadratic or an exponential function. Classroom routines for partner and group work have been established, and students know what "good listening" and "good speaking" look like and sound like. However, the teacher has also created bookmarks for students to use, with sentence starters and sentence frames to share their conjectures and rationales and to question the thinking of other students. The teacher is employing an instructional strategy called "Think-Write-Pair-Share" with scaffolds in the form of sentence frames. After a specified time for individual thinking and writing, students share their initial reasoning with a partner. A whole-class discussion ensues, with the teacher intentionally re-voicing student language and asking students to use their own words to share what they heard another student say. While the teacher informally assesses how students employ academic language in their oral statements, she also presses for "another way to say" or represent that thinking to amplify academic language.

Long-Term English Learners

The lack of English language proficiency and understanding of the language of mathematics is of particular concern for long-term English learners—students in grades six through twelve who have been enrolled in American schools for more than six years and have remained at the same English language proficiency level for two or more consecutive years, as determined by the state's annual English language development test. To address the instructional needs of long-term English learners, focused instruction such as instructed English language development (ELD) may be the most effective (Dutro and Kinsella 2010). Instructed ELD, as described by Dutro and Kinsella, focuses attention on language learning. Language skills are taught in a prescribed scope and sequence, ELD is explicitly taught, and there are many opportunities for student practice. Lessons, units, and modules are designed to build fluency and aim to help students achieve full English proficiency.

In addition to systematic ELD instruction, Dutro and Moran (2003) offer two recommendations for developing students' language in the content areas: front-loading and using "teachable moments."

> Front-loading of ELD describes a focus on language preceding a content lesson. The linguistic demands of a content task are analyzed and taught in an up-front investment of time to render the content understandable to the student. This front-loading refers not only to the vocabulary, but also to the forms or structures of language needed to discuss the content. The content instruction, like the action of a piston, switches back and forth from focus on language, to focus on content, and back to language. (Dutro and Moran 2003, 4)

The following example of Dutro and Moran's "piston" instructional strategy informally assesses and advances students' mathematical English language development.

List-Group-Label Activity

Purpose: Formative assessment of students' acquisition of academic language, as well as their ability to distinguish form and function of mathematical terms and symbols. For example, the term *polygon* reminds students of types of polygons (triangles, rectangles, rhombuses) or reminds students of components or attributes of polygons (angles, sides, parallel, perpendicular) or non-examples (circles).

Process: At the conclusion of instruction, the teacher posts a mathematical category or term that students encountered in the unit and asks students to generate as many mathematical words or symbols related to the posted term as they can.

Working with a partner or group, students compile lists of related words and agree how to best sort their lists into subgroups.

For each subgroup of terms or symbols, students must come to agreement on an appropriate label for the subgroup list and be prepared to justify their "List-Group-Label" to another student group.

Teachers also take advantage of teachable moments to expand and deepen language skills. Teachers must utilize opportunities "as they present themselves, to use precise language [**MP.6**] to fill a specific, unanticipated need for a word or a way to express a thought or idea. Fully utilizing the teachable moment means providing the next language skill needed to carry out a task or respond to a stimulus" (Dutro and Moran 2003, 4).

M. J. Schleppegrell (2007) agrees that the language of mathematical reasoning differs from informal ordinary language. Traditionally, teachers have identified mathematics vocabulary as a challenge but are not aware of the grammatical patterning embedded in mathematical language that generates difficulties. Schleppegrell identifies these linguistic structures as "patterns of language that draw on grammatical constructions that create dense clauses linked with each other in conventionalized ways" (Schleppegrell 2007, 146) yet differ from ordinary use of language. Examples include the use of long, dense noun phrases such as *the volume of a rectangular prism with sides 8, 10, and 12 cm*; classifying adjectives that precede the noun (e.g., *prime number*, *right triangle*); and qualifiers that come after the noun (e.g., *a number that can be divided by 1 and itself*). Other challenging grammatical structures that may pose difficulty include signal words such as *if*, *when*, *therefore*, *given*, and *assume*, which are used differently in mathematics than in everyday language (Schleppegrell 2007, 143–146). Schleppegrell asserts that educators need to expand their knowledge of mathematical language to recognize when and how to include grammatical structures that enable students to participate in mathematical discourse.

Other work on mathematics discourse, such as from Suzanne Irujo (cited in Anstrom et al. 2010), provides concrete classroom applications for vocabulary instruction at the elementary and secondary levels. Irujo explains and suggests three steps for teaching mathematical and academic vocabulary (Anstrom et al. 2010, 23):

- The first suggested step is for educators to analytically read texts, tests, and materials to identify potential difficulties, focusing on challenging language.
- The second step follows Dutro and Moran's findings on pre-teaching with experiential activities in mathematics; only the necessary vocabulary and key concepts are taught to introduce the central ideas.
- The third and final step is integration of the learning process. New vocabulary is pointed out as it is encountered in context, its use is modeled frequently by the teacher, and the modeling cycle is repeated, followed by guided practice, small-group practice, and independent practice. Irujo also recommends teaching complex language forms (e.g., prefixes and suffixes) through mini-lessons.

Despite the importance of academic language for success in mathematics, "in mathematics classrooms and curricula the language demands are likely to go unnoticed and unattended to" (Francis et al. 2006a, 37). Both oral and written language need to be integrated into mathematics instruction. All students, not just English learners, must be provided many opportunities to engage in mathematics discourse—to talk about mathematics and explain their reasoning. The language demands of mathematics instruction must be noted and attended to. Mathematics instruction that includes reading, writing, and speaking enhances students' learning. As lessons, units, and modules are planned, both language objectives and content objectives should be identified. By focusing on and modifying instruction to address English learners' academic language development, teachers support their students' mathematics learning.

The CA ELD standards are an important tool for designing instruction to support students' reading, writing, speaking, and listening in mathematics. The CA ELD standards help guide curriculum, instruction, and assessment for English learners who are developing the English language skills necessary to engage successfully with mathematics. California's English learners (ELs) are enrolled in a variety of school and instructional settings that influence the application of the CA ELD standards. The CA ELD standards are designed to be used by all teachers of academic content and of English language development in all settings, albeit in ways that are appropriate to each setting and to identified student needs. Additionally, the CA ELD standards are designed and intended to be used *in tandem with* the CA CCSSM to support ELs in mainstream academic content classrooms.

Neither the CA CCSSM nor the CA ELD standards should be treated as checklists. Instead, the CA ELD standards should be utilized as a tool to equip ELs to better understand mathematics concepts and solve problems. Factors affecting ELs' success in mathematics should also be taken into account. (See also the next section on Course Placement of English Learners.) There are a multitude of such factors that fall into at least one of seven characteristic types. These factors inform how educators can support ELs to achieve success in mathematics:

Van de Walle (2007) suggests specific strategies that teachers can incorporate into their mathematics instruction to support English learners:

- Let students know the purpose of the lesson and what they will accomplish during the lesson.
- Build background knowledge and link the lesson to what students already know.
- Encourage the use of each student's native language during group work while continuing to focus on English language development.
- Provide comprehensible input by simplifying sentence structure and limiting the use of non-essential vocabulary. Use visuals whenever possible.
- Explicitly teach vocabulary. Use a word wall and personal math dictionaries.
- Have students work in cooperative groups. This provides English learners with non-threatening opportunities to use language.

1. **Limited prior or background knowledge and experience with formal schooling**
   - Some ELs may lack basic mathematics skills. EL students with limited prior schooling may not have the basic computation skills required to succeed in the first year of higher mathematics. ELs who enter U.S. schools in kindergarten benefit from participation in the same instructional activities as their non-EL peers, along with additional differentiated support based on student needs. Depending upon the level and extent of previous schooling they have received, ELs who enter U.S. schools for the first time in high school may need additional support to master certain linguistic and cognitive skills and fully engage in intellectually challenging academic tasks. Regardless of their schooling background or exposure to English, all ELs should have full access to the same high-quality, intellectually challenging, and content-rich instruction and instructional materials as their non-EL peers, along with appropriate levels of scaffolding.
   - Some ELs may have prior or background knowledge, but it is important to avoid misconceptions of students' mathematics skill levels, especially when based upon their cultural background and upbringing.

2. **Cultural differences**

   - Mathematics is often considered a universal language in which numbers connect people regardless of culture, religion, age, or gender (NYU Steinhardt 2009). However, mathematics learning styles vary by country and culture, and by individual students.
   - The meanings of some symbols (such as commas and decimal points) and mathematical concepts differ according to culture and country of origin. This occurs frequently, especially when expressing currency values, measurement, temperature, and so on, and may impede an EL's understanding of the material being taught. Early on in the school year, teachers should survey their students to learn about the students' backgrounds and effectively address individual needs. It is important for teachers to inform themselves about particular aspects of their students' backgrounds, but also to see each student as an individual with distinct learning needs, regardless of cultural or linguistic influences.

3. **Linguistics**

   Everyday language is very different from academic language, and when students struggle to understand and apply these differences, they may experience difficulties in acquiring academic language. Teachers should develop all of their students' understandings of how, why, and when to use different registers and dialects of English. Some of these challenges may include understanding mathematics-specific vocabulary that is difficult to decode, associating mathematics symbols with concepts, as well as the language used to express those concepts, and grasping the complex and challenging structure of the passive voice.

4. **Polysemous words**

   Polysemous words have identical spellings and pronunciations, but different meanings that are based on context. For example, a *table* is a piece of furniture on which one can set food and dishes, but it is also a systematic arrangement of data or information. Similarly, an *operation* may be a medical procedure or a mathematical procedure; these meanings are different from each other in context, but they do have some relation to one another. The difference between *polysemes* and *homonyms* is subtle: polysemes have semantically related meanings, but homonyms do not.

5. **Syntactic features of word problems**

   - The arrangement of words in a sentence plays a major role in understanding phrases, clauses, or the entire sentence. Complex syntax is especially difficult in the reading, understanding, and solving of word problems in mathematics (NYU Steinhardt 2009). Extra support should be given to ELs regarding syntactic features.
   - Some algebraic expressions are troublesome for ELs, because if they attempt to translate the provided word order, the resulting equation may be inaccurate. For example: *A number x is 5 less than a number y*. It is logical to translate word for word when solving this problem, which would most likely result in the following translation: $x = 5 - y$. However, the correct equation would be $x = y - 5$.

6. **Semantic features**

   As shown in the following table (adapted from NYU Steinhardt 2009), many ELs may find semantic features challenging.

Feature	Examples
Synonyms	*add, plus, combine, sum*
Homophones	*sum/some, whole/hole*
Difficult expressions	If . . . then; given that . . .
Prepositions	*divided into* versus *divided by*; *above, over, from, near, to, until, toward, beside*
Comparative constructions	If Amy is taller than Peter, and Peter is taller than Scott, then Amy must be taller than Scott.
Passive structures	Five books were purchased by John.
Conditional clauses	Assuming x is true, then y . . .
Language function words	Words and phrases used to give instructions, to explain, to make requests, to disagree, and so on.

7. **Text analysis**

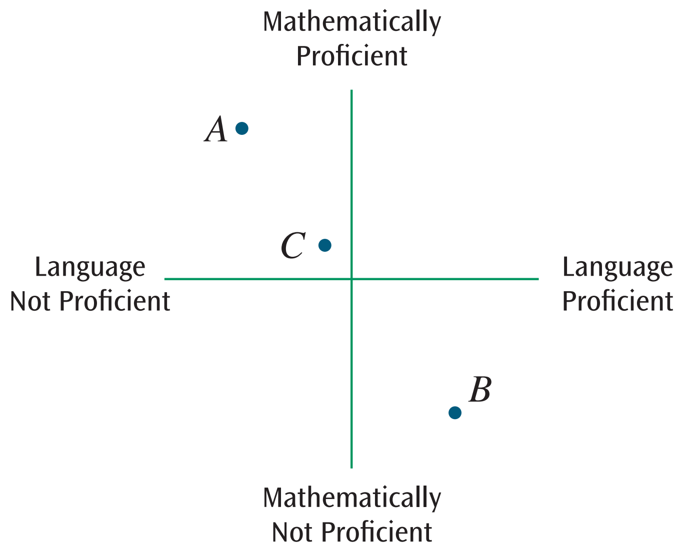
   Word problems often pose challenges because they require students to read and comprehend the text, identify the question, create a numerical equation, and then solve that equation. Reading and understanding written content in a word problem are often difficult for native speakers of English as well as ELs.

When addressing the factors that affect ELs in instruction, it is essential for teachers to know the ELD proficiency-level descriptor that applies to each student in their classroom. The *emerging*, *expanding*, and *bridging* levels identify what a student knows and can do at a particular stage of English language development and can help teachers differentiate their instruction appropriately. The seven factors discussed above remain barriers for EL students if they are not addressed by teachers. Schools and districts are responsible for ensuring that all ELs have full access to an intellectually rich and comprehensive curriculum, via appropriately designed instruction, and that they make steady and accelerated progress in English language development, particularly in secondary grades.

Course Placement of English Learners

Educators must pay careful attention to placement and assessment practices for students who have studied mathematics in other countries and may be proficient in higher-level mathematics but lack proficiency with the English language. Indeed, a student's performance on mathematics assessments may be affected by his or her language proficiency. For example, in figure UA-3, results for students A, B, and C on the same test may look very similar even though the students' language and mathematical proficiency levels vary considerably. The design of the assessment needs to be mindful of this problem, and the results need to be interpreted with students' language proficiency factored in. If possible, mathematics assessments should be done in the student's primary language so that lack of English language proficiency does not affect the test results.

Figure UA-3. Intersecting Continua of Mathematics Proficiency with Language Proficiency



(Graphic adapted from Asturias 2010)

For English learners who may know the mathematical content but have difficulty on assessments due to lack of English language proficiency, Burden and Byrd (2009) list the following strategies for adapting assessments:

- *Level of support.* Increase the amount of scaffolding that is provided during the assessment.
- *Product.* Adapt the type of response to decrease reliance on academic language.
- *Participation.* Allow for cooperative group work and group self-assessment using student-created rubrics for performance tasks.
- *Range.* Decrease the number of assessment items.
- *Time.* Provide extra time for English learners to complete tasks.
- *Difficulty.* Adapt the problem, the task, or the approach to the problem.

Celedón-Pattichis (2004) advises that the initial placement of English learners is highly important because "these placements tend to follow students for the rest of their academic lives" (Celedón-Pattichis 2004, 188). When placement of highly proficient students is not based upon their mathematical competence, but rather on their language proficiency, the students may (1) lose academic learning time and the opportunity to continue with their study of higher-level mathematics; and (2) experience a decline in their level of mathematics achievement because of little practice. On the other hand, when low-performing students are placed in courses that are too difficult for their knowledge or language proficiency level, they are likely to become discouraged.

Similarly, students who have studied mathematics in other countries may experience significant differences in how mathematical concepts are represented in California classrooms. Notational differences include how students read and write numbers, use a decimal point, and separate digits in large numbers. There also may be differences in the designation of billions and trillions. For example:

> A student schooled in the United States will read 10,782,621,751 as "10 billion, 782 million, 621 thousand, 751." In some students' countries of origin, the number is read as "10 mil 782 *millones*, 621 mil, 751"; or it is read as "10 thousand 782 million, 621 thousand, 751." (Perkins and Flores 2002, 347)

Differences also occur in how students compute problems by algorithm. For example, students may mentally compute the steps in an algorithm and only write the answer or display the intermediate steps differently, as with long division. Additional difficulties occur as students confront U.S. currency (Perkins and Flores 2002).

These differences may become apparent when parents who have been educated in other countries assist their children at home. There is a strong need for a meaningful dialogue between parents and teachers in which learning about different learning methods and approaches can occur for all. For example, when students or parents possess different ways of performing arithmetic operations, teachers can use these different approaches as learning opportunities instead of dismissing them. This is particularly important for immigrant children (or children of immigrant parents), who are often navigating two worlds. As Cummins (2000) states, "Conceptual knowledge developed in one language helps to make input in the other language comprehensible" (Cummins 2000, 39).

Planning Instruction for Standard English Learners

The Los Angeles Unified School District (LAUSD) defines Standard English Learners (SELs) as "students for whom Standard English is not native and whose home language differs in structure and form from Standard and academic English" (LAUSD 2012, 83). The Academic English Mastery Program (AEMP) and the Multilingual and Multicultural Department of LAUSD have identified six access strategies to help SELs succeed:

1. *Making cultural connections* — the use of "cultural knowledge, prior experience, frames of reference and performance styles" of students to make learning more relevant, effective, and engaging (LAUSD 2012, 85).
2. *Contrastive analysis* — comparing and contrasting the linguistic features of the primary language and Standard English (LAUSD 2012, 162). During a content lesson, the teacher may demonstrate the difference in languages by repeating the student response in Standard English. This recasting then may be used at a later date as an exemplar to examine the differences. In the following example, note the differences in subject–verb agreement, plurals, and past tense:
   - **Non-Standard English.** *There was three runner. The winner finish the race in three minute.*
   - **Standard English.** *There were three runners. The winner finished the race in three minutes.*
3. *Cooperative learning* — working in pairs or small groups on tasks that are challenging enough to truly require collaboration, or as a way to provide strategic peer support to specific students.
4. *Instructional conversations* — academic conversations, often student-led, that allow students to use language to analyze, reflect, and think critically. These conversations may also be referred to as *accountable talk* or *handing off*.
5. *Academic language development* — explicit teaching of vocabulary and language patterns needed to express the students' thinking. Like English learners, SELs benefit from the use of sentence frames (communication guides); unlike the supports for English learners, the guides are based on Standard English and academic vocabulary and not on English language proficiency levels.
6. *Advanced graphic organizers* — visual representation to help students organize thoughts.

For additional guidance, see chapter 4, Theoretical Foundations and the Research Base of the California English Language Development Standards, in the California English Language Development Standards (CDE 2013b).

Planning Instruction for At-Risk Learners

Mathematical focus and in-depth coverage of the CA CCSSM are as necessary for students with mathematics difficulties as they are for more proficient students (Gersten et al. 2009). As soon as students begin to fall behind in their mastery of mathematics standards, immediate intervention is warranted. Interventions must combine practice in material not yet mastered with instruction in new skill areas. Students who are behind will find it challenging to catch up with their peers and stay current as new topics are introduced. The need for remediation is temporary and cannot be allowed to exclude these students from full instruction. In a standards-based environment, students who are struggling to learn or master mathematics need the richest and most organized type of instruction. For some students, Tier 3 interventions may be necessary.

Students who have fallen behind, or who are in danger of doing so, may need more than the normal schedule of daily mathematics. Systems must be devised to provide these students with ongoing tutorials. It is important to offer special tutorials during or outside of the regular school day; however, to ensure access for all students, extra help and practice should occur in additional periods of mathematics instruction during the school day. Instructional time might be extended in summer school, with extra support focused on strengthening and rebuilding gaps in foundational concepts and skills.

Requiring a student with intensive learning challenges to remain in a course for which he or she lacks the foundational skills to master major concepts is an inefficient use of student learning time. To ensure that students can successfully complete full courses, course and semester structures and class schedules should be re-examined and revised or re-created as needed. Targeted intervention, especially at the middle school level or earlier, can increase students' chances of being successful in higher mathematics. Early intervention in mathematics is both powerful and effective (Newman-Gonchar, Clarke, and Gersten 2009).

Grouping as an Aid to Instruction

As a tool, grouping should be used flexibly to ensure that all students master the standards—and instructional objectives should always be based on the CA CCSSM. Small-group instruction may be utilized as a temporary measure for students who have not learned the prerequisite content (Emmer and Evertson 2009). For example, a teacher may discover that some students are having trouble understanding and using the Pythagorean Theorem. Without this understanding, the students will have serious difficulties in higher-level mathematics. It is perfectly appropriate to group these students, find time to re-teach the concept or skill in a different way, and provide additional practice. These students should also participate with a more heterogeneous mix of students in other classroom activities and groups in which a variety of mathematics problems are discussed.

Teachers rely on their experiences and judgment to determine when and how to incorporate grouping strategies into the classroom. To promote maximum learning when grouping students, educators must ensure that progress monitoring is ongoing, formative assessment is frequent, high-quality instruction is always provided to all students, and that students are frequently moved into appropriate instructional groups according to their needs.

Planning Instruction for Advanced Learners

In the context of this framework, *advanced learners* are students who demonstrate, or are capable of demonstrating, performance in mathematics at a level significantly above the performance that is typical for their age group. In California, each school district sets its own criteria for identifying gifted and talented students. The percentage of students identified varies, and each district may choose whether to identify students as "gifted" on the basis of their ability in mathematics and other subject areas. The criteria should take into account students who are struggling with language barriers. The criteria should also include alternative measures to identify students who are highly proficient in mathematics or have the capacity to become highly proficient in mathematics but may have a learning disability.

The National Mathematics Advisory Panel (2008) looked at research on effective mathematics instruction for gifted students and found only a few studies that met the panel's criteria for evaluating research. This lack of rigorous research limited the panel's findings and recommendations, and the panel called for more high-quality research to study the effectiveness of instructional programs and strategies for gifted students. Based on the research available, the panel reported the following findings.

National Mathematics Advisory Panel Recommendations for Gifted Students

- The studies that were reviewed provided some support for the value of differentiating the mathematics curriculum for students who have sufficient motivation, especially when acceleration is a component (i.e., pace and level of instruction are adjusted).
- A small number of studies indicated that individualized instruction, in which pace of learning is increased and often managed via computer by instructors, produces gains in learning.
- Gifted students who are accelerated by other means not only gained time and reached educational milestones (e.g., college entrance) earlier, but also appeared to achieve at levels at least comparable to those of their equally able same-age peers on a variety of indicators, even though they were younger when demonstrating their performance on various achievement benchmarks.
- Gifted students appeared to become more strongly engaged in science, technology, engineering, or mathematical areas of study. Additionally, there is no evidence in the research literature that gaps or holes in knowledge have occurred as a result of student acceleration.

Source: National Mathematics Advisory Panel 2008.

Based on these findings and general agreement in the field of gifted education, the panel stated, "combined acceleration and enrichment should be the intervention of choice" for mathematically gifted students (National Mathematics Advisory Panel 2008, 53). The panel recommended that mathematically gifted students be allowed to learn mathematics at an accelerated pace and encouraged schools to develop policies that support challenging work in mathematics for gifted students. (See appendix D, Course Placement and Sequences, for additional guidance.)

Several research studies have demonstrated the importance of setting high standards for all students, including advanced learners. The CA CCSSM provide students with goals worth reaching and identify the point at which skills and knowledge should be mastered. The natural corollary is that when standards are mastered, advanced students should either move on to standards at higher grade levels, be provided with enrichment activities that connect to or go beyond the standards, or delve deeper into mathematical concepts and connections across domains. Enrichment or extension leads students to complex, technically sound applications. Activities and challenging problems should be designed to contribute to deeper learning or new insights.

Accelerating the learning of advanced students requires the same careful, consistent, and continual assessment of their progress that is needed to support the learning of average and struggling students. Responding to the results of such assessments allows districts and schools to adopt innovative approaches to teaching and learning to best meet the instructional needs of their students.

In a classroom based on the CA CCSSM, the design of instruction demands dynamic, carefully constructed, mathematically sound lessons, units, and modules created by groups of teachers who pool their expertise to help all children learn. These teams must devise innovative methods for using regular assessments of student progress in conceptual understanding, procedural skill and fluency, and application to ensure that each student progresses toward mastery of the mathematics standards.

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