Braille Mathematics Standards

Adopted by the California State Board of Education

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Contents

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A Message from the State Superintendent of Public Instruction and the President of the State Board of Education

California is making great strides to improve our educational system. As a result of our high-quality content standards and statewide accountability system, we have seen increased student achievement. We need to ensure that all students share in this progress, including those students who are blind or visually impaired.

In 2002, the California Legislature enacted Assembly Bill 2326, which called for the establishment of a task force to develop Braille Reading Standards. We convened this task force, and it issued its recommendations to the State Board of Education in 2004.

In 2005, the Legislature enacted Assembly Bill 897. That legislation called for the development of Braille Mathematics Standards and required the State Board to adopt both Braille Reading and Braille Mathematics Standards for pupils who are blind or visually impaired by June 2006.

The Braille Mathematics and Reading Standards do not establish content standards. Including instructional principles and prerequisites appropriate for students who learn through the sense of touch does not change nor conflict with what the Board previously adopted for all students. It merely provides additional technical information pertaining to braille learners.

The braille standards are carefully aligned with the English-language arts and mathematics content standards for students who use print. We are proud that California is the first state in the nation to address the unique needs of students who use braille for learning their state-adopted reading and mathematics content standards. Supporting content standards-aligned instruction in each content area for students who use braille is evidence of our commitment to providing equal opportunity for all students.

We wish to express our appreciation to the task force members for their dedication and assistance in producing these braille standards. These standards will help our blind and visually impaired students to acquire the skills they need for future education, employment, and independent living and to become successful members of our society.

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Introduction

"A thorough grounding in mathematics enhances educational and occupational opportunities for all people, whether sighted or visually impaired. In day-to-day routines, a practical understanding of mathematics allows a person to function more successfully and independently." (Kapperman, Heinze, and Sticken, 2000)

Why Braille Mathematics Standards?

Teaching mathematics to students who are blind or visually impaired is essential for the same reasons that it is essential for sighted students. However, mathematics can be especially challenging for blind students because so many aspects and concepts of mathematics are visual and spatial in nature.

In 2001 when President Bush signed the No Child Left Behind (NCLB) legislation, the U.S. Department of Education made it clear that the Individuals with Disabilities Education Act (IDEA) would be reauthorized to incorporate NCLB's structure. IDEA states that ". . . to the extent possible, children with disabilities are entitled to the same educational experience as their nondisabled peers."

The California Legislature has acknowledged the importance of literacy for *all* students and passed Assembly Bill (AB) 2326, which created a task force to develop a comprehensive set of braille reading standards. Students who are blind or visually impaired, and their teachers, now have standards which are closely aligned with those used for sighted students.

Now, with the passage of Assembly Bill 897, mathematics standards for braille readers will also be in place. This legislation recognizes the importance of braille and tactile graphics as the media by which students who are blind or visually impaired can best organize, utilize, and process the concepts described by mathematics and science. Braille mathematics standards are essential to ensure that functionally blind students become literate in mathematics. Multisensory experiences are needed for a blind child to maximize the use of tactile information when learning and applying mathematical concepts. One of the many obvious reasons that mathematics instruction is necessary for all students is the fact that knowledge of basic algebra is now needed in order to pass the California High School Exit Examination and graduate with a diploma. Other reasons include the fact that day-to-day calculations are needed to maintain a normal life, such as following a recipe, calculating an appropriate tip for a waiter, and taking measurements for new window coverings. All of these activities are dependent upon the mastery of basic mathematics concepts. In addition, consideration must be given to the blind students who desire to continue their education in the fields of mathematics and science.

It is critical to have a qualified teacher who understands the nonvisual approach necessary for teaching functionally blind students. A blind child uses multisensory experiences to develop and organize computations, solutions, and spatial relationships that are expressed in mathematics. Since a blind child cannot take in his or her surroundings at a glance, touching is essential. Because of this, a blind child will need additional learning time in order to have the opportunity for tactile exploration of shapes, objects, or graphics. That does not mean that the blind child is a slower learner; it simply means this child must use a different (nonvisual) method of learning. Parental and educator expectations for blind children must be at a high level. “What separates a fundamental belief in the ability of blind children from a generalized belief is the matter of clear expectations” (Schroeder, 2004).

Tactile exploration must begin in early childhood, with families playing a key role. The introduction of simple, tactile shapes and objects and braille materials should begin at an early age in the same way these common shapes and printed materials are introduced at an early age to sighted children. Children who are blind deserve the same learning opportunities as their sighted peers. They simply need to have these learning experiences presented in a nonvisual methodology.

Unique Challenges for Blind Learners

The Nemeth Code for Braille Mathematics and Science is the current standard for tactile mathematics textbooks and allows students who are blind to braille all the necessary mathematical symbols up to and including the highest levels of mathematics (Kapperman, 1994; Wittenstein, 1993). However, according to Osterhaus (http://www.tsbvi.edu), “ . . . often the Nemeth Code is not taught to the blind students as they progress through their lower level math classes. This creates great difficulties as they progress into Algebra and most students MUST use the Nemeth Code (or some other tactual code) to be successful in higher mathematics.” In addition, the possibility exists that the movement to unify the English braille code may result in a new tactile mathematics code in the future. Whichever tactile mathematics code becomes the standard for blind learners, mathematics must be presented in a tactile mode for those who cannot access print. Auditory representations of advanced mathematical concepts cannot truly convey the essential and precise information required in mathematics (Cullers, 2003 Keynote address, American Printing House for the Blind).

These challenges necessitate a set of braille mathematics standards aligned with the mathematics standards designed for sighted students. Without these adapted standards, as stated in AB 897, “Functionally blind pupils in California are at a disadvantage in mathematics performance due to the lack of Braille mathematics standards in the state.”

Braille Mathematics Standards

These braille mathematics standards begin with a very straightforward philosophy:

1. The teaching and learning of mathematics in a tactile mode is as essential for a blind child as the teaching and learning of print mathematics is for a sighted child.
2. Although there are some similarities to traditional mathematics instruction in the teaching and learning of mathematics in a tactile mode, there are also

> challenging and unique aspects for the blind learner. Traditional mathematics relies heavily on visual, spatial, and abstract concepts that are particularly challenging to the blind, tactile learner. Therefore, instruction and materials in mathematics for blind learners must take these differences into consideration and ensure learners have access to precise mathematical information in a tactile mode.

The standards contained in this document represent a strong consensus of the task force members on the skills, knowledge, and abilities that all blind students should master in order to master mathematics. These standards are carefully aligned with the *Mathematics Content Standards for California Public Schools* while emphasizing the unique differences in learning mathematics through the sense of touch.

Issues of Concern

Through the process of reaching consensus on these standards, the task force created by AB 2326 and continued through AB 897 recognized several issues that may create obstacles to the implementation of these standards. Among these obstacles are:

Time for instruction. It is vital for children learning mathematics in a tactile mode to have at least as much direct instructional time in this medium as children learning through sight have. In many cases blind children do not have sufficient access to a teacher of blind students who not only is knowledgeable in the braille mathematics code and in how to teach it but also is well versed in the specialized methods and materials for teaching mathematics in a tactile mode. This lack of knowledgeable teachers severely limits the kind of continuous feedback that is vital in the emergence of mathematical skills.

Attitudes. Some professionals, parents, and children who are blind believe braille is a second-class medium unable to provide the same access to learning as print provides. These negative and inaccurate attitudes can lead to decisions to substitute less efficient media and devices.

Service delivery. Most children who are blind are served by itinerant teachers who travel from school to school serving children in their home schools. This service delivery model can create an obstacle if the child does not have access to a classroom teacher with knowledge of the braille math code and of the teaching methodology of tactile mathematics. In such cases the teacher of the blind students must act as a collaborator and consultant to the teacher of mathematics, and this team approach must incorporate the necessary tactile adaptations in order to create an appropriate environment for instruction. Some blind children are served in specialized, self-contained programs, and their teachers need to be knowledgeable about traditional mathematics instruction, the implementation of state standards, and the methodology of teaching mathematics in a tactile mode.

Teacher Training. Teachers of children who are blind need access to ongoing in-service training to enhance and refresh their university preparation activities. Teachers need support, training, and time to provide these vital services.

Technology. Some people incorrectly believe that talking computers and recordings can replace braille and even render braille obsolete. However, much of the best assistive technology available enhances the use and production of braille. Access to information auditorily does not replace print or braille. It supplements those essential media. In mathematics, tactile representations are critical to the child's concept development and growth.

Age at onset of blindness. Children become blind at different times in their lives. Therefore, they may need to learn beginning braille mathematics code at any age and at any grade level, providing additional challenges for students and teachers.

Braille production standards. The quality of braille mathematics materials varies widely. Access to certified Nemeth Code transcribers varies widely as well. There must be a commitment to "dot-perfect" braille and appropriate tactile graphics of shapes, diagrams, charts, graphs, and maps in the schools. It is never sufficient to use braille translation software alone without the knowledge and proofreading skills of certified braille transcribers. Braille readers deserve the same level of accuracy in materials as do print readers. Simply stated, instructional materials for blind children must be as accurate and error-free as instructional materials are for sighted children.

For blind children to attain the mathematical skills required for high school graduation and beyond, these obstacles must be overcome.

An Essential Discipline

As stated in the California *Mathematics Content Standards*, "With the adoption of these content standards in mathematics, California is going beyond reform. We are redefining the state's role in public education. For the first time, we are stating—explicitly—the content that students need to acquire at each grade level from kindergarten to grade twelve. These standards are rigorous. With student mastery of this content, California schools will be on a par with those in the best educational systems in other states and nations. The content is attainable by all students, given sufficient time, except for those few who have severe disabilities. We regard the standards as firm but not unyielding; they will be modified in future years to reflect new research and scholarship" (p. iv).

Now, with the *Braille Mathematics Standards,* California has asserted that blind children have the same rights to rigorous standards and appropriate education in mathematics as do sighted children.

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Getting Ready for Braille Mathematics

Many of the strategies used to develop a readiness in blind children to learn mathematics are the same as those used to get a child ready to read literary braille.

Early learning experiences set the stage for the development of mathematical skills. Children with normal vision are exposed, by direct instruction and through incidental learning, to a wide variety of experiences, beginning at birth. At least 80 percent of the information they take in from the world around them is taken in through vision (Hill & Blasch, 1980). For children who are blind or visually impaired to have equivalent experiences upon which they can build their concepts, they must also be exposed to the world around them—but in ways that will enable them to learn through senses other than vision.

Children with little or no usable vision experience the world through their ears, their fingers, their skin, their noses, their mouths, and their movements. Because of this difference in learning, children who are blind or visually impaired will not generally develop on their own the same kinds of concepts about the world as their peers with sight do. Hence, learning experiences from infancy and the reinforcement of those experiences must be carefully constructed. What is learned must be carefully assessed and monitored by those knowledgeable about how children who are visually impaired learn. Otherwise, children who are blind and visually impaired are at risk of developing such significantly different concepts about the world that confusion and misunderstanding result, leading to later difficulties in all areas of learning, including the formation of mathematical concepts.

Children with sight can learn concepts such as “more or less” and “big or little” through frequent incidental opportunities to compare and contrast form, size, and amount. Thus, for example, a sighted child can come to understand the concept of “big” as including the concept “tall” because he can see that his “big sister” is taller than he is. A blind child might come to interpret “big” only as meaning “older” because he cannot easily observe his sister’s height, but he hears that she is older than he. In fact, without direct, hands-on instruction, he may have no real understanding of “tall” either.

In addition to relying more strongly on senses other than sight, children who are blind and visually impaired often have other differences in learning style:

- Learning without sight takes more time. Children who are blind and visually impaired need the time to explore objects physically that sighted children can take in at a glance.
- Children who are blind and visually impaired will need help integrating what they experience tactually with what they hear, smell, and taste.
- One-on-one time will be needed with an adult for a child who is blind or visually impaired to learn the names of objects, understand terms for movements, and acquire other labels for the world that sighted children might acquire incidentally.

- For reasons not wholly understood, many blind and visually impaired children have “tactual defensiveness,” an unwillingness to use their hands for exploration. This reluctance to touch must be overcome through patience, special techniques, and sensitive encouragement on the part of adults.
- Often children who are blind and those who are visually impaired may learn to tune out much of the language they hear because it is based on what the speakers have seen. The language does not fit these children’s experiences, so it does not make sense to them, and they ignore it.
- A teacher must help students integrate their knowledge with what they hear others say. This assistance requires an especially close and sensitive bond—one in which the child trusts the teacher yet does not become dependent on the teacher for learning.

Preschool

Preschool programs set the stage for later school experiences and provide opportunities for children to learn “school behaviors,” such as sharing, taking turns, paying attention in a group, and following directions. Preschool programs that include children who are blind and visually impaired must also be organized to provide maximum opportunities for hands-on exploration, acquisition of fine and gross motor skills, kinesthetic development, development of language appropriate to their understanding for use in everyday activities (“pragmatic” language), and activities that promote interdependence and interaction with their peers.

Families of young children who are blind and visually impaired must work in partnership with teachers of students with visual impairments and other preschool teachers to ensure that the education their children are receiving is consistent and meets each student’s individual needs. Collaboration is essential to provide meaningful experiences that promote early acquisition of mathematical concepts and readiness for reading and writing braille numerals and mathematical symbols.

Learning Environment

Young children who are blind or visually impaired require:

- A learning environment that is organized, structured, and predictable
- A learning environment that emphasizes hands-on experiences, activities that promote exploration with the senses, real-life experiences, and interactions that nurture independence and relationships with peers
- A learning environment that is calm, is free of visual and auditory clutter, and moves at a pace appropriate to the students’ needs
- Immersion in a “braille-rich world,” as sighted children are immersed in a “print-rich world,” with braille numerals on objects where incidental print numerals are found; many different kinds of objects to count and compare; shapes to explore; and items such as rulers, clocks, measuring cups and spoons, and thermometers designed for blind users.

Learning Opportunities

The curriculum for preschoolers who are blind and visually impaired should provide:

- Activities, such as cooking, sand and water tables and other “messy play,” and artwork, to engage students’ senses and provide opportunities for exploring measurement, height, weight, and capacity
- Opportunities for climbing, swinging, running, jumping, and riding on ride-on toys, which not only promote physical activity but also help children develop spatial awareness and positional concepts
- Books that include things to touch, including braille, textures, raised line drawings, and tactile graphics
- Play materials such as puzzles and building sets that also promote development of spatial concepts and lead to understanding of part/whole relationships
- Repeated opportunities to count, count, count, and develop one-to-one correspondence and many activities to learn the vocabulary of mathematics: more, less, equal, empty, full, shorter, longer, taller, same, different, some, all, none, before, after, bigger, smaller, middle sized, heavier, lighter, holds more, and holds less
- Access to a braillewriter and tools for students to make raised-line drawings so that in “scribbling,” they can learn to make shapes, long lines, short lines, curves, and so forth
- The opportunity for students, when they are ready, to learn tactile discrimination, names of shapes, braille numerals, numeral names, and how to make and interpret tactile graphs

Preschool Staff

The staff in a preschool that includes blind and visually impaired children should:

- Receive information about normal development in blind children and monitor the children’s concept acquisition frequently to ensure it is meaningful and accurate.
- Help sighted students interact appropriately with the blind or visually impaired children.
- Monitor their students’ learning and ensure that students have opportunities to explore objects, discuss their properties, and learn counting strategies that promote accuracy.
- Recognize the importance of families’ participation in early mathematical experiences and encourage the students’ families to get the students involved in hands-on activities, including such chores as table setting, that promote counting and one-to-one correspondence.

The Wider Community

Preschool-age blind and visually impaired students should have access to:

- Opportunities for them and their families to meet and get to know blind children and adults who use mathematics every day in their lives
- Opportunities for their families, friends, and general education teachers to learn the braille mathematical code, especially braille numerals and signs of operation

These tenets should be a part of the education of every child who is blind or visually impaired—not only in preschool but in kindergarten through twelfth grade as well.

Mathematics Instruction Prerequisites for All Children

All children, including those with visual impairments, should demonstrate the specific skills noted below to learn mathematics. If students do not demonstrate these prerequisite skills, then they should be taught as appropriate to prepare them for learning.

- Cognitive ability of five years of age or above
- Expressive vocabulary—oral, signed, or with the use of an augmentative communication device—of several thousand words
- Understanding that abstract symbols, such as braille numerals, represent numbers and can be used to record mathematical calculations
- Attention span of at least ten minutes
- Interest in counting objects and comparing and contrasting things

Concept Development

- Understanding that braille is a way of reading by using the fingers and that braille symbols stand for concepts
- Curiosity about exploring the world, observing the objects in it, and learning about their properties
- Interest in and attentiveness to comparing and contrasting individual objects and groups of objects, sorting and classifying objects, measuring, and telling time
- Ability to remain engaged in a task for ten minutes while seated at a table
- Ability to follow one-step (preferably, two-step or more) directions
- Knowledge of left and right on one’s own body and on a page
- Understanding of “same” and “different” in a variety of contexts
- Interest in initiating activities

Tactile Skills

- Willingness to touch a variety of materials, including lines of braille on a page

- Ability to sort materials into two or more categories by touch (e.g., all these shapes are big; all these shapes are small)
- Ability to match objects from a given set of concrete objects, based on one attribute (e.g., match circle to circle, square to square, triangle to triangle, regardless of size or texture)
- Ability to sort by one attribute and to state or demonstrate what that attribute is (e.g., all these are circles; all these are other shapes)
- Ability to identify like shapes in various positions (e.g., recognizes two triangles as the same even though one has the apex pointing up and the other has the apex pointing down)

Hand Skills

- Skill and dexterity in making a variety of hand movements: pushing, pulling, twisting, turning, poking, tracing, squeezing, separating, joining, picking up, putting down, holding, cutting, and pasting
- Ability to use each hand independently in a coordinated manner to complete a task
- Sufficient finger strength and dexterity to form braille characters by using fingers correctly on a braillewriter

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Overview of the Braille Mathematics Standards

These braille mathematics standards are written in “tracking form” so that teachers, parents, and administrators can easily see the differences between learning mathematics using print (visual medium) and learning mathematics using braille (tactile medium). The braille mathematics standards are integrated with the *Mathematics Content Standards for California Public Schools* (adopted by the State Board of Education in 1997). Sections that are added are underlined.

A quick review of the braille mathematics standards dramatically demonstrates the need for disability-specific instruction on a daily basis for students who are blind at early ages. Because of the additional skills and symbols required to learn mathematics in braille, many students will take longer to acquire the skills compared to students who learn mathematics in print. Many mathematical concepts are difficult to acquire without vision. Abstract concepts and functions, easily acquired through visual means, must be presented through verbal and physical modeling by credentialed teachers of students with visual impairments.

There are significantly more mathematics standards for braille readers (kindergarten through grade six) than there are for sighted readers. The braille math standards that have been added address the mechanics of reading and setting up braille math problems, use of braille math symbols, use of the abacus as a calculation tool, and use of tactile graphing devices. In a technologically advanced society, it is essential for students with visual impairments to have early and continuous experience interpreting and preparing tactile graphics. These are critical skills that students who are blind need to master. The additional standards clearly demonstrate that braille learners must receive ongoing, systematic, daily instruction by trained teachers of the blind throughout the students’ educational program.

A great deal of thought and discussion took place during the development of these standards with respect to the order in which the abacus and braille mathematical symbols are taught and at what grade levels. Although there is great variation in practice, the decision should depend on the individual student’s needs and strengths, the standards being taught, and the textbooks being used. For example, one might think that teaching the abacus is outdated or unnecessary. The task force agreed, however, that the abacus is an important and essential tool for mathematical calculations, as is the braillewriter, the slate and stylus, or the talking calculator.

In middle school and high school, the effective use of specialized technology (e.g., braille electronic notetakers, computer and online sources of information, screen readers, scanners, braille/print translation software, embossers, and tactile graphing devices) becomes essential for accessing information and acquiring advanced mathematical concepts. It is important to note that students must learn a myriad of new braille math symbols for algebra, geometry, trigonometry, mathematical analysis, and calculus. Particularly at the high school level, the teacher of students with visual impairments must work in close coordination with the general education mathematics teachers. It is essential that blind students have the specialized braille skills and knowledge of mathematics to compete with their sighted classmates. With the advent of high stakes testing for all high school students in California, blind students deserve and require the tools and services that ensure success.

The creation of braille mathematics standards sends a clear message: Students who are blind or visually impaired and their teachers are to be held accountable for meeting the mathematics content standards to the same extent as are sighted students.

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KINDERGARTEN

By the end of kindergarten, students understand small numbers, quantities, and simple shapes in their everyday environment. They count, compare, describe and sort objects, and develop a sense of properties and patterns.

Number Sense

1.0 Students understand the relationship between numbers and quantities (i.e., that a set of objects has the same number of objects in different situations regardless of its position or arrangement) by using tactile manipulatives:

- 1.1 Compare two or more sets of objects (up to ten objects in each group) and identify which set is equal to, more than, or less than the other.
- 1.2 Count, recognize, display, name, and order a number of objects (up to 30).
- 1.3 Know that the larger numbers describe sets with more objects in them than the smaller numbers have.

2.0 Students understand and describe simple additions and subtractions:

- 2.1 Use concrete objects to determine the answers to addition and subtraction problems (for two numbers that are each less than 10).

3.0 Students use estimation strategies in computation and problem solving that involve numbers that use the ones and tens places:

- 3.1 Recognize when an estimate is reasonable.

4.0 Students read and braille the digits through and the braille number sign (numeric indicator).

Algebra and Functions

1.0 Students sort and classify objects with properties that are easy to discern tactilely:

- 1.1 Identify, sort, and classify objects by one attribute and identify objects that do not belong to a particular group (e.g., all these shapes are circles, those are squares).

Measurement and Geometry

1.0 Students understand the concept of time and units to measure it; they understand that objects have properties, such as length, weight, and capacity, and that comparisons may be made by referring to those properties:

- 1.1 Compare the length, weight, and capacity of concrete objects by making direct comparisons with reference objects (e.g., note which object is shorter, longer, taller, lighter, heavier, or holds more).

1.2 Demonstrate an understanding of concepts of time (e.g., morning, afternoon, evening, today, yesterday, tomorrow, week, year) and tools that measure time (e.g., braille or talking clock, braille calendar).

1.3 Name the days of the week.

1.4 Name the time (to the nearest hour) of everyday events (e.g., lunch time is 12 o'clock; bedtime is 8 o'clock at night).

2.0 Students identify common objects in their environment and describe the geometric features:

2.1 Identify and describe common geometric objects (e.g., circle, triangle, square, rectangle, cube, sphere, cone).

2.2 Compare familiar plane and solid objects by common attributes (e.g., position, shape, size, roundness, number of corners).

Statistics, Data Analysis, and Probability

1.0 Students collect information about objects and events in their environment:

1.1 Pose information questions; collect data; and record the results using objects and tactile graphs.

1.2 Identify, describe, and extend simple patterns (such as circles or triangles) by referring to their shapes, sizes, or thicknesses.

Mathematical Reasoning

1.0 Students make decisions about how to set up a problem:

1.1 Determine the approach, materials, and strategies to be used.

1.2 Use tools and strategies, such as manipulatives, to model problems.

2.0 Students solve problems in reasonable ways and justify their reasoning:

2.1 Explain the reasoning used with concrete objects.

2.2 Make precise calculations using manipulatives and check the validity of the results in the context of the problem.

GRADE ONE

By the end of grade one, students understand and use the concept of ones and tens in the place value number system. Students add and subtract small numbers with ease. They measure with simple units and give and follow directions to locate objects in space. They describe data and analyze and solve simple problems.

Number Sense

1.0 Students understand and use numbers up to 100:

- 1.1 Count, read, and braille whole numbers to 100.
- 1.2 Compare and order whole numbers to 100 by using the braille symbols for less than, equal to, or greater than (<, =, >).
- 1.3 Represent equivalent forms of the same number through the use of physical models and number expressions (to 20) (e.g., 8 may be represented as 4 + 4, 5 + 3, 2 + 2 + 2 + 2, 10 - 2, 11 - 3).
- 1.4 Count and group concrete objects in ones and tens (e.g., three groups of 10 and 4 equals 34, or 30 + 4).
- 1.5 Identify and know the value of real coins and, using real coins, show different combinations of coins that equal the same value.

2.0 Students demonstrate the meaning of addition and subtraction and use these operations to solve problems:

- 2.1 Know the addition facts (sums to 20) and the corresponding subtraction facts and commit them to memory.
- 2.2 Use the inverse relationship between addition and subtraction to solve problems.
- 2.3 Identify one more than, one less than, 10 more than, and 10 less than a given number.
- 2.4 Count by 2s, 5s, and 10s to 100.
- 2.5 Show the meaning of addition (putting together, increasing) and subtraction (taking away, comparing, finding the difference).
- 2.6 Solve addition and subtraction problems with one- and two-digit numbers (e.g., 5 + 58 = __).
  - 2.6.1 Read braille addition and subtraction problems correctly in equation (horizontal)format and spatial(vertical) format.
  - 2.6.2 Know the braille symbols for + and - .
  - 2.6.3 Know the braille mathematical punctuation indicator and the separation line (below which the answer to aproblem is written in spatial format).

2.6.4 Recognize the braille symbols used as placeholders (e.g.,braille full cell and the braille underscore).

2.6.5 Braille problems and answers in equation and spatial formats using correct spacing and alignment.

2.7 Find the sum of three one-digit numbers.

3.0 Students use various estimation strategies andmathematicalaids(e.g.,the Cranmer abacus,base10 blocks, and counting abacus) in computation and problem solving that involve numbers that use the ones, tens, and hundreds places:

3.1 Make reasonable estimates when comparing larger or smaller numbers.

3.1.1 Read,braille,and solve on a braille writer all problems using both the equation (read left to right) and spatial (read top to bottom)formats.

Algebra and Functions

1.0 Students use number sentences in braille with operational symbols and expressions to solve problems:

1.1 Braille and solve number sentences from problem situations that express relationships involving addition and subtraction.

1.2 Understand the meaning of, and the braille for, the symbols +, -, =.

1.3 Create problem situations that might lead to given number sentences involving addition and subtraction.

Measurement and Geometry

1.0 Students use direct comparison and nonstandard units to describe the measurements of objects:

1.1 Compare the length, weight, and volume of two or more objects by using direct comparison or a nonstandard unit.

1.2 Using braille and talking clocks, tell time to the nearest half hour and relate time to events (e.g., before/after, shorter/longer).

2.0 Students identify common geometric figures, classify them by common attributes, and describe their relative position or their location in relation to other objects:

2.1 Identify, describe, and compare triangles, rectangles, squares, and circles, including the faces of three-dimensional objects.

2.2 Classify familiar plane and solid objects by common attributes, such as position, shape, size, roundness, or number of corners, and explain which attributes are being used for classification.

2.3 Give and follow directions about the location of objects and people.

2.4 Arrange and describe the location of objects by proximity, position, and direction (e.g., near, far, below, above, up, down, behind, in front of, next to, left or right of).

Statistics, Data Analysis, and Probability

1.0 Students organize, represent, and compare data by category on simple tactile graphs and charts:

1.1 Sort objects and data by common attributes and describe the categories into which they are sorted.

1.2 Represent and compare data (e.g., largest, smallest, most often, least often) by using tactile graphs,such as bar graphs, object graphs, and raised dot and line graphs.

2.0 Students sort objects and create and describe patterns by numbers, shapes, sizes, or rhythms.

2.1 Describe, extend, and explain ways to get to the next element in simple repeating patterns (e.g., rhythmic, numeric, size, and shape).

Mathematical Reasoning

1.0 Students make decisions about how to set up a problem:

1.1 Determine the approach, materials, and strategies to be used.

1.2 Use manipulatives to model problems.

2.0 Students solve problems and justify their reasoning:

2.1 Explain the reasoning used and justify the procedures selected.

2.2 Make precise calculations and check the validity of the results from the context of the problem or by using manipulatives.

3.0 Students note connections between one problem and another.

GRADE TWO

By the end of grade two, students understand place value and number relationships in addition and subtraction, and they use simple concepts of multiplication. They measure quantities with appropriate units. They classify shapes and see relationships among them by paying attention to their geometric attributes. They collect and analyze data and verify the answers.

Number Sense

1.0 Students understand the relationship between numbers, quantities, and place value in whole numbers up to 1,000:

- 1.1 Count, read, and braille whole numbers to 1,000 and identify the place value for each digit. Know the braille mathematical comma.
- 1.2 Use words, models, and expanded forms (e.g., 45 = 4 tens + 5) to represent numbers (to 1,000).
- 1.3 Order and compare whole numbers to 1,000 by using the braille symbols <, =, >.

2.0 Students estimate, calculate, and solve problems involving addition and subtraction of two- and three-digit numbers:

- 2.1 Understand and use the inverse relationship between addition and subtraction (e.g., an opposite number sentence for 8 + 6 = 14 is 14 - 6 = 8) to solve problems and check solutions.
- 2.2 Find the sum or difference of two whole numbers up to three digits long.
- 2.3 Use mental arithmetic to find the sum or difference of two two-digit numbers.
- 2.4 Use the Cranmer abacus to make addition and subtraction calculations (see AppendixA).

3.0 Students model and solve simple problems involving multiplication and division:

- 3.1 Use repeated addition, arrays, and counting by multiples to do multiplication.
- 3.2 Use repeated subtraction, equal sharing, and forming equal groups with remainders to do simple division.
- 3.3 Know the multiplication tables of 2s, 5s, and 10s (to “times 10”) and commit them to memory.

4.0 Students understand that fractions and decimals may refer to parts of a set and parts of a whole:

- 4.1 Recognize in braille, name, and compare unit fractions from 1/12 to 1/2.
- 4.2 Recognize physical models of fractions of a whole and parts of a group (e.g., one-fourth of a pie, two-thirds of 15 balls).

4.3 Know that when all fractional parts are included, such as four-fourths, the result is equal to the whole and to one.

5.0 Students model and solve problems by representing, adding, and subtracting amounts of money:

5.1 Solve problems using combinations of real coins and bills.

5.2 Know and use the braille decimal notation and the braille mathematical dollar and cent symbols for money.

6.0 Students use estimation strategies in computation and problem solving that involve numbers that use the ones, tens, hundreds, and thousands places:

6.1 Recognize when an estimate is reasonable in measurements (e.g., closest inch).

Algebra and Functions

1.0 Students model, represent, and interpret number relationships to create and solve problems involving addition and subtraction:

1.1 Use the commutative and associative rules to simplify mental calculations and to check results.

1.2 Relate problem situations to number sentences involving addition and subtraction.

1.3 Solve addition and subtraction problems by using data from simple charts, tactile graphs, and number sentences.

Measurement and Geometry

1.0 Students understand that measurement is accomplished by identifying a unit of measure, iterating (repeating) that unit, and comparing it to the item to be measured:

1.1 Measure the length of objects by iterating (repeating) a nonstandard or standard unit.

1.2 Use different units to measure the same object and predict whether the measure will be greater or smaller when a different unit is used.

1.3 Measure the length of an object to the nearest inch and/or centimeter.

1.4 Using a braille or talking clock, tell time to the nearest quarter hour and know relationships of time (e.g., minutes in an hour, days in a month, weeks in a year).

1.5 Determine the duration of intervals of time in hours (e.g., 11:00 a.m. to 4:00 p.m.).

2.0 Students identify and describe the attributes of common figures in the plane and of common three-dimensional objects:

- 2.1 Describe and classify plane and solid geometric shapes (e.g., circle, triangle, square, rectangle, sphere, pyramid, cube, rectangular prism) according to the number and shape of faces, edges, and vertices.
- 2.2 Put shapes together and take them apart to form other shapes (e.g., two congruent right triangles can be arranged to form a rectangle).

Statistics, Data Analysis, and Probability

1.0 Students collect numerical data and record, organize, tactilely display, and interpret the data on bar graphs and other representations:

- 1.1 Record numerical data in systematic ways, keeping track of what has been counted.
- 1.2 Represent the same data set in more than one way (e.g., raised line bar graphs and other raised line graphs).
- 1.3 Identify features of data sets (range and mode).
- 1.4 Ask and answer simple questions related to data representations.

2.0 Students demonstrate an understanding of patterns and how patterns grow and describe them in general ways:

- 2.1 Recognize, describe, and extend patterns and determine a next term in linear patterns (e.g., 4, 8, 12 . . .; the number of ears on one horse, two horses, three horses, four horses).
- 2.2 Solve problems involving simple number patterns.

Mathematical Reasoning

1.0 Students make decisions about how to set up a problem:

- 1.1 Determine the approach, materials, and strategies to be used.
- 1.2 Use manipulatives to model problems.

2.0 Students solve problems and justify their reasoning:

- 2.1 Defend the reasoning used and justify the procedures selected.
- 2.2 Make precise calculations and check the validity of the results in the context of the problem.

3.0 Students note connections between one problem and another.

GRADE THREE

By the end of grade three, students deepen their understanding of place value and their understanding of and skill with addition, subtraction, multiplication, and division of whole numbers. Students estimate, measure, and describe objects in space. They use patterns to help solve problems. They represent number relationships and conduct simple probability experiments.

Number Sense

1.0 Students understand the place value of whole numbers:

- 1.1 Using accepted braille mathematics code and formatting rules,count, recognize, and write whole numbers to 10,000.
- 1.2 Compare and order whole numbers to 10,000.
- 1.3 Identify the place value for each braille digit in numbers to 10,000.
  - 1.3.1 Understand that numbers are written in equation format (horizontal) and spatial format (vertical) by using braille digits to represent countable, measurable,and discrete components contained within the number as awhole.
  - 1.3.2 Read and verbalize tactile representations of each digit contained within the whole number up to10,000 by using proper mathematical vocabulary and syntax;for example, 2,165 may be read and spoken as “two thousand one hundred sixty-five” or as “two thousands,one hundred,six tens,five ones.”
  - 1.3.3 Braille each digit contained within whole number s to10,000.
  - 1.3.4 Read and braille whole numbers to10,000 following accepted braille mathematics code and formatting.
  - 1.3.5 Using an abacus,set and read whole numbers to 10,000.
- 1.4 Round off numbers to 10,000 to the nearest ten, hundred, and thousand.
- 1.5 Use expanded notation to represent braille numbers (e.g., 3,206 = 3,000 + 200 + 6).

2.0 Students calculate and solve problems involving addition, subtraction, multiplication, and division:

- 2.1 Find the sum or difference of two whole numbers between 0 and 10,000.
  - 2.1.1 Using braille writer or slate and stylus and following accepted braille mathematics code and formatting, find the sum or difference of two whole numbers between 0 and 10,000.
  - 2.1.2 Using an abacus, setup and find the sum or difference of two whole numbers between 0 and 10,000.

2.2 Memorize to automaticity the multiplication table for numbers between 1 and 10.

2.2.1 Recognize and braille the multiplication symbol within mathematic equations by following accepted braille mathematics code and formatting rules.

2.3 Use the inverse relationship of multiplication and division to compute and check results.

2.3.1 Recognize and write equations using the braille multiplication and division symbols and following accepted braille mathematics code and formatting rules.

2.4 Solve simple problems involving multiplication of multi-digit numbers by one- digit numbers (3,671 x 3 = __).

2.4.1 Using a braille writer or slate and stylus,setup and solve simple problems involving multiplication of multi-digit numbers by one-digit numbers (3,671 x 3 =), following accepted braille mathematics code and formatting rules.

2.4.2 Using an abacus,setup and solve simple problems involving multiplication of multi-digit numbersbyone-digitnumbers (3.671 x 3 =__).

2.5 Solve division problems in which a multi-digit number is evenly divided by a one-digit number (135 ÷ 5 = __).

2.5.1 Using a braille writer or slate and stylus,solve division problems in which a multi-digit number is evenly divided by a one-digit number, following accepted braille mathematics code and formatting rules.

2.5.2 Using an abacus, setup and solve division problems in which a multi-digit number is evenly divided by a one-digit number.

2.6 Understand the special properties of 0 and 1 in multiplication and division.

2.7 Determine the unit cost when given the total cost and number of units.

2.7.1 Recognize and braille the dollar sign,cents sign,and decimal point within numbers expressing dollar amounts by following accepted braille mathematics code and formatting rules.

2.8 Using a braille writer or slate and stylus,setup and solve problems that require two or more of the skills mentioned above.

2.8.1 Using an abacus, setup and solve problems that require addition, subtraction, multiplication, and division of numbers containing the decimal point.

2.8.2 Using an abacus, setup and solve problems that require two or more of the skills mentioned above.

3.0 Students understand the relationship between whole numbers, simple fractions, and decimals:

3.1 Compare fractions represented by drawings and concrete materials to show equivalency and to add and subtract simple fractions in context (e.g., 1/2 of a pizza is the same amount as 2/4 of another pizza that is the same size; show that 3/8 is larger than 1/4).

3.1.1 Systematically examine and interpret elements of simple,tactile representations depicting fractional parts of whole objects.The representations may be real but must not be limited to this medium. The examination and interpretation must relate two dimensional drawings to each other.

3.2 Add and subtract simple fractions (e.g., determine that 1/8 + 3/8 is the same as 1/2).

3.2.1 Recognize, braille, and find the sum or difference between two or more simple fractions, following accepted braille mathematics code and formatting rules (e.g.,determine that 1/8+3/8 is the same as 1/2).

3.2.2 Using an abacus, setup and solve addition and subtraction problems that contain fractions.

3.3 Solve problems involving addition, subtraction, multiplication, and division of money amounts in decimal notation and multiply and divide money amounts in decimal notation by using whole-number multipliers and divisors.

3.4 Know and understand that fractions and decimals are two different representations of the same concept (e.g., 50 cents is 1/2 of a dollar, 75 cents is 3/4 of a dollar).

Algebra and Functions

1.0 Students select appropriate tactile symbols, operations, and properties to represent, describe, simplify, and solve simple number relationships:

1.1 Represent tactilely relationships of quantities in the form of mathematical expressions, equations, or inequalities by following accepted braille code and formatting rules.

1.2 Solve problems involving numeric equations or inequalities by using a variety of tools,such as tactile graphics, a braille writer,a slate and stylus,an abacus, or other mathematical manipulatives.

1.3 Select appropriate tactile or braille operational and relational symbols to make an expression true (e.g., if 4 __ 3 = 12, what operational symbol goes in the blank?).

1.4 Express simple unit conversions in tactile form using accepted braille code and formatting rules (e.g., inches = feet x 12).

1.5 Recognize and use the commutative and associative properties of multiplication (e.g., if 5 x 7 = 35, then what is 7 x 5? and if 5 x 7 x 3 = 105, then what is 7 x 3 x 5?).

1.5.1 Recognize, braille, and use the braille parentheses symbols when employing the commutative and associative properties of multiplication, following accepted braille mathematics code and formatting rules.

2.0 Students represent simple functional relationships:

2.1 Solve simple problems involving a functional relationship between two quantities (e.g., find the total cost of multiple items given the cost per unit) by using appropriate tactile tools and representational strategies, such as a braille writer, slate and stylus, abacus, other mathematical manipulatives, or tactile graphic materials.

2.2 Extend and recognize by touch a linear pattern by its rules (e.g., the number of legs on a given number of horses may be calculated by counting by 4s or by multiplying the number of horses by 4).

Measurement and Geometry

1.0 Students choose and use appropriate units and measurement tools to quantify by touch the properties of objects:

1.1 Choose the appropriate tactile tools and units (metric and U.S.) and estimate and measure the length, liquid volume, and weight/mass of given objects.

1.2 Estimate or determine the area and volume of solid figures by covering them with squares or by counting the number of cubes that would fill them.

1.3 Find the perimeter of a polygon with integer sides.

1.4 Carry out simple unit conversions within a system of measurement (e.g., centimeters and meters, hours and minutes).

2.0 Students describe and compare by touch the attributes of plane and solid geometric figures and use their understanding to show relationships and solve problems:

2.1 Identify, describe, and classify polygons by touch (including pentagons, hexagons, and octagons) by using tactile three-dimensional objects and two-dimensional graphic representations.

2.2 By touch identify attributes of triangles (e.g., two equal sides for the isosceles triangle, three equal sides for the equilateral triangle, right angle for the right triangle).

2.3 By touch identify attributes of quadrilaterals (e.g., parallel sides for the parallelogram, right angles for the rectangle, equal sides and right angles for the square).

2.4 By touch identify right angles in geometric figures or in real objects and determine whether other angles are greater or less than a right angle.

2.5 By touch identify, describe, and classify common three-dimensional geometric objects (e.g., cube, rectangular solid, sphere, prism, pyramid, cone, cylinder).

2.6 By touch identify common solid objects that are the components needed to make a more complex solid object.

Statistics, Data Analysis, and Probability

1.0 Students conduct simple probability experiments by determining the number of possible outcomes and make simple predictions:

1.1 Identify whether common events are certain, likely, unlikely, or improbable.

1.2 Record the possible outcomes for a simple event (e.g., tossing a coin) and systematically keep track of the outcomes when the event is repeated many times, following accepted braille code and formatting rules.

1.3 Summarize and display the results of probability experiments in a clear and organized way (e.g., use a bar graph or a line plot), following accepted braille code and formatting rules within the tactile graphic representation.

1.4 Use the results of probability experiments to predict future events (e.g., use a tactile line plot to predict the temperature forecast for the next day).

Mathematical Reasoning

1.0 Students make decisions about how to approach problems:

1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.

1.2 Determine when and how to break a problem into simpler parts.

2.0 Students use strategies, skills, and concepts in finding solutions:

2.1 Use estimation to verify the reasonableness of calculated results.

2.2 Apply strategies and results from simpler problems to more complex problems.

2.3 Use a variety of methods based on touch, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

2.4 Express the solution clearly and logically by following accepted braille mathematical code and formatting rules and using terms and clear language; support solutions with evidence in both verbal and symbolic work.

2.5 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

2.6 Make precise calculations and check the validity of the results from the context of the problem.

3.0 Students move beyond a particular problem by generalizing to other situations:

3.1 Evaluate the reasonableness of the solution in the context of the original situation.

3.2 Note the method of deriving the solution by using an appropriate tactile format and demonstrate a conceptual understanding of the derivation by solving similar problems.

3.3 Develop generalizations of the results obtained and apply them in other circumstances.

GRADE FOUR

By the end of grade four, students understand large numbers and addition, subtraction, multiplication, and division of whole numbers. They describe and compare simple fractions and decimals. They understand the properties of, and the relationships between, plane geometric figures. They collect, represent, and analyze data to answer questions.

Number Sense

1.0 Students understand the place value of whole numbers and decimals to two decimal places and how whole numbers and decimals relate to simple fractions. Students use the concepts of negative numbers:

- 1.1 Recognize and braille whole numbers in the millions, following accepted braille code and formatting rules.
- 1.2 By touch order and compare whole numbers and decimals to two decimal places.
- 1.3 Round whole numbers through the millions to the nearest ten, hundred, thousand, ten thousand, or hundred thousand.
- 1.4 Decide when a rounded solution is called for and explain why such a solution may be appropriate.
- 1.5 Explain different interpretations of fractions, for example, parts of a whole, parts of a set, and division of whole numbers by whole numbers; explain equivalence of fractions using accepted braille code and formatting rules and tactile graphics materials or real objects (see Standard 4.0).
- 1.6 Write tenths and hundredths in decimal and fraction notations, following accepted braille code and formatting rules, and know the fraction and decimal equivalents for halves and fourths (e.g., 1/2 = 0.5 or .50; 7/4 = 1 3/4 = 1.75).
- 1.7 Write the fraction represented by a drawing of parts of a figure, using accepted braille code and formatting rules; represent a given fraction by creating and using tactile drawings; and relate a fraction to a simple decimal on a prefabricated number line.
- 1.8 Use concepts of negative numbers (e.g., on a tactile number line, in counting, in temperature, in "owing").
    - 1.8.1 Recognize and braille the symbols signifying positive and negative numbers by following accepted braille code and formatting rules.
- 1.9 Identify on a tactile number line the relative position of positive fractions, positive mixed numbers, and positive decimals to two decimal places.

2.0 Students extend their use and understanding of whole numbers to the addition and subtraction of simple decimals:

2.1 Estimate and compute the sum or difference of whole numbers and positive decimals to two places by following accepted braille code and formatting rules. Students must be able to properly setup and solve these problems by using a braille writer or slate and stylus and, where appropriate, an abacus.

2.2 Round two-place decimals to one decimal or the nearest whole number and judge the reasonableness of the rounded answer.

3.0 Students solve problems involving addition, subtraction, multiplication, and division of whole numbers and understand the relationships among the operations:

3.1 Demonstrate an understanding of, and the ability to use, standard algorithms for the addition and subtraction of multi-digit numbers, following accepted braille mathematics code and formatting rules.

3.1.1 Using anabacus, find the sum of or difference between multi-digit numbers.

3.2 Demonstrate an understanding of, and the ability to use, standard algorithms for multiplying a multi-digit number by a two-digit number and for dividing a multi-digit number by a one-digit number; use relationships between them to simplify computations and to check results. The student must be able to appropriately document the computational processes contained in this standard by following accepted braille mathematics code and formatting rules.

3.2.1 Using an abacus, accurately solve problems involving multiplication of a multi-digit number by a two-digit number and division of a multi-digit number by a one-digit number.

3.3 Solve problems involving multiplication of multi-digit numbers by two-digit numbers, using a braille writer, slate and stylus, or abacus.

3.4 Solve problems involving division of multi-digit numbers by one-digit numbers, using a braille writer, slate and stylus, or abacus.

4.0 Students know how to factor small whole numbers:

4.1 Understand that many whole numbers break down in different ways (e.g., 12 = 4 x 3 = 2 x 6 = 2 x 2 x 3).

4.2 Know that numbers such as 2, 3, 5, 7, and 11 do not have any factors except 1 and themselves and that such numbers are called prime numbers.

Algebra and Functions

1.0 Students use and interpret variables, mathematical symbols, and properties to write and simplify expressions and sentences:

- 1.1 Use tactile letters, boxes, or other symbols to stand for any number in simple expressions or equations (e.g., demonstrate an understanding and the use of the concept of a variable).
- 1.2 Interpret and evaluate braille mathematical expressions that now use parentheses.
- 1.3 Use parentheses to indicate which operation to perform first when writing expressions containing more than two terms and different operations, following accepted braille mathematics code and formatting rules.
- 1.4 Use and interpret formulas (e.g., area = length x width or $A = lw$) to answer questions about quantities and their relationships.
- 1.5 Understand that an equation such as $y = 3x + 5$ is a prescription for determining a second number when a first number is given.

2.0 Students know how to manipulate equations:

- 2.1 Know and understand that equals added to equals are equal.
- 2.2 Know and understand that equals multiplied by equals are equal.

Measurement and Geometry

1.0 Students understand perimeter and area:

- 1.1 By touch measure the area of rectangular shapes by using appropriate units, such as square centimeter (cm^2), square meter (m^2), square kilometer (km^2), square inch (in^2), square yard (yd^2), or square mile (mi^2).
- 1.2 Recognize that rectangles that have the same area can have different perimeters.
- 1.3 Understand that rectangles that have the same perimeter can have different areas.
- 1.4 Understand and use formulas to solve problems involving perimeters and areas of rectangles and squares. Use those formulas to find the areas of more complex figures by dividing the figures into basic shapes.The student must be able to interpret both two-dimensional tactile representations and real objects when completing these calculations.

2.0 Students use two-dimensional coordinate grids to represent points and graph lines and simple figures:

- 2.1 By touch place the points corresponding to linear relationships on braille graph paper (e.g., place 10 points on the graph of the equation $y = 3x$ and connect them by using a tactile straight line).

2.2 Understand that the length of a horizontal line segment equals the difference of the *x*-coordinates.

2.3 Understand that the length of a vertical line segment equals the difference of the *y*-coordinates.

3.0 Students demonstrate an understanding of plane and solid geometric objects and use this knowledge to show relationships and solve problems:

3.1 By touch identify lines that are parallel and perpendicular.

3.2 By touch identify the radius and diameter of a circle.

3.3 By touch identify congruent figures.

3.4 By touch identify figures that have bilateral and rotational symmetry.

3.5 Know the definitions of a right angle, an acute angle, and an obtuse angle. Understand that 90°, 180°, 270°, and 360° are associated, respectively, with 1/4, 1/2, 3/4, and full turns.

3.6 Visualize, describe, and make tactile models of geometric solids (e.g., prisms, pyramids) in terms of the number and shape of faces, edges, and vertices; interpret two-dimensional representations of three-dimensional objects; and create tactile patterns (of faces) for a solid that, when cut and folded, will make a model of the solid.

3.7 Know the definitions of different triangles (e.g., equilateral, isosceles, scalene) and identify their attributes.

3.8 Know the definition of different quadrilaterals (e.g., rhombus, square, rectangle, parallelogram, trapezoid).

Statistics, Data Analysis, and Probability

1.0 Students organize, represent, and interpret numerical and categorical data and clearly communicate their findings:

1.1 Formulate survey questions; systematically collect and represent data on a tactile number line; and coordinate tactile graphs, braille tables, and charts.

1.2 Identify the mode(s) for sets of categorical data and the mode(s), median, and any apparent outliers for numerical data sets.

1.3 Interpret by touch one- and two-variable data graphs to answer questions about a situation.

2.0 Students make predictions for simple probability situations:

2.1 Represent all possible outcomes for a simple probability situation in an organized way (e.g., tactile tables, grids, tree diagrams). Students must use tactile representations and may verbalize interpretations as apart of the descriptive and recording processes.

- 2.2 Express outcomes of experimental probability situations verbally and numerically (e.g., 3 out of 4; 3/4), following accepted braille mathematics code and formatting rules.

Mathematical Reasoning

1.0 Students make decisions about how to approach problems:

- 1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.
- 1.2 Determine when and how to break a problem into simpler parts.

2.0 Students use strategies, skills, and concepts in finding solutions:

- 2.1 Use estimation to verify the reasonableness of calculated results.
- 2.2 Apply strategies and results from simpler problems to more complex problems.
- 2.3 Use a variety of methods, such as words, numbers, tactile symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.
- 2.4 Express the solution clearly and logically, following accepted braille mathematics code and formatting rules, by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.
- 2.5 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.
- 2.6 Make precise calculations and check the validity of the results from the context of the problem.

3.0 Students move beyond a particular problem by generalizing to other situations:

- 3.1 Evaluate the reasonableness of the solution in the context of the original situation.
- 3.2 In braille note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.
- 3.3 Develop generalizations of the results obtained and apply them in other circumstances.

GRADE FIVE

By the end of grade five, students increase their facility with the four basic arithmetic operations applied to fractions, decimals, and positive and negative numbers. They know and use common measuring units to determine length and area and know and use formulas to determine the volume of simple geometric figures. Students know the concept of angle measurement and use a protractor and compass to solve problems. They use grids, tables, graphs, and charts to record and analyze data.

Number Sense

1.0 Students compute with very large and very small numbers, positive integers, decimals, and fractions and understand the relationship between decimals, fractions, and percents. They understand the relative magnitudes of numbers:

1.1 Estimate, round, and manipulate very large (e.g., millions) and very small (e.g., thousandths) numbers.

1.2 Interpret percents as a part of a hundred; find decimal and percent equivalents for common fractions and explain why they represent the same value; compute a given percent of a whole number.

1.2.1. Recognize and braille the percent, decimal,and fraction symbols in isolation and within equations, following accepted braille mathematics code and formatting rules.

1.2.2 Setup and solve problems that require the use of decimals and percentages as a part of the original equation or solution by using a braille writer or slate and stylus.

1.2.3 Using an abacus, setup and compute problems requiring the decimal point and/or percentage as a part of the original problem or solution.

1.2.4 Using a braille writer, slate and stylus, or abacus,find decimal and percent equivalents for common fractions.

1.3 Understand and compute positive integer powers of nonnegative integers; compute examples as repeated multiplication, following accepted braille mathematics code and formatting rules.

1.4 Determine the prime factors of all numbers through 50 and braille the numbers as the product of their prime factors by using exponents to show multiples of a factor (e.g., $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$).

1.4.1 Recognize and braille the symbols used to indicate exponents within the context of mathematic expressions of x to the power of y (x^y).

1.5 Identify and represent on a tactile number line decimals, fractions, mixed numbers, and positive and negative integers.

2.0 Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals:

2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results. Use a braille writer, slate and stylus, or abacus to perform the computations and verify the reasonableness of the results.

2.2 Demonstrate proficiency with division, including division with positive decimals and long division with multi-digit divisors.

2.2.1 Use a braille writer or slate and stylus to demonstrate proficiency with division, including division with positive decimals and long division with multi-digit divisors.

2.2.2 Use an abacus to demonstrate proficiency with division, including division with positive decimals and division with multi-digit divisors.

2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.

2.3.1 Use a braille writer or slate and stylus to solve simple problems, including ones arising in concrete situations involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less),and express answers in the simplest form. Follow accepted braille mathematics code and formatting rules.

2.3.2 Use an abacus to setupandsolve simple problems, including ones arising in concrete situations involving the addition and subtraction of fractions and mixed numbers(like and unlike denominators of 20 or less),and express answers in the simplest form.

2.4 Understand the concept of multiplication and division of fractions.

2.5 Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.

Algebra and Functions

1.0 Students use variables in simple expressions, compute the value of the expression for specific values of the variable, and plot and interpret the results:

1.1 Use information taken from a tactile graph or equation to answer questions about a problem situation.

1.2 Use a letter to represent an unknown number; braille and evaluate simple algebraic expressions in one variable by substitution.

1.3 Know and use the distributive property in equations and expressions with variables.

1.4 Identify and tactilely graph ordered pairs in the four quadrants of the coordinate plane.

1.5 Solve problems involving linear functions with integer values; braille the equation; and tactilely graph the resulting ordered pairs of integers on a grid.

Measurement and Geometry

1.0 Students understand and compute the volumes and areas of simple objects:

1.1 Derive and use the formula for the area of a triangle and of a parallelogram by comparing it with the formula for the area of a rectangle (i.e., two of the same triangles make a parallelogram with twice the area; a parallelogram is compared with a rectangle of the same area by cutting and pasting a right triangle on the parallelogram).

1.2 Construct a cube and rectangular box from two-dimensional patterns and use these patterns to compute the surface area for these objects.

1.3 Understand the concept of volume and use the appropriate units in common measuring systems (i.e., cubic centimeter [cm^3], cubic meter [m^3], cubic inch [in^3], cubic yard [yd^3]) to compute the volume of rectangular solids.

1.4 Differentiate between, and use appropriate units of measures for, two- and three-dimensional objects (i.e., find the perimeter, area, volume).

2.0 Students identify, describe, and classify the properties of, and the relationships between, plane and solid geometric figures:

2.1 Measure, identify, and draw angles, perpendicular and parallel lines, rectangles, and triangles by using appropriate tools (e.g., straightedge, ruler, compass, protractor, tactile drawing software).

2.2 Know that the sum of the angles of any triangle is 180^o and the sum of the angles of any quadrilateral is 360^o and use this information to solve problems.

2.3 Visualize and draw two-dimensional views of three-dimensional objects made from rectangular solids.

Statistics, Data Analysis, and Probability

1.0 Students tactilely display, analyze, compare, and interpret different data sets, including data sets of different sizes:

1.1 Know the concepts of mean, median, and mode; compute and compare simple examples to show that they may differ.

1.2 Organize and display single-variable data in appropriate tactile graphs and representations (e.g., histogram, circle graphs) and explain which types of graphs are appropriate for various data sets.

1.3 Use fractions and percentages to compare data sets of different sizes.

1.4 Identify ordered pairs of data from a tactile graph and interpret the meaning of the data in terms of the situation depicted by the graph.

1.5 Know how to braille ordered pairs correctly; for example, (*x, y*).

Mathematical Reasoning

1.0 Students make decisions about how to approach problems:

1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.

1.2 Determine when and how to break a problem into simpler parts.

2.0 Students use strategies, skills, and concepts in finding solutions:

2.1 Use estimation to verify the reasonableness of calculated results.

2.2 Apply strategies and results from simpler problems to more complex problems.

2.3 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

2.4 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

2.5 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

2.6 Make precise calculations and check the validity of the results from the context of the problem.

3.0 Students move beyond a particular problem by generalizing to other situations:

3.1 Evaluate the reasonableness of the solution in the context of the original situation.

3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

3.3 Develop generalizations of the results obtained and apply them in other circumstances.

GRADE SIX

By the end of grade six, students have mastered the four arithmetic operations with whole numbers, positive fractions, positive decimals, and positive and negative integers; they accurately compute and solve problems. They apply their knowledge to statistics and probability. Students understand the concepts of mean, median, and mode of data sets and how to calculate the range. They analyze data and sampling processes for possible bias and misleading conclusions; they use addition and multiplication of fractions routinely to calculate the probabilities for compound events. Students conceptually understand and work with ratios and proportions; they compute percentages (e.g., tax, tips, interest). Students know about π and the formulas for the circumference and area of a circle. They use letters for numbers in formulas involving geometric shapes and in ratios to represent an unknown part of an expression. They solve one-step linear equations.

Number Sense

1.0 Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages:

1.1 Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a braille number line, following accepted braille mathematics code and formatting rules.

1.2 Interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show the relative sizes of two quantities using appropriate braille notations and following accepted braille mathematics and formatting code (*a/b, a* to *b, a:b*).

1.3 Use proportions to solve problems (e.g., determine the value of *N* if 4/7 = *N*/21, find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.

1.3.1 Setup and solve problems on a braille writer by using proportions (e.g.,determine the value of *N* if 4/7=*N*/21, find the length of side ofa polygon similar to a known polygon).Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.

1.4 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips,using both a braille writer and an abacus.

2.0 Students calculate and solve problems involving addition, subtraction, multiplication, and division:

2.1 Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation,using both a braille writer and an abacus.

2.2 Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g., 5/8 ÷ 15/16 = 5/8 x 16/15 = 2/3), using both a braille writer and an abacus.

2.3 Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations, using a braille writer or abacus.

2.4 Determine the least common multiple and the greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction) by using an abacus or braille writer and following accepted braille mathematics code and formatting rules.

Algebra and Functions

1.0 Students write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, solve simple linear equations, and graph and interpret their results:

1.1 Write and solve one-step linear equations in one variable by setting the problem upon a braille writer and following accepted braille mathematics code and formatting rules.

1.2 Write and evaluate an algebraic expression for a given situation, using up to three variables, on a braille writer, following accepted braille mathematics code and formatting rules.

1.3 Apply algebraic order of operations and the commutative, associative, and distributive properties to evaluate expressions; and justify each step in the process,using a braille writer and following accepted braille mathematics code and formatting rules.

1.4 Solve problems manually by using the correct order of operations or by using a talking scientific calculator.

1.4.1 Solve problems by using a braille writer and following accepted braille mathematics code and formatting rules or by using a talking scientific calculator.(Refer to the resources section for information concerning the talking scientific calculator.)

2.0 Students analyze and use tables, tactile graphs, and rules to solve problems involving rates and proportions:

2.1 Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches).

2.2 Demonstrate an understanding that *rate* is a measure of one quantity per unit value of another quantity.

2.3 Solve problems involving rates, average speed, distance, and time,using both a braille writer and an abacus.

3.0 Students investigate geometric patterns and describe them algebraically:

3.1 Use variables in expressions describing geometric quantities (e.g., $P = 2w + 2l$, $A = 1/2bh$, $C = \pi d$ the formulas for the perimeter of a rectangle, the area of a triangle, and the circumference of a circle, respectively), reading and writing the min braille and following accepted braille mathematics code and formatting rules.

3.2 Express in symbolic form simple relationships arising from geometry.

Measurement and Geometry

1.0 Students deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems:

1.1 Understand the concept of a constant such as π; know the formulas for the circumference and area of a circle.

1.2 Know common estimates of π (3.14; 22/7) and use these values to estimate and calculate the circumference and the area of circles; compare with actual measurements.

1.3 Know and use the formulas for the volume of triangular prisms and cylinders (area of base x height); compare these formulas and explain the similarity between them and the formula for the volume of a rectangular solid.

2.0 Students identify and describe the properties of tactile two-dimensional figures:

2.1 By touch identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms.

2.2 Use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle by setting the problems up on a braille writer and following accepted braille mathematics code and formatting rules.

2.3 Create tactile representations of quadrilaterals and triangles from given information about them (e.g., a quadrilateral having equal sides but no right angles, a right isosceles triangle).

Statistics, Data Analysis, and Probability

1.0 Students compute and analyze statistical measurements for data sets:

1.1 Compute the range, mean, median, and mode of data sets.

1.2 Understand how additional data added to data sets may affect these computations of measures of central tendency.

1.3 Understand how the inclusion or exclusion of outliers affects measures of central tendency.

1.4 Know why a specific measure of central tendency (mean, median, mode) provides the most useful information in a given context.

2.0 Students use data samples of a population and describe the characteristics and limitations of the samples:

2.1 Compare different samples of a population with the data from the entire population and identify a situation in which it makes sense to use a sample.

2.2 Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.

2.3 By touch analyze data displays and explain why the way in which the question was asked might have influenced the results obtained and why the way in which the results were displayed might have influenced the conclusions reached.

2.4 By touch identify data that represent sampling errors and explain why the sample (and the display) might be biased.

2.5 Identify claims based on statistical data and, in simple cases, evaluate the validity of the claims.

3.0 Students determine theoretical and experimental probabilities and use these to make predictions about events:

3.1 Represent all possible outcomes for compound events in an organized way using tactual representations (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome in braille by following accepted braille mathematics code and formatting rules.

3.2 Use data to estimate the probability of future events (e.g., batting averages or number of accidents per mile driven).

3.3 Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if P is the probability of an event, $1-P$ is the probability of an event not occurring.

3.4 Understand that the probability of either of two disjoint events occurring is the sum of the two individual probabilities and that the probability of one event following another, in independent trials, is the product of the two probabilities.

3.5 Understand the difference between independent and dependent events.

Mathematical Reasoning

1.0 Students make decisions about how to approach problems:

1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.

1.3 Determine when and how to break a problem into simpler parts.

2.0 Students use strategies, skills, and concepts in finding solutions:

2.1 Use estimation to verify the reasonableness of calculated results.

2.2 Apply strategies and results from simpler problems to more complex problems.

2.3 Estimate unknown quantities by using tactile representations and solve for them by using logical reasoning and arithmetic and algebraic techniques.

2.4 Use a variety of methods based on braille and tactile representations, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

2.5 Express the solution clearly and logically by using the appropriate braille mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

2.6 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy by following accepted braille mathematics code and formatting rules.

2.7 Make precise calculations and check the validity of the results from the context of the problem by using a braille writer and accepted braille mathematics code and formatting rules or by using an abacus.

3.0 Students move beyond a particular problem by generalizing to other situations:

3.1 Evaluate the reasonableness of the solution in the context of the original situation.

3.2 Note in braille the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

3.3 Develop generalizations of the results obtained and the strategies used and apply them in new problem situations.

GRADE SEVEN

By the end of grade seven, students are adept at manipulating numbers and equations and understand the general principles at work. Students understand and use factoring of numerators and denominators and properties of exponents. They know the Pythagorean theorem and solve problems in which they compute the length of an unknown side. Students know how to compute the surface area and volume of basic three-dimensional objects and understand how area and volume change with a change in scale. Students make conversions between different units of measurement. They know and use different representations of fractional numbers (fractions, decimals, and percents) and are proficient at changing from one to another. They increase their facility with ratio and proportion, compute percents of increase and decrease, and compute simple and compound interest. They graph linear functions and understand the idea of slope and its relation to ratio.

Number Sense

1.0 Students know the properties of, and compute with, rational numbers expressed in a variety of forms:

- 1.1 Using accepted braille mathematics code and formatting, read, braille, and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation.
- 1.2 Using a braille writer and following accepted braille mathematics code and formatting, add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.
- 1.3 Using a Cranmer abacus or a braille writer and following accepted braille mathematics code and formatting, convert fractions to decimals and percents and use these representations in estimations, computations, and applications.
- 1.4 Differentiate between rational and irrational numbers.
- 1.5 Know that every rational number is either a terminating or a repeating decimal and be able to convert terminating decimals into reduced fractions.
- 1.6 Using a Cranmer abacus or a braille writer and following accepted braille mathematics code and formatting, calculate the percentage of increases and decreases of a quantity.
- 1.7 Using a Cranmer abacus or a braille writer and following accepted braille mathematics code and formatting, solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.

2.0 Students use exponents, powers, and roots and use exponents in working with fractions. Using accepted braille mathematics code and formatting, students

Demonstrate they can read and braille correctly the braille symbols for exponents, negative exponents, roots, and absolute value:

2.1 Using accepted braille mathematics code and formatting, demonstrate they understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.

2.2 Using a Cranmer abacus or a braille writer and following accepted braille mathematics code and formatting, add and subtract fractions by using factoring to find common denominators.

2.3 Using a braille writer and following accepted braille mathematics code and formatting, multiply, divide, and simplify rational numbers by using exponent rules.

2.4 Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.

2.5 Demonstrate they understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.

Algebra and Functions

1.0 Students express quantitative relationships by using algebraic terminology, expressions, equations, inequalities, and graphs:

1.1 Using a braille writer and following accepted braille mathematics code and formatting, use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).

1.2 Using a braille writerand following accepted braille mathematics code and formatting, use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$.

1.3 Using a braille writer and following accepted braille mathematics code and formatting, simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.

1.4 Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.

1.5 Represent quantitative relationships on tactile graphs and interpret the meaning of a specific part of a graph in the situation represented by the graph.

2.0 Using a braille writer and following accepted braille mathematics code and formatting, students interpret and evaluate expressions involving integer powers and simple roots:

  2.1 Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Using a braille writer and following accepted braille mathematics code and formatting, simplify and evaluate expressions that include exponents.

  2.2 Using a braille writer and following accepted braille mathematics code and formatting, multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.

3.0 Using braille graphs and graphing devices, students graph and interpret linear and some nonlinear functions:

  3.1 Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.

  3.2 On braille graphs and graphing devices, plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and an equilateral triangle base of varying lengths).

  3.3 On braille graphs and graphing devices, graph linear functions, noting that the vertical change (change in *y*-value) per unit of horizontal change (change in *x*-value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.

  3.4 On braille graphs and graphing devices, plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities.

4.0 Students solve simple linear equations and inequalities over the rational numbers:

  4.1 Using a braillewriter and following accepted braille mathematics code and formatting, solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

  4.2 Using a braille writer and following accepted braille mathematics code and formatting, solve multistep problems involving rate, average speed, distance, and time or a direct variation.

Measurement and Geometry

1.0 Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems:

  1.1 Using accepted braille mathematics code and formatting and measurement and geometry materials designed for blind users, compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).

1.2 Using accepted braille mathematics code and formatting and measurement and geometry materials designed for blind users, construct and read drawings and models made to scale.

1.3 Using a braille writer and following accepted braille mathematics code and formatting, use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.

2.0 Using a braille writer and following accepted braille mathematics code and formatting, students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area, and volume are affected by changes of scale. Students demonstrate they can read and braille correctly the braille symbols for rectangles; parallelograms; trapezoids; squares; isosceles, equilateral, and scalene triangles; circles; prisms; rectangular solids; cubes; and cylinders:

2.1 Using a braille writer and following accepted braille mathematics code and formatting, use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

2.2 Using a braille writer and following accepted braille mathematics code and formatting, estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.

2.3 Using a braille writer and following accepted braille mathematics code and formatting, compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.

2.4 Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or [1 ft^2] = [144 in^2], 1 cubic inch is approximately 16.38 cubic centimeters or [1 in^3] = [16.38 cm^3]).

3.0 Students know the Pythagorean theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures:

3.1 Using accepted braille mathematics code and formatting, braille graphs and graphing devices, and measurement and geometry materials designed for blindusers, identify and construct basic elements of geometric figures (e.g., altitudes, mid-points, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.

3.2 Demonstratethey understand and can use coordinate tactile graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.

3.3 Demonstrate they know and understand the Pythagorean theorem and its converse and can use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically can verify the Pythagorean theorem by direct measurement.

3.4 Using accepted braille mathematics code and formatting, read and braille the correct symbols for “congruent” and demonstrate an understanding of conditions that indicate two geometric figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.

3.5 Using mathematical drawing tools designed for blind users, construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.

3.6 Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).

Statistics, Data Analysis, and Probability

1.0 Students collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set by hand and through the use of an electronic spreadsheet software program, using a screen reading program or braille display:

1.1 Using braille graphs and graphing devices, know and construct various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.

1.2 Represent tactilely two numerical variables on a tactile scatter plot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).

1.3 Demonstrate they understand the meaning of, and are able to compute, the minimum, the lower quartile, the median, the upper quartile, and the maximum of a data set.

Mathematical Reasoning

1.0 Students make decisions about how to approach problems:

   1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

   1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.

   1.3 Determine when and how to break a problem into simpler parts.

2.0 Students use strategies, skills, and concepts in finding solutions:

   2.1 Use estimation to verify the reasonableness of calculated results.

   2.2 Apply strategies and results from simpler problems to more complex problems.

   2.3 Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

   2.4 Make and test conjectures by using both inductive and deductive reasoning.

   2.5 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

   2.6 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

   2.7 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

   2.8 Make precise calculations and check the validity of the results from the context of the problem.

3.0 Students determine a solution is complete and move beyond a particular problem by generalizing to other situations:

   3.1 Evaluate the reasonableness of the solution in the context of the original situation.

   3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

   3.3 Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.

GRADES EIGHT THROUGH TWELVE

Introduction

The standards for grades eight through twelve are organized differently from those for kindergarten through grade seven. In this section strands are not used for organizational purposes as they are in the elementary grades because the mathematics studied in grades eight through twelve falls naturally under discipline headings: algebra, geometry, and so forth. Many schools teach this material in traditional courses; others teach it in an integrated fashion. To allow local educational agencies and teachers flexibility in teaching the material, the standards for grades eight through twelve do not mandate that a particular discipline be initiated and completed in a single grade. The core content of these subjects must be covered; students are expected to achieve the standards however these subjects are sequenced.

Braille standards are provided for algebra I, geometry, algebra II, trigonometry, mathematical analysis, probability and statistics, and calculus. It is not necessary to develop separate braille standards for linear algebra and advanced placement probability and statistics because students using braille will already know the required braille notation for these advanced courses from the basic mathematics courses. Many of the more advanced subjects are not taught in every middle school or high school. Moreover, schools and districts have different ways of combining the subject matter in these various disciplines. For example, many schools combine some trigonometry, mathematical analysis, and linear algebra to form a precalculus course. Some districts prefer offering trigonometry content with algebra II.

Table 1, “Mathematics Disciplines, by Grade Level,” reflects typical grade-level groupings of these disciplines in both integrated and traditional curricula. The boxes containing a single “X” reflect the minimum requirement for mastery by all students. The boxes containing a double “XX” depict content that is typically considered elective but that should also be mastered by students who complete the other disciplines in the lower grade levels and continue the study of mathematics.

Many other combinations of these advanced subjects into courses are possible. What is described in this section are standards for the academic content by discipline; this document does not endorse a particular choice of structure for courses or a particular method of teaching the mathematical content.

When students delve deeply into mathematics, they gain not only conceptual understanding of mathematical principles but also knowledge of, and experience with, pure reasoning. One of the most important goals of mathematics is to teach students logical reasoning. The logical reasoning inherent in the study of mathematics allows for applications to a broad range of situations in which answers to practical problems can be found with accuracy.

Table 1
Mathematics Disciplines, by Grade Level

Discipline	Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
Algebra I	X	X	X	X	X
Geometry	X	X	X	X	X
Algebra II	X	X	X	X	X
Probability and Statistics	X	X	X	X	X
Trigonometry			XX	XX	XX
Linear Algebra			XX	XX	XX
Mathematical Analysis			XX	XX	XX
Advanced Placement Probability and Statistics				XX	XX
Calculus				XX	XX

By grade eight, students' mathematical sensitivity should be sharpened. Students need to start perceiving logical subtleties and appreciate the need for sound mathematical arguments before making conclusions. As students progress in the study of mathematics, they learn to distinguish between inductive and deductive reasoning; understand the meaning of logical implication; test general assertions; realize that one counterexample is enough to show that a general assertion is false; understand conceptually that although a general assertion is true in a few cases, it is not true in all cases; distinguish between something being proven and a mere plausibility argument; and identify logical errors in chains of reasoning.

Mathematical reasoning and conceptual understanding are not separate from content; they are intrinsic to the mathematical discipline students master at more advanced levels.

In coordination within struction from a teacher of mathematics, braille readers require instruction from the teacher of the visually impaired in the areas of (a) braille mathematical symbols; (b) interpretation of raised line drawings; (c) tactile graphing; and (d) use of modified tools necessary to complete tactile constructions.

The grades eight through twelve standards identify more than 100 braille symbols not introduced to students at earlier grade levels. Whereas print offers unlimited ways of writing symbols, all braille is created from combinations of six dots. One misplaced or omitted dot completely changes the meaning of a sign or symbol. Some braille mathematics symbols require modifier signs; some require beginning and terminating signs.The teacher of the visually impaired must carefully check reference books to present dot-perfect symbols according to braille mathematics code and formatting rules.

The teacher of the visually impaired must also determine how best to assist the student in preparation of the student's mathematics work so that a print-reading mathematics teacher can read what the student has written.Since blind students’ technological skills vary, the teacher may choose to (a) overwrite a braille copy produced by the student to generate a math code braille copy of classwork and homework; (b) edit a student's print copy to generate an untranslated grade 0 (computer braille) copy of classwork and homework originally written in math code; or (c)edit a student's print copy to generate an untranslated grade 0 copy of classwork and homework written with the modifier braille math symbols removed.

Algebra I

Symbolic reasoning and calculations with symbols are central in algebra. Through the study of algebra, a student develops an understanding of the symbolic language of mathematics and the sciences. In addition, algebraic skills and concepts are developed and used in a wide variety of problem-solving situations.

The braille algebraI standards identify more than 30 braille symbols not introduced to students at earlier grade levels.The algebraI student requires instruction from the teacher of the visually impaired to learn the new symbols and to interpret raised line drawings and use tactile graphing devices developed specifically for blind students.This requirement includes practice sessions to model, correct, and facilitate method of recording answers.

(See the Grades Eight Through Twelve Introduction for information about braille symbols and methods of preparing students' braille work so that a print-reading mathematics teacher can read what the student has written.)

1.0 Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable:

  1.1 Students use properties of numbers to demonstrate whether assertions are true or false.

    1.1.1 Students use a braille writer or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) parentheses, brackets, and braces; (b) repeating number; (c) radicals; (d) exponents; (e) the Greek letter *pi* (π); (f) does not equal; (g) crossed out; (h) formatting indicators that show italicized,bold,and colored letters; (i) approximately; (j) under bar; (k) over bar; (l) modifier; (m) subscript numbers; (n) superscript numbers; (o) arrows; and (p) other applicable symbols as needed.

2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

  2.1 Students read the accepted braille mathematics code for braces written above or below a series of numbers or letters, including sketched braces and braces embedded in a linear series.

3.0 Students solve equations and inequalities involving absolute values.

  3.1 Students use a braille writer or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols to produce print copies: (a) less than; (b) greater than; (c) less than or equal to; (d) greater than or equal to; (e) absolute ;and (f) other applicable symbols as needed.

4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x-5) + 4(x-2) = 12$.

5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

6.0 Students graph a linear equation and compute the *x*- and *y*- intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).

6.1 Students graph three points on a tactile graphing device by using tactile indicators for points (e.g., tacks if the graphing device is a rubber graph board).

6.2 Students mark and differentiate the *x* and *y* axes, the origin, points, and regions by using flat tacks, push pins, and other optional markers.

6.3 Students are able to indicate the region defined by linear inequality by using a tactile region indicator (e.g.,flat thumbtacks, four placed close together).

7.0 Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.

8.0 Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.

8.1 Students read and use the accepted braille mathematics code for parallel and perpendicular symbols.

9.0 Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.

9.1 Using a tactile graphing device, students graph three points on each line.

9.2 Using a tactile graphing device, students use tactile indicators for each region thats how the solution sets.

10.0 Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.

11.0 Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

12.0 Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.

13.0 Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.

14.0 Students solve a quadratic equation by factoring or completing the square.

15.0 Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

16.0 Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.

17.0 Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.

18.0 Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.

19.0 Students know the quadratic formula and are familiar with its proof by completing the square.

20.0 Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.

21.0 Students graph quadratic functions and know that their roots are the x-intercepts.

21.1 Using a tactile graphing device, students graph the vertex and two points on each side of the parabola.

22.0 Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.

23.0 Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.

24.0 Students use and know simple aspects of a logical argument:

24.1 Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.

24.2 Students identify the hypothesis and conclusion in logical deduction.

24.3 Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

25.0 Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements:

25.1 Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.

25.2 Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step.

25.3 Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.

Geometry

The geometry skills and concepts developed in this discipline are useful to all students. Aside from learning these skills and concepts, students will develop their ability to construct formal, logical arguments and proofs in geometric settings and problems.

The geometry student requires instruction from the teacher of the visually impaired to use the modified tools necessary to complete tactile constructions.These tools include modified compass, protractor, tracing wheel, rubber matting,and raised line drawings as appropriate. An additional 30 braille symbols are introduced.

(See the Grades Eight Through Twelve Introduction for information about braille symbols and methods of preparing students' braille work so that a print-reading mathematics teacher can read what the student has written.)

1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.

  1.1 Students use a braille writer or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) modifiers and their terminators; (b) directly over; (c) bar over; (d) bar under; (e) shape; (f) left arrow; (g) line between arrow; (h) right arrow; (i) ray; and (j) other applicable symbols as needed.

2.0 Students braille geometric proofs, including proofs by contradiction.

  2.1 Students use a braille writer or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) congruence; (b) approximately; and (c) other applicable symbols as needed.

3.0 Students construct and judge the validity of a logical argument and give counter examples to disprove a statement.

  3.1 Students use a braille writer or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) negation; (b) not equal to; and (c) other applicable symbols as needed.

4.0 Students prove basic theorems involving congruence and similarity.

  4.1 Students use a braille writer or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) congruence; (b) angle indicator; (c) is similar to; and (d) other applicable symbols as needed.

5.0 Students prove that triangles are congruent or similar, and they are able to use the concept of corresponding parts of congruent triangles.

 5.1 Students use a braille writer or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the triangle symbol.

6.0 Students know and are able to use the triangle inequality theorem.

7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles.

 7.1 Students use a braille writer or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) is parallel to; (b) is perpendicular to; (c) parallelogram; (d) circle; and (e) other applicable symbols as needed.

8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures.

 8.1 Students use a braille writer or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) arc; (b) *pi* (π); and (c) other applicable symbols as needed.

9.0 Students compute the volumes and surface areas of prisms, pyramids, cylinders, cones, and spheres; and students commit to memory the formulas for prisms, pyramids, and cylinders.

10.0 Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids.

 10.1 Students use a braille writer or slate and stylus, followingacceptedbraille mathematics code and formatting rules, to read and braille the following symbols: (a) degrees; (b) triangle; and (c) other applicable symbols as needed.

11.0 Students determine how changes in dimensions affect the perimeter, area, and volume of common geometric figures and solids.

12.0 Students find and use measures of sides and of interior and exterior angles of triangles and polygons to classify figures and solve problems.

13.0 Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles.

14.0 Students prove the Pythagorean theorem.

15.0 Students use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles.

16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line, by using such tactile drawing devices as a raised line drawing kit, tracing wheel, or crayon compass.

16.1 Students use tactile drawing devices to produce a raised line.

16.2 Students use a modified compass to draw circles and bisect lines and angles.

17.0 Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.

18.0 Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them. For example, $\tan(x) = \sin(x)/\cos(x)$, $(\sin(x))^2 + (\cos(x))^2 = 1$.

19.0 Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side:

19.1 Students use a braille writer or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) colon used in a ratio; and (b) other applicable symbols as needed.

20.0 Students know and are able to use angle and side relationships in problems with special right triangles, such as 30°, 60°, and 90° triangles and 45°, 45°, and 90° triangles.

21.0 Students prove and solve problems regarding relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons of circles.

22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, including rotations, translations, and reflections, by using appropriate tactile materials:

22.1 Students use a braille writer or slate and stylus, following accepted braille mathematics code and formatting rules,to read and braille the following: (a) vector; (b) left and right angle brackets; (c) prime indicator; (d) similar to; and (e) other applicable symbols as needed.

Algebra II

This discipline complements and expands the mathematical content and concepts of algebra I and geometry. Students who master algebra II will gain experience with algebraic solutions of problems in various content areas, including the solution of systems of quadratic equations, logarithmic and exponential functions, the binomial theorem, and the complex number system.

One-third of the algebra II standards involve graphing or reading graphs.The algebraI I student requires instruction from the teacher of the visually impaired to read complex graphs.The student will need instruction about preparation of a modified graph to show mastery of the essential concepts of the complex graph. Additional braille symbolsare introduced.

(See the Grades Eight ThroughTwelve Introduction for information about braille symbols and methods of preparing students' braille work so that a print-reading mathematics teacher can read what the student has written.)

1.0 Students solve equations and inequalities involving absolute value.

2.0 Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs (moldedrubber graph board), or with matrices.

  2.1 Students graph three points on each line and use tactile indicators for each region that shows the solution sets.

3.0 Students are adept at operations on polynomials, including long division.

4.0 Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes.

5.0 Students demonstrate knowledge of how real and complex numbers are related both arithmetically and graphically. In particular, they can plot complex numbers as points in the plane.

  5.1 Students plot three complex numbers by using a tactile graphing device.

6.0 Students add, subtract, multiply, and divide complex numbers.

7.0 Students add, subtract, multiply, divide, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator.

8.0 Students solve and graph quadratic equations by factoring, completing the square, or using the quadratic formula. Students apply these techniques in solving word problems. They also solve quadratic equations in the complex number system.

9.0 Students demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as *a, b,* and *c* vary in the equation $y = a(x-b)^2 + c$.

10.0 Students tactilely graph quadratic functions and determine the maxima, minima, and zeros of the function.

  10.1 Students graph by using a tactile graphing device the maxima, minima, and zeros of the quadratic function.

11.0 Students prove simple laws of logarithms:

  11.1 Students understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

  11.2 Students judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step.

12.0 Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.

13.0 Students use the definition of logarithms to translate between logarithms in any base.

14.0 Students understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.

15.0 Students determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.

16.0 Students demonstrate and explain how the geometry of the graph of a conic section (e.g., asymptotes, foci, eccentricity) depends on the coefficients of the quadratic equation representing it.

17.0 Given a quadratic equation of the form $ax2 + by2 + cx + dy + e = 0$, students can use the method for completing the square to put the equation into standard form and can recognize whether the graph of the equation is a circle, ellipse, parabola, or hyperbola. Students can then graph the equation.

  17.1 Given a quadratic equation of the form $ax^2+by^2+cx+dy+e=0$, students graph three or more points by using a tactile graphing device.

18.0 Students use fundamental counting principles to compute combinations and permutations.

19.0 Students use combinations and permutations to compute probabilities.

20.0 Students know the binomial theorem and use it to expand binomial expressions that are raised to positive integer powers.

21.0 Students apply the method of mathematical induction to prove general statements about the positive integers.

22.0 Students find the general term and the sums of arithmetic series and of both finite and infinite geometric series.

22.1 Students use a braille writer or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) infinity; (b) Greek lower case *sigma;* (c) Greek capital letter *sigma;* (d) factorial; (e) Greek lower case *theta;* and (f) other applicable symbols as needed.

23.0 Students derive the summation formulas for arithmetic series and for both finite and infinite geometric series.

24.0 Students solve problems involving functional concepts, such as composition, defining the inverse function and performing arithmetic operations on functions.

25.0 Students use properties from number systems to justify steps in combining and simplifying functions.

Trigonometry

Trigonometry uses the techniques that students have previously learned from the study of algebra and geometry. The trigonometric functions studied are defined geometrically rather than in terms of algebraic equations. Facility with these functions as well as the ability to prove basic identities regarding them is especially important for students intending to study calculus, more advanced mathematics, physics and other sciences, and engineering in college.

The trigonometry student requires instruction from the teacher of the visually impaired in the areas of tactile measurement, interpretation of trigonometric functions (drawn within circles as opposed to only the triangle in previous mathematics disciplines) and the raised line drawings representing the functions, and modifications for advanced tactile graphing (e.g., circles, arcs, sinewaves). Additional braille symbols are introduced, including various capital and lower case Greek letters.

(See the Grades Eight Through Twelve Introduction for information about braille symbols and methods of preparing students' braille work so that a print-reading mathematics teacher can read what the student has written.)

1.0 Students understand the notion of angle and how to measure it, in both degrees and radians. They can convert between degrees and radians.

2.0 Students know the definition of sine and cosine as *y*- and *x*-coordinates of points on the unit circle and are familiar with the graphs of the sine and cosine functions.

3.0 Students know the identity $\cos^2(x) + \sin^2(x) = 1$:

3.1 Students prove that this identity is equivalent to the Pythagorean theorem (i.e., students can prove this identity by using the Pythagorean theorem and, conversely, they can prove the Pythagorean theorem as a consequence of this identity).

3.2 Students prove other trigonometric identities and simplify others by using the identity $\cos^2(x) + \sin^2(x) = 1$. For example, students use this identity to prove that $\sec^2(x) = \tan^2(x) + 1$.

4.0 Students tactilely graph functions of the form $f(t) = A \sin(Bt + C)$ or $f(t) = A \cos(Bt + C)$ and interpret *A, B,* and *C* in terms of amplitude, frequency, period, and phase shift.

4.1 Students graph the maxima and minima (crest and trough) for three cycles on a tactile graphing device by using a tactile indicator for points (e.g.,tacks if the graphing device is a rubber graph board).

5.0 Students know the definitions of the tangent and cotangent functions and can tactilely graph them.

5.1 Using a braille writer or slate and stylus and following accepted braille mathematics code and formatting rules,read and braille the following symbols: (a) infinity; (b) factorial; (c) Greek lower case *alpha;* (d)

Greek

Lower case *beta;* (e) Greek lower case *lamda;* (f) Greek lower case *omega;* (g) Greek lower case *sigma;* (h) Greek lower case *theta;* (i) Greek capital letter *delta;* (j) Greek capital letter *phi;* (k) Greek capital letter *sigma;* and (l) other applicable symbols as needed.

6.0 Students know the definitions of the secant and cosecant functions and can tactilely graph them.

6.1 Students graph two or more points on a tactile graphing device by using a tactile indicator for points (e.g.,tacks if the graphing aid is a rubber graph board), indicating the points on the *x*-axis and *y*-axis. (See there sources section for information about American Printing House for the Blind [APH] graphing aids for mathematics.)

7.0 Students know that the tangent of the angle that a line makes with the *x*-axis is equal to the slope of the line.

8.0 Students know the definitions of the inverse trigonometric functions and can tactilely graph the functions.

8.1 Students graph three or more points on a tactile graphing aid by using a tactile indicator for points (e.g., tacks if the graphing aid is a rubber graph board) indicating the range and domain points.

9.0 Students compute, by hand, the values of the trigonometric functions and the inverse trigonometric functions at various standard points.

10.0 Students demonstrate an understanding of the addition formulas for sines and cosines and their proofs and can use those formulas to prove and/or simplify other trigonometric identities.

11.0 Students demonstrate an understanding of half-angle and double-angle formulas for sines and cosines and can use those formulas to prove and/or simplify other trigonometric identities.

12.0 Students use trigonometry to determine unknown sides or angles in right triangles.

13.0 Students know the law of sines and the law of cosines and apply those laws to solve problems.

14.0 Students determine the area of a triangle, given one angle and the two adjacent sides.

15.0 Students are familiar with polar coordinates. In particular, they can determine polar coordinates of a point given in rectangular coordinates and vice versa.

16.0 Students represent equations given in rectangular coordinates in terms of polar coordinates.

17.0 Students are familiar with complex numbers. They can represent a complex number in polar form and know how to multiply complex numbers in their polar form.

18.0 Students know DeMoivre’s theorem and can give n^{th} roots of a complex number given in polar form.

19.0 Students are adept at using trigonometry in a variety of applications and word problems.

Mathematical Analysis

This discipline combines many of the trigonometric, geometric, and algebraic techniques needed to prepare students for the study of calculus and strengthens their conceptual understanding of problems and mathematical reasoning in solving problems. These standards take a functional point of view toward those topics. The most significant new concept is that of limits. Mathematical analysis is often combined with a course in trigonometry or perhaps with one in linear algebra to make a year-long precalculus course.

The mathematical analysis student requires instruction from the teacher of the visually impaired in the are a of tactile graphing for the polar coordinate system, a symptotes, and the critical points needed to indicate the curve of equations.

(See the Grades Eight Through Twelve Introduction for information about braille symbols and methods of preparing students' braille work so that a print-reading mathematics teacher can read what the student has written.)

1.0 Students are familiar with, and can apply, polar coordinates and vectors in the plane. In particular, they can translate between polar and rectangular coordinates and can interpret polar coordinates and vectors graphically.

    1.1 Students plot three or more points in the polar coordinate system by using a tactile graphing device.

2.0 Students are adept at the arithmetic of complex numbers. They can use the trigonometric form of complex numbers and understand that a function of a complex variable can be viewed as a function of two real variables. They know the proof of DeMoivre's theorem.

3.0 Students can give proofs of various formulas by using the technique of mathematical induction.

4.0 Students know the statement of, and can apply, the fundamental theorem of algebra.

5.0 Students are familiar with conic sections, both analytically and geometrically:

    5.1 Students can take a quadratic equation in two variables; put it in standard form by completing the square and using rotations and translations, if necessary; determine what type of conic section the equation represents; and determine its geometric components (foci, asymptotes, and so forth).

    5.2 Students can take a geometric description of a conic section—for example, the locus of points whose sum of its distances from (1, 0) and (-1, 0) is 6—and derive a quadratic equation representing it.

6.0 Students find the roots and poles of a rational function and can tactilely graph the function and locate its asymptotes.

6.1 Students graph three or more points on a tactile graphing device by using a tactile indicator for points (e.g.,tacks if the graphing aid is a rubber graph board) to indicate the a symptotes. (See their sources section for information on contacting the American Printing House for the Blind [APH]about graphing aids for mathematics.)

7.0 Students demonstrate an understanding of functions and equations defined parametrically and can tactilely graph them.

7.1 Students graph three or more points on a tactile graphing device by using a tactile indicator for points.

8.0 Students are familiar with the notion of the limit of a sequence and the limit of a function as the independent variable approaches a number or infinity. They determine whether certain sequences converge or diverge.

8.1 Students use a braille writer or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the Greek capital letter *sigma* and the infinity symbol.

Linear Algebra

Separate braille standards for linear algebra are not necessary because students using braille will already know the required braille notation for this course from the basic mathematics courses. Refer to the print standards for linear algebra standards.

Probability and Statistics

This discipline is an introduction to the study of probability, interpretation of data, and fundamental statistical problem solving. Mastery of this academic content will provide students with a solid foundation in probability and facility in processing statistical information.

1.0 Students know the definition of the notion of *independent events* and can use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.

2.0 Students know the definition of *conditional probability* and use it to solve for probabilities in finite sample spaces.

3.0 Students demonstrate an understanding of the notion of *discrete random variables* by using them to solve for the probabilities of outcomes, such as the probability of the occurrence of five heads in 14 coin tosses.

4.0 Students are familiar with the standard distributions (normal, binomial, and exponential) and can use them to solve for events in problems in which the distribution belongs to those families.

4.1 Students use a braille writer or slate and stylus and a note taker to produce print copies, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) Greek lower case *beta;* (b) Greek capital letter *sigma;* (c) Greek lower case *mu;* (d) Greek lower case *omega;* (e) logarithmic functions; (f) integral; and (g) factorial.

5.0 Students determine the mean and the standard deviation of a normally distributed random variable.

6.0 Students know the definitions of the *mean, median,* and *mode* of a distribution of data and can compute each in particular situations.

7.0 Students compute the variance and the standard deviation of a distribution of data.

8.0 Students organize and describe distributions of data by using a number of different methods, including frequency tables, histograms, standard line and bar graphs, stem-and-leaf displays, scatter plots, and box-and-whisker plots.

8.1 Studentsorganizeanddescribedistributionsofdatabyusinganumberof different methods, including the use of a tactile graphing aid, braille writer, slate and stylus, and braille note taker, to prepare graphs and tables.

Advanced Placement Probability and Statistics

Separate braille standards for advanced placement probability and statistics are not necessary because students using braille will already know the required braille notations for this course from the basic mathematics courses. Refer to the print standards for this advanced placement course.

Calculus

When taught in high school, calculus should be presented with the same level of depth and rigor as are entry-level college and university calculus courses. These standards outline a complete college curriculum in one variable calculus. Many high school programs may have insufficient time to cover all of the following content in a typical academic year. For example, some districts may treat differential equations lightly and spend substantial time on infinite sequences and series. Others may do the opposite. Consideration of the College Board syllabi for the Calculus AB and Calculus BC sections of the Advanced Placement Examination in Mathematics may be helpful in making curricular decisions. Calculus is a widely applied area of mathematics and involves a beautiful intrinsic theory. Students mastering this content will be exposed to both aspects of the subject.

The calculus student requires instruction from the teacher of the visually impaired to learn how to access graphing calculators visually or auditorily to meet the requirements of the instructor and the demands of the class. Additional braille symbols are introduced.

(See the Grades Eight Through Twelve Introduction for information about braille symbols and methods of preparing students' braille work so that a print-reading mathematics teacher can read what the student has written.)

1.0 Students demonstrate knowledge of both the formal definition and the graphical interpretation of limit of values of functions. This knowledge includes one-sided limits, infinite limits, and limits at infinity. Students know the definition of convergence and divergence of a function as the domain variable approaches either a number or infinity:

 1.1 Students prove and use theorems evaluating the limits of sums, products, quotients, and composition of functions.

 1.2 Students use accessible graphical calculators to verify and estimate limits. (Refer to the resources section.)

 1.3 Students prove and use special limits, such as the limits of $(\sin(x))/x$ and $(1-\cos(x))/x$ as x tends to 0.

  1.3.1 Students use a braille writeror slate and stylus, following accepted braille mathematics code and formatting rules,to read and braille the following symbols: (a) the Greek lower case *delta;* (b) the Greek lower case *theta;* (c) union; (d) infinity; (e) limits of functions; (f) left and right braces; and (g) other applicable symbols as needed.

2.0 Students demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function.

3.0 Students demonstrate an understanding and the application of the intermediate value theorem and the extreme value theorem.

4.0 Students demonstrate an understanding of the formal definition of the derivative of a function at a point and the notion of differentiability:

   4.1 Students demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function.

   4.2 Students demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change. Students can use derivatives to solve a variety of problems from physics, chemistry, economics, and so forth that involve the rate of change of a function.

   4.3 Students understand the relation between differentiability and continuity.

   4.4 Students derive derivative formulas and use them to find the derivatives of algebraic, trigonometric, inverse trigonometric, exponential, and logarithmic functions.

5.0 Students know the chain rule and its proof and applications to the calculation of the derivative of a variety of composite functions.

6.0 Students find the derivatives of parametrically defined functions and use implicit differentiation in a wide variety of problems in physics, chemistry, economics, and so forth.

7.0 Students compute derivatives of higher orders.

8.0 Students know and can apply Rolle's theorem, the mean value theorem, and L'Hôpital's rule.

9.0 Students use differentiation to sketch, by hand, graphs of functions. They can identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing.

   9.1 Using a tactile graphing device, students read and plot the maxima and minima of a function, identify intervals,and calculate the inflection point.

10.0 Students know Newton's method for approximating the zeros of a function.

11.0 Students use differentiation to solve optimization (maximum-minimum problems) in a variety of pure and applied contexts.

12.0 Students use differentiation to solve related rate problems in a variety of pure and applied contexts.

13.0 Students know the definition of the definite integral by using Riemann sums. They use this definition to approximate integrals.

   13.1 Students use a braille writer or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) the Greek capital letter *sigma;* (b) the *sigma* notation, including braille modifying symbols of directly over and directly under; and (c) other applicable symbols as needed.

14.0 Students apply the definition of the integral to model problems in physics, economics, and so forth, obtaining results in terms of integrals.

   14.1 Students use a braille writer or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the integral symbol and the upper and lower limits of integration and other applicable symbols as needed.

15.0 Students demonstrate knowledge and proof of the fundamental theorem of calculus and use it to interpret integrals as anti-derivatives.

16.0 Students use definite integrals in problems involving area, velocity, acceleration, volume of a solid, area of a surface of revolution, length of a curve, and work.

17.0 Students compute, by hand, the integrals of a wide variety of functions by using techniques of integration, such as substitution, integration by parts, and trigonometric substitution. They can also combine these techniques when appropriate.

18.0 Students know the definitions and properties of inverse trigonometric functions and the expression of these functions as indefinite integrals.

19.0 Students compute, by hand, the integrals of rational functions by combining the techniques in standard 17.0 with the algebraic techniques of partial fractions and completing the square.

20.0 Students compute the integrals of trigonometric functions by using the techniques noted above.

21.0 Students understand the algorithms involved in Simpson's rule and Newton's method. They use calculators or computers or both to approximate integrals numerically.

22.0 Students understand improper integrals as limits of definite integrals.

23.0 Students demonstrate an understanding of the definitions of convergence and divergence of sequences and series of real numbers. By using such tests as the comparison test, ratio test, and alternate series test, they can determine whether a series converges.

24.0 Students understand and can compute the radius (interval) of the convergence of power series.

25.0 Students differentiate and integrate the terms of a power series in order to form new series from known ones.

Appendix A

ABACUS POSITION PAPER

Recording the process of calculation and “showing one’s work” can be particularly challenging for braille readers who cannot easily produce pencil work on paper. Various techniques have been used over the years to assist students who are blind in performing these tasks, including the recording of the calculation process by means of a braillewriter, slate and stylus, cubarithm, tape recorder, Taylor slate, and adaptations to the various forms of the abacus. All of these methods attempt to replicate the function, of pencil and paper used by sighted students. None of these methods are analogous to calculators.

Presented below is a position paper from the American Printing House for the Blind (APH) Accessible Tests Department regarding the use of one such adapted device, the Cranmer abacus, as a reasonable accommodation in assessment. This position paper can also be found online at http://www.aph.org/accessible-tests/position-papers/abacus-in-test-taking/

Abacus: Position Paper

By Terrie Terlau and Fred Gissoni

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Definition and Description

The mathematical abacus is a frame with beads or balls that can be slid on wires or in slots for calculating or teaching arithmetic (*The American Heritage Dictionary of the English Language,* 1996). The abacus has been used as a calculation device in Europe, Japan, China, and the Middle East since the third century A.D.

The Cranmer abacus was developed as a calculation device for persons who are blind or visually impaired and is currently produced by the American Printing House for the Blind (APH: *Abacuses,* 2001). The Cranmer abacus frame is made of high impact plastic, measures 6-1/8 x 3-1/4 x 7/16 inches, and contains thirteen vertical rods and one horizontal cross bar. Four beads can be moved vertically on each of the thirteen rods below the cross bar and one bead can be moved vertically along the rods above the cross bar.

Abacus Functionality

When calculating with the Cranmer abacus, vertical rods represent units, tens, hundreds, etc. Numbers are recorded and manipulated by moving beads toward the cross bar on their respective rods.

The abacus is a passive device. It is not a calculator or a slide rule. The abacus does not perform mathematical operations. It does not contain information that would enable an abacus user to achieve calculation results without a solid knowledge of mathematical concepts and relationships. Abacus users produce calculations as a result of their understanding of the behavior of numbers, not because of any inherent property of the abacus.

Both abacus and pencil-and-paper users must learn strategies for performing mathematical operations. The primary difference in the activity of abacus and pencil-and-paper users is that pencil-and-paper users apply and record steps in these operations by writing while abacus users apply and record these processes by moving abacus beads.

Persons who are blind or visually impaired and who have had appropriate abacus instruction can use the abacus to perform addition, subtraction, multiplication, division, and square and cube roots. The abacus does not permit permanent storage of problem solutions because beads must be rearranged to perform subsequent problems. After each calculation using an abacus, answers can be recorded in a variety of formats, including Braille, large print, voice recording, word processing, or dictation into an electronic device.

Position Statement

Whenever a test-taker is allowed to use a pencil and paper for working calculations, an abacus should be considered an equivalent substitution.

References

APH: *Abacuses*. (2001). Retrieved January 15, 2002

The American Heritage Dictionary of the English Language (3rd edition).

Appendix B

TACTILE GRAPHICS

The mathematics content standards adopted by the California State Board of Education are based on what students actually need to know and be able to do. To help students who are blind meet California's high expectations and attain mastery of the mathematics content standards, mathematics teachers, teachers of the visually impaired, and braille transcribers must work together to provide appropriate instruction and learning resources, including readily usable and understandable tactile representations of mathematical diagrams

Mathematical diagrams that include grids, graphs, pictographs, number lines, three-dimensional graphics, measurements, line graphs, dot and line plots, thermometers, angles, clocks, money, and counting symbols must be presented in ways that will best serve the visually impaired reader's needs. If the information is not repetitive, not more meaningful as text, or does not require visual discrimination, then a tactile graphic must be produced. A variety of strategies are available to help transcribers design and produce tactile graphics, including collage, swell paper, slate and stylus, craft ink, sculpture, and embossed braille images from computer files. When feasible, the exploration of three-dimensional objects should be in conjunction with the use of tactile graphics.

Unfortunately, some transcribers and braille organizations do not have the resources or skills required to make tactile graphics usable and understandable to braille readers. Transcribers of braille textbooks who do have the skills and resources to interpret and represent print graphics into braille graphic format are often not consistent in their presentation.

Students who read braille must have access to books that include graphical information in a format that is consistent with those in the text print versions provided to their sighted classmates. With standards for format and layout, presentation style, and consistent symbols, braille readers will be able to enjoy both complete and consistent graphic representation. Recognizing the need to set standards for tactile graphic format and layout and ensure consistency of symbols and styles, the Braille Authority of North America (BANA) formed a Technical Committee on Tactile Graphics to develop standards for production. After the BANA committee researched tactile graphics and related topics, reviewed results, and consulted with other experts, the committee joined the Canadian Braille Authority in a combined effort to conduct additional research before developing the standards.

The resulting unique standards for the production of tactile graphics are embodied in *The BANA Guidelines for Production of Tactile Graphics,* which is expected to be available in early summer of 2006. The new publication will serve as a braille "code" for tactile graphics and will address the issue of presentation consistency.

Contact:

Lucia Hasty, Chair
BANA Technical Committee on Tactile Graphics
(719) 577-4710

The Clearinghouse for Specialized Media and Technology (CSMT), a unit of the California Department of Education, provides California public schools with state-adopted mathematics textbooks in braille format. In addition, CSMT maintains a large inventory of mathematics manipulatives and other learning tools, including abacuses, models, measuring tools, geometric forms, tactile graphics kits and tools, flash cards, games, computer software, tangible graphs, thermometer and money handling resources that are provided by the American Printing House for the Blind (APH) Federal Quota program.

Contact:

California Department of Education
Clearinghouse for Specialized Media and Technology
1430 N Street, Suite 3207
Sacramento, CA 95814
(916) 445-5103
(916) 323-2202 Fax

Appendix C

ASSISTIVE TECHNOLOGY

Assistive technology is not a luxury for people with disabilities, especially those with impaired vision; it is a necessity. Assistive technology enables persons with visual impairments to perform ordinary functions they could not otherwise do. That is why the Individuals with Disabilities Education Act (IDEA) requires that the individualized education program (IEP) team consider whether the child with a disability requires assistive technology and services (20 U.S.C. Section 1414[d][3][B][v]).

California *Education Code* Section 60061, created by Senate Bill 842 in 2004, requires every publisher or manufacturer of instructional materials offered for adoption or sale in California to comply with Section 508 of the Rehabilitation Act. Internet resources and digital multimedia programs intended for use by the general population of pupils, pupils in kindergarten and grades one to twelve inclusive, shall at least meet the standards for accessibility as set forth in Section 508 of the Rehabilitation Act of 1973, as amended (29 U.S.C. Sec. 794d), and regulations implementing that Act as set forth in Part 1194 of Title 36 of the *Code of Federal Regulations.*

As defined in the Individuals with Disabilities Education Improvement Act (IDEIA, PL 180-446, Section 602 (1)(A)), the term *assistive technology device* means any item, piece of equipment, or product system—whether acquired commercially off the shelf, modified from other technology, or custom built—that is used to increase, maintain, or improve functional capabilities of a child with a disability. The term *assistive technology service* means any service that directly assists a child with a disability to select, acquire, or use an assistive technology device and includes evaluation, purchasing, coordinating, and training.

The rapidly growing number of users and digital, electronic documents is testimony to the claim that the Internet and the technology used to access it are becoming an everyday part of life for all of us. Digital technology provides instant access for the blind and visually impaired to an enormous volume of information that was previously unavailable.

Though electronic digital files can produce both audible output and braille, it is especially essential for blind people to master the braille code in order to benefit fully from mathematics instruction. Auditory information is insufficient for acquiring a thorough understanding of mathematical concepts. Imagine trying to solve a complex algebra equation simply by listening and speaking, without the equation in front of you to read for yourself (in braille or in print).

For the first time ever, many blind and partially sighted students should have access to the same core curriculum instructional materials at the same time as their sighted peers because of the new federally mandated National Instructional Materials Accessibility Standards (NIMAS). This universal design that provides access is an integral part of assistive technology.

It must be noted that NIMAS files to be provided by publishers of instructional materials are presently not designed to be student-ready. Educators nationwide are expecting that third-party software, technology companies, and a cadre of skilled individuals will create the capacity to produce electronic resources from NIMAS files that will help students access the general curriculum in a timely manner. At this time, it is vastly more difficult to create math and science materials in accessible digital formats than to create literary materials.

IEP teams are required to consider assistive technology as part of a multifaceted approach to addressing the needs and strengths of students with disabilities. It is critical that the degree of technology assistance is determined to be appropriate for the student's learning potential, motivation, chronological age, developmental level, goals, and objectives.

Assistive technology is as much a process as a product. Assistive technology is a tool for access (for example, to the school environment and core curriculum) and for independence in communication and mobility. It will therefore need to change as the student's needs change and as technology continues to evolve. The need for assistive technology should be addressed as a part of every comprehensive assessment for students with visual impairments in all areas related to their disability.

When most people think about assistive technology for the blind and visually impaired for doing mathematics and other tasks, the very expensive “high-tech” variety usually first comes to mind. Complicated, specialized equipment and support services, such as portable electronic braille note-takers, Global Positioning Systems (GPS), computerized screen reading text recognition systems, electronic closed circuit television magnifiers, and talking calculators are a few examples. Although in some cases these high-tech devices may be the only appropriate choice that enables the blind or visually impaired student to accomplish desired tasks and permits them to compete with nondisabled peers, sometimes less-expensive “low-tech” alternatives may also provide effective solutions.

Low-tech equipment may include, but is not limited to, items such as a specially modified abacus; a standard tape recorder; raised line drawing kits; counting sticks; two- and three-dimensional geometric models; base-10 blocks; dark-lined paper; felt pens; Unifix cubes; geo-boards; and braille rulers, protractors, compasses, and charts.

Blind and visually impaired students, like students with other disabilities, do not constitute a homogeneous group. Varying degrees of central vision, peripheral vision, light perception, and color perception occur. Factors such as the age of onset and whether the vision loss is stable or unstable play a major role in the concept development of students with visual impairment.

How much functional vision a visually impaired student has determines the extent to which the student is educated using the senses of sight, hearing, and touch. Students with deteriorating vision may initially use residual sight, but as their vision decreases, they may rely more on auditory and tactile means of acquiring information.

Visual function is influenced by print size, font style, color-contrast ratios, illumination level, and the accuracy with which the brain interprets the visual image it receives. Therefore, a wide range of assistive technology aids must be available for the many different visual impairments of students. Access to and ease of use of a specific medium, for example, Web pages, are often determined by document length, design, and structure.

Although assistive technology helps level the playing field, it does not make it equal. As innovations improve the speed, power, and versatility of technology and as the costs continue to decrease, assistive devices will become even more useful and contribute even more significantly to the success of blind and visually impaired students in the sighted world.

Some California assistive technology resources that may be helpful are as follows:

California Department of Education

Clearinghouse for Specialized Media and Technology
http://www.cde.ca.gov/re/pn/sm/index.asp

Special Education Division Assistive Technology

State Special Schools and Services Division
California School for the Blind, Fremont Assistive Technology Program

Technology Services Division, Education Technology Office
Provides financial and technical support through state federal funds for educational technology for all students in California. http://www.cde.ca.gov/ls/et/index.asp

Other resources

California Assistive Technology System
Provides assistance, information, and referrals

Assistive Technology Network
http://www.atnet.org

Matrix Guide Assistive Technology

Accessing Assistive Technology

The High Tech Center Training Unit of the California Community Colleges
http://www.htctu.fhda.edu

Appendix D

BRAILLE MATHEMATICS MATERIALS AND RESOURCES

Name	Available at	Cost
Base ten blocks	Primary Concepts	$69.95 wood
Wooden geometric solids	Primary Concepts	$10.95 wood
Attribute blocks	Primary Concepts	$19.95 medium, $12.95 pocket
Unifix cubes	Primary Concepts	$11.95 set of 100
On the Way to Literacy: Giggly Wiggly Snickety Snick, Jellybean Jungle, The Gumdrop Tree	American Printing House for the Blind (APH)	On quota
Tactile Treasures	APH	On quota
Hands On, sorting trays	APH (find trays in catalog under (“replacement parts”)	On quota
Giant textured beads	APH	On quota
Textured pegs	APH	On quota
Sound matching board II	APH	On quota
Work-Play tray and dividers	APH	On quota
Various materials in the mathematics section, such as Focus on Mathematics, analog clock, geometric forms, number line device, flash cards, hundreds board, abacus, rulers,	APH	On quota
Tactile demonstration thermometer	APH	On quota
IntelliTactiles	APH	On quota
Calendar kits	APH	On quota
Tangible graphs	APH	On quota
Teaching touch	APH	On quota
Plexiglas blocks	APH	On quota
Let’s See inserts and formboards (replacement parts)	APH	On quota
Light Box Level II sticks, opaque set	APH	On quota
Card set, raised outline shapes	APH	On quota
Sensory Stimulation Kit Melody bells (two sets)	APH	On quota
SSK weight bags	APH	On quota
Score card sets, several	APH	On quota
Cubarithm slate	APH	On quota
Unifix cubes	Various vendors	
Thermo-pen and Flexi paper	LS & S, other vendors	About $100 for the pen

Wikki Stix	Educational Teaching Aids (formerly Exceptional Teaching Aids)	
Work-Play trays with sticky-back fuzzy Velcro attached	Trays: APH Velcro: craft shops	
Steel cookie sheets	Cookie sheets: housewares Magnet strips: craft shops	
Math Flash	APH	On quota
Tactile Talking Tablet	View Plus	

Resources

AFB Press. American Foundation for the Blind, 11 Penn Plaza, Suite 300, New York, NY 10001; (800) 232-3044; FAX (412) 741-0609; http://www.afb.org

APH. American Printing House for the Blind, P.O. Box 6084, 1839 Frankfort Ave., Louisville, KY 40206-0085; (800) 223-1839; http://www.aph.org

CCB. California Council of the Blind, 578 B St., Hayward, CA 94541; (510) 537-7877; FAX (510) 537-7830; http://www.ccbnet.org

Craig, R. (1987). *Learning the Nemeth Braille Code*. Louisville, KY: American Printing House for the Blind.

CSB. California School for the Blind, 500 Walnut Ave., Fremont, CA 94536; (510) 794-3800; http://www.csb-cde.ca.gov

CTEVH. California Transcribers and Educators of the Visually Handicapped, 714 N. Vermont Ave., Los Angeles, CA 90039; (323) 666-2211; http://www.ctebvi.org

ETA. Exceptional Teaching Aids, 5673 W. Las Pasitas Blvd, Pleasanton, CA 94588; (800) 549-6999; (925) 598-0092.

FS. Freedom Scientific, 11800 31st Court North, St. Petersburg, FL 33716-1805; (800) 444-4443; FAX (727) 803-8001; http://www.freedomscientific.com/

Livingston, R. (1997). *Use of the Cranmer Abacus* (Second edition). Austin, TX: Texas School for the Blind and Visually Impaired.

NBP. National Braille Press, 88 St. Stephen St., Boston, MA 02115; (800) 548-7323; http://www.nbp.org

NFBC. National Federation of the Blind of California, 175 E. Olive Ave., Burbank, CA 91502; (818) 558-6524; e-mail *nfbcal@sbcglobal.net*

Nemeth, A. (1972). *The Nemeth Braille Code for Mathematics and Scientific Notation*. Louisville, KY: American Printing House for the Blind.

Primary Concepts, P.O. Box 10043, Berkeley, CA 94709; (800) 660-8646

Roberts, H. (1978). *An Introduction to Braille Mathematics*. Washington, DC: National Library Service.

SCALARS Publishing, P.O. Box 382834, Germantown, TN 38183-2834; (901) 737-0001

TSBVI. Texas School for the Blind and Visually Impaired, 1100 West 45th St., Austin, TX 78756; http://www.tsbvi.edu

Glossary

abacus—There are several different abaci used by persons with visual impairments. The abacus is a calculation tool used by sighted and blind persons to solve various mathematical functions. The abacus consists of rows of beads that can be manipulated in an up and down manner to solve math problems. The abacus replaces the paper and pencil for a person who is blind.

accepted braille codes and formatting rules—As of 2006 the accepted code used for braille mathematics is the Nemeth Code.

APH—American Printing House for the Blind. APH is the major supplier of educational materials for visually impaired children.

assistive technology—Any item, piece of equipment, or system that is used to increase, maintain, or improve functional capabilities of individuals with disabilities.

blindness—The inability to see; absence or severe reduction of vision. See also *functionally blind* and *legally blind*.

braille—A tactile code system, consisting of raised dots organized in cells, used for reading and writing by persons who are blind. Each braille cell consists of up to six dots, which are arranged in different patterns to represent letters, numbers, symbols, and words.

braille notetakers—A small electronic talking device that is configured like a braillewriter but uses a series of commands to produce braille. This device can be used with a standard computer to print out assignments in print or in braille.

braillewriter—A machine used to produce embossed braille symbols.

Clearinghouse for Specialized Media and Technology (CSMT)—A unit of the California Department of Education. The CSMT administers the American Printing House federal quota program as well as reader services for blind teachers, and it provides instructional resources in special formats for students who are blind.

compensatory skills—Any technique, habit, or activity (such as daily living, social, and emotional skills) that must be developed to overcome a severe visual impairment.

contracted braille—Sometimes referred to as grade two braille. Contractions are signs that represent whole words, parts of words, or letter combinations. There are 189 contractions in the braille code.

federal quota program—A federal program administered by the American Printing House for the Blind (APH) and its ex officio trustee in each state that provides adapted educational materials and equipment to eligible students who meet the definition of blindness.

functionally blind—A student whose primary channels for learning are tactile and auditory.

individualized education program (IEP)—A written plan for a special education student that is developed and implemented in accordance with the IEP team and that is designed to meet the assessed needs of the student. Federal law includes specific requirements for the instruction of braille to blind students.

Individuals with Disabilities Education Improvement Act (IDEIA)—The IDEIA ensures a free, appropriate public education in the least restrictive environment for all students and youths with disabilities.

legally blind—Central visual acuity of 20/200 or less in the better eye after best correction with conventional spectacle lenses or visual acuity better than 20/200 if there is a visual field defect in which the widest diameter of the visual field is no greater than 20 degrees. In the United States this definition has been established primarily for economic and legal purposes.

Nemeth Code—A braille code for mathematics and scientific notation.

other braille codes—In addition to the Nemeth Code, there is the literary braille code made up of rules for the use of contracted and uncontracted braille, a computer braille code, a foreign language code, and a music code.

quota funds—Funds earmarked by federal legislation for students who are registered by the American Printing House for the Blind. Each state receives specialized funds for books and materials for the blind. In California quota funds are managed by CSMT.

SEACOE—Special Education Administrators of County Offices of Education

SELPA—Special education local plan area. A consortium of school districts or county offices of education that provides a full continuum of services for students with disabilities.

slate and stylus—A note-taking device. The slate is a flat implement made out of metal or plastic that has rows of braille cells on it; braille is produced by pushing the stylus through the holes in the slate to make braille dots.

tactile graphics—Representations of drawings, charts, and graphs in a tactile format that are created by specialized tools and computer software designed for individuals who are blind or visually impaired.

talking calculator—An electronic device used for mathematical and scientific calculation that provides auditory output for the user.

textbook format—Specialized braille rules that specify how braille pages, tables, graphs, and pictures in texts will be organized.

uncontracted braille-The braille symbols that represent the alphabet; sometimes referred to as grade one braille.

UEBC-Unified English braille code.